Connections in High School Mathematics Textbooks that Support Understanding of Key Features of Graphs of Functions

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Abstract

This study aims to evaluate a particular high school mathematics series of textbooks for its potential to adequately support teachers in emphasizing function-related connections. The investigation was limited to topics surrounding analysis of graphs of functions, specifically, domain and range; minima, maxima, and vertex; slope and increasing or decreasing intervals; and, intercepts and zeros. The chosen textbooks are McGraw-Hill's Algebra 1, Algebra 2, and Precalculus textbooks with a copyright date of 2016. Each textbook was reviewed in sequential order. Items relating to the topics of interest were placed in a connection category and evaluated for nine different qualities.

Data collected about the types of connections and the quality of those items reveals that the chosen textbooks have numerous connections among representations, but fewer connections to the real-world and across disciplines. Furthermore, a greater number of contextual items are included in Algebra 1 and Algebra 2 than in Precalculus.

Future investigations may aim to compare the series evaluated in this study to another series of mathematics textbooks. Additionally, teacher interactions with the textbook might provide more useful insights.

Dedication

I dedicate this work to my three children, Cain, Clover, and Barrett. In the future, I hope you each find something you want to work toward and have the confidence and drive to do it.

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I. Introduction

Student ability to make connections is a central theme in standards and mathematics reform. The National Council of Teachers of Mathematics (2000) asserts that students' understanding of mathematics is improved when the same concepts can be viewed from a variety of mathematical perspectives (p.288). The Common Core State Standards Initiative (2016) recognizes that mathematics is a coherent body of knowledge made up of interconnected concepts and has designed the standards around logical progressions from grade to grade. The Texas Essential Knowledge and Skills (TEKS) call for connections among various mathematical representations, from course to course, and to real-world applications (SBOE Rules - TAC, 2012). The Texas P-16 Initiative (2008) reiterates the notion that mathematics cannot be viewed solely as a series of stand-alone courses or a set of separate skills. Mathematics knowledge is essential for success in numerous areas of study. Ultimately, the ability to make connections affects college readiness and student attraction to science, technology, engineering and mathematics (STEM) fields. Because of the importance, the Texas College and Career Readiness standards dedicate an entire section to connections, emphasizing connections among the strands of mathematics and connections of mathematics to nature, real-world situations, and everyday life (Texas P-16 Initiative, 2008). Although standards call for emphasis on connections, there is still a question about whether curriculum materials adequately address this aspect of the standards.

Making Connections and Knowledge Transfer

Mathematics is a nexus of topics and concepts. When students fail to establish meaningful connections within mathematics, incomplete concept images result. Without complete concept understanding, students may struggle with knowledge transfer (Boaler, 1993b). To clarify, *connections* is a term for linkages relating mathematical concepts from level to level, from topic to topic, among representations, across content areas or disciplines, and to the real-world. *Knowledge transfer* refers to the ability of a learner to make connections between learning experiences and new situations, and it is necessary for real-world and interdisciplinary mathematics applications. Ability to transfer knowledge means that a learner can identify conceptual similarities among various problems that may have surface-level differences and correctly apply the appropriate mathematics to novel situations.

Many students struggle with knowledge transfer. The deficiency with mathematical knowledge transfer across disciplines is a well-documented problem. In one study, Planinic, Milin-Sipus, Katic, Susac, and Ivanjek (2012) found that student knowledge is very compartmentalized and stronger links between subjects are needed even if mathematics skills are not the only cause of student difficulties in physics. Another similar study by McDermott, Rosenquist, and Van Zee (1987) found that student facility with graphing plays a role in helping students transfer mathematics knowledge to kinematics, and that student difficulty in transferring knowledge arises from their inability to make connections between graphical representations and the subject they represent. Similarly, Hoban, Finlayson, and Nolan (2013) noted students' struggles in transferring mathematics knowledge to chemistry and asserted that students who have more ability to explain a mathematics concept are more favorably disposed toward transfer. Similarly, Childers and Vidakovic (2014) found that students who are better able to communicate a personal definition of function are more likely to do well on both explicit questions and contextual word-problems about vertex of quadratic functions. Viirman (2008) found that engineering students demonstrated a higher level of conceptual understanding of functions than mathematics education students. Because engineering work demands the ability to transfer knowledge, Viirman's findings further support the idea that ability to transfer knowledge is tied to conceptual understanding.

Additional background research on the concept of functions suggests that although it is considered a unifying topic in mathematics curriculum, the concept of function is difficult for many students (Jones, 2006; Childers & Vidakovic, 2014; Akkoc & Tall, 2005; Viirman, 2008). Most students focus on individual properties of functions and the various representations without connecting the ideas together (Akkoc & Tall, 2005) which results in an underdeveloped concept image. Weak understanding can be a reason for lack of transfer.

STEM and College Readiness

In addition to difficulty with the function concept and mathematical knowledge transfer, studies also indicate that in the United States, there is concern about student attraction to and/or retention in STEM programs and students' general lack of college readiness. STEM jobs are increasing faster than a capable workforce is being produced. Carnevale, Smith, and Melton (2011) projected that in Texas alone, the 584,120 STEM jobs of 2008 will increase to 715,380 STEM jobs by 2018. They also asserted that the national need for STEM employees is growing similarly. If there is a shortage of STEM workers, Americans will fail to compete successfully in the global economy (Carnevale, Smith, & Melton, 2011).

Furthermore, by 2020, estimates suggest that 65% of all jobs will require some form of education or training beyond high school. (Carnevale, Smith, & Strohl, 2013). The trouble is that students are struggling with post-secondary education because they are underprepared. Analysis by Attewell, Lavin, Domina, and Levey (2006) indicates that 40% of college undergraduates enroll in at least one remedial course. Additionally, the National Center for Education Statistics (Malkus & Sparks, 2013) reports that over a quarter of college freshmen take at least one developmental mathematics course.

The problems with knowledge transfer, underprepared college students, and low participation in STEM programs are not mutually exclusive. While inability to transfer knowledge is not the sole culprit, it is one of the main reasons students opt out of STEM programs in college (Chen, 2013). As research suggests, a more developed student capability to make connections can result in better understanding of mathematical concepts, which can lead to improved knowledge transfer (Boaler, 1993b; McDermott et. al, 1987; Planinic et. al, 2012) and in turn increased college and career readiness; but how can the capability to make connections be fostered?

Factors Influencing Teaching and Learning

Studies show that teaching methods have the largest impact on student improvement (Boaler, 1993a). While particular teaching methods can have a significant influence on students' understandings of mathematical concepts, they are not foolproof or singular. Not all teachers have been trained or had sufficient time and support in developing better teaching methods (Remillard & Bryans, 2004). Research indicates that because improving teaching methods requires a large time investment and additional commitment, altering teaching methods on a large scale is difficult (Ball & Cohen, 1996).

Additionally, no matter what teaching methods are used, there is another major influential factor to consider -- the textbook. Although the individual teacher controls how material is taught, multiple studies show that the textbook often has a significant role in determining what material is covered in mathematics classrooms (Weiss, Pasley, Smith, Banilower, & Heck, 2003); this is especially true in classrooms led by newer teachers (Taylor, 2013). According to TEA demographics of the past five years, 55% or more of Texas mathematics teachers have less than ten years of teaching experience, with over half of those teachers having less than four years of teaching experience. If it is assumed that patterns from research persist, then a significant number of Texas teachers is likely to rely heavily on the content of a mathematics textbook. If teachers are the key factors influencing student understanding, and textbooks are the key factors influencing teachers, then it logically follows that the contents of mathematics textbooks can have influence over learning that occurs in mathematics classrooms.

Within the past year in Texas high schools, new mathematics standards emphasizing connections have been implemented and new textbooks have been adopted. Because of these recent changes, it is worth investigating both the content of the textbooks and their potential for supporting teachers in their duty to better prepare students.

Purpose of The Study

The purpose of this study is to evaluate the potential for a chosen mathematics textbook series to support teaching and learning of function related concepts associated with STEM fields. The goal is not only to uncover deficiencies in the series, but also to recognize valuable content. With those two main objectives in mind, this curriculum study was driven by two guiding questions.

1. What number and types of connections related to understanding key features of graphs of functions can be found in a specific traditional Texas high school textbook series of Algebra 1, Algebra 2, and Precalculus?
2. What potential does this specific textbook series have for supporting teachers and helping students develop their ability to make connections related to understanding functions?

Significance of Study

Although similar curriculum studies have been done on other mathematics textbooks, this study of the McGraw-Hill textbook series is unique; not only is it the first for the selected series but also the evaluation instrument is original. This investigation into a new textbook curriculum provides information about types of connections present in the textbooks and their potential for supporting teacher and student learning. The results can inform teachers and school leaders about deficiencies and offer insight about the gaps that need to be filled when using the chosen series. As Fan and Zhu (2007) noted in their research, textbooks have been found to have specific deficiencies that cause considerable gaps between textbook content and curriculum standards. Recognizing textbook deficiencies is an important step in overcoming their negative impact on the teaching and learning of mathematics in the classroom (Fan & Zhu, 2007).

This study also lays the foundation for future curriculum investigations. Future studies may seek comparison with other traditional textbooks from different publishers and/or standards-based curriculums. It is also likely that, with the function-related content of the textbook already summarized, future studies will investigate teacher interactions with the content (Dole & Shield, 2013).

Summary

The importance of connections in mathematics is evident in every set of standards previously mentioned. Student ability to make connections plays a significant role in transferring mathematics content knowledge to novel situations, to the real-world, and across disciplines. Studies have documented student difficulty transferring mathematics knowledge. This unresolved knowledge-transfer problem is a cause for concern. It has affected student readiness for college and participation in STEM related programs. The number of STEM workers needed in the United States is on the rise while the number of capable workers is not increasing at the same rate. Improving teaching methods has been shown to be most influential on student performance; however, according to many researchers, the textbook also has a significant impact. Further details about related research are included in the literature review section.

II. Literature Review

Student difficulties with making connections and knowledge transfer of mathematics concepts have been well documented in research literature. A review of several studies reveals issues in students' ability to solve contextual mathematics problems. In general, results of other studies show that students generally struggle with the concept of functions. Some of the reviewed literature about functions uncovers student conceptions of the function definition (Akkoc & Tall, 2005; Jones, 2006; Viirman, 2008) while some literature relates more specifically to key features of functions (Childers & Vidakovic, 2014). Additional reviewed research details findings about factors affecting students' understanding and ability to transfer mathematics knowledge. While learning environment and teaching methods (Boaler, 1993a) have been identified as two major influencing factors, information from the reviewed research literature also indicates that textbooks are significant (Ball & Feiman-Nemser, 1988; Budiansky, 2001; NCES, 2003; Pehkonen, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003). Frameworks for evaluating the quality potential of mathematics textbooks (AAAS, 2000; Dole & Shield, 2013; EQuIP, 2013; Schmidt, 2012) and textbooks in general (Price, 2012; Williams, 2007) are also included in this review.

Research Literature About Knowledge Transfer

Britton (2002) developed an instrument to study the transfer of mathematics skills to disciplines like physics, microbiology, and computer science. Britton piloted her instrument on 47 paid volunteers who were in their first year of college. Thirty of the students were science majors, 16 were engineering majors, and one was an arts student. Students were given 40 minutes to answer the questions. Mathematics questions were scored together and the other subjects or “transfer” subjects were scored together. Results of the trials indicated that students were more successful on the mathematics questions than on the compilation of transfer subjects. These results implied that mathematics transfer to other contexts is difficult and requires a solid understanding of mathematics and an ability to interpret the language used in other contexts.

Planinic et al. (2012) conducted a study to compare students’ understandings of the slope concept in two different contexts: mathematics and physics. They developed two pairs of questions that were administered as a part of a longer test to 114 Croatian students aged 15 to 16 years. Each pair consisted of one mathematics question and one physics question which required students to use the slope concept. The results of this study indicated that students were more frequently able to correctly answer the mathematics questions. Fewer students were able to correctly answer the related physics questions. Additionally, most students who did correctly answer the physics questions were also able to correctly answer the mathematics questions but the converse was not necessarily true.

Hoban et. al (2013) performed a very similar small-scale study using pairs of mathematics and chemistry questions. However, their investigation involved fewer than thirty college students majoring in a science degree. The researchers’ main goal was to investigate students' abilities in transferring mathematical knowledge to chemistry. Their study consisted of parallel mathematics and chemistry questions. Several of the items used in their study required students to have a fully developed understanding of the slope concept, from a basic algebra-level understanding up to a calculus-level understanding. The results of their study were not consistent with other studies. The researchers found that students who performed well on the mathematics questions were also the ones who were able to explain their understanding of slope and successfully transfer their knowledge to the chemistry problems. This study might seem to contradict other studies (Britton, 2002; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012) that found fewer students were capable of answering the transfer questions, but its significance is minor because it was performed on a very small group of older college students who were in a science field. These students have more experiences than some of the younger, more inexperienced students who participated in other studies. Thus, the results may not be reproducible with a larger number of participants or with a different group of students.

Research Literature About Students' Understanding of Functions

Jones (2006) stated that the topic of functions is not only the most central concept in math, but also among the most difficult for students to completely comprehend. Her study examined several aspects of the concept of function including historical roots, various types of representations, textbook portrayal, and student understandings. Jones' report explores action, process, and object levels of understanding functions and notes several student misconceptions about functions.

For the textbook analysis, Jones examined eight different textbooks of various levels. Four of the analyzed textbooks were Pre-Algebra or Algebra, three were Calculus, and one was Discrete Math. Jones observed that each of the algebra textbooks presented material on functions in different ways and emphasized different perceptions on the subjects. She also noted that some textbook authors attempted to help students make connections, and other authors made no effort to explicitly make connections. While the algebra textbooks seemed to define function from the perspective of a relation, the calculus textbooks seemed to focus more on the idea of functions as rules or formulas. Jones suggested that the discrepancy in definitions from level to level may result in greater disconnect in student understanding; however, she also discovered that students were more likely to base their understanding of functions on observed examples rather than learned definitions. This tendency leads to incorrect assumptions and misconceptions.

Similarly, Akkoc and Tall (2005) addressed the disconnected nature of students' understanding of function. Their study involved questionnaires given to 114 grade-13 Turkish students. The questionnaires asked students to give a definition for the term *function*. Only 10.5% of the responses were considered to be in line with the colloquial definition. Nine of the 114 participants were interviewed about various representations of functions. The students were asked to determine if given representations could be considered functions. The results revealed a spectrum of incomplete understandings.

Additionally, Akkoc and Tall suggested that curriculum designers wrongly assume that students can abstract the concept of function after studying various representations. Akkoc and Tall pointed out that although the mathematics curriculum may develop logically, students' cognitive thought does not develop in the same way. They further assert that curriculum should emphasize the process of mathematical thinking and not the product of mathematical thought (Akkoc & Tall, 2005).

Viirman (2008) specifically studied mathematics education students' and engineering students' conceptual understanding of functions. The main purpose of the study was to determine student understandings of the function concept and how students personally define the term *function*. The 34 participants were college students enrolled in a calculus course. Fourteen participants were mathematics education students who had previously taken more than one semester of math. Twenty participants were engineering students who had only previously taken one algebra course. Students were given questionnaires and asked to create a mind map of free-associations about the function concept. They were also asked to determine which of a variety of representations were functions and provide a rating of their certainty. Additionally, participants were asked to provide their own definition of function and construct a function with given characteristics.

The results showed that eight of the 14 mathematics education students were not able to give a meaningful definition of function even though they had taken more hours of math. Of the 20 engineering students only two could not give a definition. Mathematics education students also showed less confidence in their answers and less developed conceptions of the function concept. The engineering students were more confident and showed a more developed understanding. As previously stated, because engineering work demands the ability to transfer knowledge, Viirman's findings support the idea that ability to transfer knowledge is tied to conceptual understanding.

Another study explored how students understand the concept of the vertex of quadratic functions. The study attempted to reveal how students handle both explicit computation and real-world application problems. Sixty-six undergraduate students enrolled in a math-modeling course participated in the study. On a class exam, students were asked to answer questions explicitly asking about the concavity and vertex of a given function and context questions asking about maximizing profit of a function given as part of a word problem. On a separate class exam, students were asked to write about the meaning of vertex. Data was analyzed using the action-process-object-schema framework and the results showed that the students lacked the ability to connect the explicit algebraic calculation to the real-world context. Additionally, the students with a more correct personal definition were more likely to perform well on both types of questions (Childers & Vidakovic, 2014).

Study About Teaching Methods and Environment

Boaler (1993a) reports on her study investigating the relationship between learners' abilities to transfer knowledge and the environment or method of learning. Her study involved a comparison of students from four eighth grade classrooms in the United Kingdom. Two of the classes were located at one school while two were at another for a total of 50 students from each school. The schools were demographically similar; however, the learning environment at each campus was different. One school integrated mathematical process and content by regularly emphasizing open-ended questions that encouraged communication and negotiation. The other school separated the process and content, more traditionally teaching content in class with booklets and providing further investigational activities as assignments to be done at home in isolation from other students and in absence of a nourishing learning environment. Based on an analysis of other research findings, Boaler hypothesized that the students in the integrated content/context classroom would be more equipped to handle a variety of contextual questions and would therefore demonstrate a better ability to transfer their mathematical knowledge. Her qualitative study involved a review of data collected from interviews, observations and assessments given on two separate occasions. Boaler's results suggested that the integrated content/process environment leads to better transfer of mathematics knowledge. However, Boaler was not quick to read too much into this one particular study and suggested that more research needs to be done before definitive conclusions can be drawn because other factors may play a role in student learning.

The Teaching and Learning Potential of Mathematics Textbooks

Literature about curriculum is numerous and in general agreement that the textbook has an effect on what is taught and learned in mathematics classrooms. Budiansky (2001) criticized textbooks for being filled with disconnected facts and noted that teachers rely heavily on ancillary materials for lesson planning and implementation. Additionally, analysis of data from the Third International Mathematics and Science Study (TIMSS) shows that at least 90% of lessons in all the countries used some type of textbook or worksheet (NCES, 2003). Although not all teachers are convinced of the quality of their mathematics textbook (Ball & Feiman-Nemser, 1988), in some countries the textbook is as an authority (Pehkonen, 2004). Among all of the curriculum investigations, there were some that very specifically spoke of the potential of textbooks to support teaching and learning.

Project 2061 is a research and development initiative focused on all Americans becoming literate in science, mathematics, and technology (American Association for the Advancement of Science [AAAS], 2000). Through Project 2061, AAAS sets forth a systematic method for evaluating textbook contents. Their analysis technique used six learning benchmarks selected from national and state standards. Each benchmark was then subsequently evaluated on 24 criteria. The criteria include a variety of support items like specifying prerequisite knowledge, skill building items like connecting benchmark ideas, and reflection items like assessing through applications. Overall, they evaluated 13 middle-grades and Algebra 1 textbooks resulting in individual reports on the strengths and weaknesses of each textbook. The overall results point to more shortcomings than strengths in areas of connections and helping build deep understandings of the mathematical content.

Similarly, Schmidt (2012) advanced a methodology for measuring the content of textbooks in relation to their potential for providing opportunities for students to learn math. Schmidt's study focused on data collected from the TIMSS textbook classification system. Schmidt developed four mathematics topic exposure indices to measure textbook content and coverage. The measures were largely based on page counts and weighted by grade placement and performance expectations. All of the measures indicated that although student exposure to mathematics increases from middles grades to tenth grade, there is a reduced cognitive demand in the high school textbooks. Schmidt concludes that the textbook is a source of opportunity for both instruction and student performance. He also emphasizes that the interplay between teacher and textbook is significant.

Dole and Shield (2013) also assessed the potential for textbooks to assist in teaching and learning that emphasizes interconnectedness and deep learning of mathematics. Although their study cited Remillard (2000), recognizing that there are limitations in analyzing textbooks independent of how they are used by teachers, Dole and Shield (2013) did not consider how textbooks are used. Their aim was merely to focus on the structure and content of the textbook, and they described their work as an acceptable first-level analysis.

The Dole and Shield study was an analysis of proportionality found in five series of textbooks. Each textbook series was reviewed sequentially in order to establish the degree of connections among related topics. The content was then rated based on a framework adapted from Project 2061 (AAAS, 2000). Essentially, five main curriculum content goals related to proportionality were identified. Each of the five content goals was then assigned three indicators that described the content sought in the textbooks. The indicators were determined by what was believed to have contributed to the attainment of the specified content goal. After reviewing the material, each indicator was rated as high, medium, low, or no evidence. Once the data were compiled, a judgment about each textbook series and its specific strengths and weaknesses was made.

Achieve, an education reform organization (EQuIP, Educators Evaluating the Quality of Instructional Products, 2013), provides another method for evaluating mathematics materials. Achieve's purpose is to make college and career readiness a priority across the country. Their initiative, Educators Evaluating the Quality of Instructional Products (EQuIP, 2013), identifies high-quality materials aligned to the Common Core State Standards. In order to identify these exemplar lessons, EQuIP developed rubrics for each subject. The rubric for mathematics includes four rating categories called dimensions. The four dimensions are alignment, key shifts, instructional supports, and assessment. Each dimension has a description of the expectation and allows evaluators to assign a rating from zero to three. A 3-rating means that the lesson meets most or all of the criteria in the dimension. A 0-rating means that the material does not meet the criteria in the dimension. The rubric also requires an overall rating that depends on the scores of the four dimensions. The overall rating is a descriptor that characterizes the quality of the lesson. For example, an overall rating of E implies that the reviewed lesson exemplifies the quality standard and exemplifies most of the criteria across all dimensions of the rubric, while N suggests that the material is not aligned and does not address criteria (EQuIP, 2013).

Evaluation of Textbooks in General

Price (2012) provides another in depth study of textbooks that examines the quality of open source materials compared to that of traditionally published textbooks. Her study was not specific to math, but her report includes useful information for textbooks of any subject. Price employed an evaluation rubric based on general design principles described in "The Non-Designers Design Book" by Williams (2007). This study also addressed teacher and student perceptions about essential and unnecessary characteristics textbooks. Findings suggested that essential textbook features included content that was written for the intended grade level and useful graphics. Additionally, Price found that color graphics, real-world examples, and application boxes were very helpful to students and teachers. The unnecessary textbook features included content coverage that far exceeds what can be covered in one year, expensive paper and binding choices, full-color graphics on every page, and extra ancillary content.

Summary

In summary, studies performed on student populations below the second year of college suggested that students had difficulties with knowledge transfer. Multiple studies (Britton, 2002; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012) reported testing conceptually parallel problems in a variety of contexts to determine if students were capable of transferring their knowledge. However, the studies did not investigate specific causes for the difficulty.

Recently adopted textbooks have claimed to address the new TEKS. However, the research literature (Budiansky, 2001; Fan & Zhu, 2007) pointed to textbook weaknesses in making explicit connections and in adequately addressing curricular standards. Other research literature discussed the influence of textbooks on classroom instruction (NCES, 2003; Pehkonen, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003) and the quality of textbooks (AAAS, 2000; Dole & Shield, 2013; EQuIP, 2013; Schmidt, 2012). If textbooks are instrumental in teacher instruction but do not adequately address the standards for connections, what chance is there to improve the knowledge transfer of mathematics? The quality of textbooks is important. This study was designed to address the potential of a specific textbook series to support the well-connected teaching and learning of function related concepts.

III. Design and Methodology

This study investigates the types of connections and quality of items relating to analyzing graphs of functions that can be found in a series of high school math textbooks. This study builds from three separate ideas inferred from the research literature. First, a well-connected concept understanding is more likely to lead to improved ability to transfer mathematics knowledge to other applications. Second, STEM programs and careers require a high degree of transfer. Third, analyzing graphs of functions appears in the requirements for Algebra 1, Algebra 2, and Precalculus, and involves skills that may be useful in STEM work. From these three ideas, two research questions emerged. Although the research questions were stated in the introduction, they will be restated here for convenience and discussion.

1. What number and types of connections related to understanding key features of graphs of functions can be found in a specific traditional Texas high school textbook series of Algebra 1, Algebra 2, and Precalculus?
2. What potential does this specific textbook series have for supporting teachers and helping students develop their ability to make connections related to understanding functions?

In order to answer the research questions a qualitative curriculum study was designed and carried out. A qualitative study was selected because the research questions are focused on quality of content and subjective judgment. Although the first question called for a quantity, it is based on a quantity of a qualitative characteristic. Additionally, in accordance with qualitative methods set forth by Corbin and Strauss (2015), the guiding questions direct the research toward generating a hypothesis instead of testing a hypothesis. The study does not hypothesize about the quality of the textbook and then subsequently test the quality. Rather it aims to create a hypothesis about the textbook quality based on the collected data. Details about the study design and methods of data collection are described in the following subsections.

Evolution of The Research Design

As can be expected during qualitative research, the study evolved and changed during data collection based on findings and perceptions. Additional modifications were made for other reasons including time constraints and lack of incidence. Because of the evolving nature of the qualitative study, the implemented actions are not identical to the planned actions. Justifications for initial choices and for deviations from the original plan are provided.

Justifications for chosen textbooks. The planned study was to focus on teachers' editions of a specific traditional high school sequence of Algebra 1, Geometry, Algebra 2, and Precalculus mathematics textbooks of a single publisher. The chosen series was published by McGraw-Hill with a copyright date of 2016 and was tailored for Texas schools. The textbooks were adopted for several high schools by a large independent school district in South Texas beginning in the 2015-2016 school year. It is undetermined how many Texas schools or districts use this series of textbooks; however, the local impact of this textbook could be substantial. The adopting district is among the top 5% of largest school districts in Texas, serving more than 35,000 students who will likely use textbooks from this series before a future adoption cycles in something new.

Although it was intended that all four textbooks would be reviewed, ultimately only three were included because of their relevance. Due to lack of pertinent content in the textbook and absence of Geometry TEKS associated with functions, the Geometry textbook was eliminated from evaluation.

Topics under investigation. The study was restricted to topics associated with analyzing graphs of functions. As noted in the review of related work, the function concept is difficult for students to understand (Akkoc & Tall, 2005; Childers & Vidakovic, 2014; Jones, 2006; Viirman, 2008). The ability to interpret graphs was noted as a deficiency in knowledge transfer studies (Hoban et. al, 2013; Planinic et. al, 2012) and it was suggested that a better mathematical understanding of key features could aid in transfer (McDermott, Rosenquist, & Van Zee, 1987).

For feasibility, asymptotes and end-behavior were eliminated from consideration for the following reasons:

1. They were viewed as less urgent for knowledge transfer to real-world and interdisciplinary applications.
2. Neither concept was mentioned in the reviewed related work.
3. End behavior was not mentioned in the Algebra 1 or Algebra 2 TEKS so tracing development of the concept was not possible.

Furthermore, after a preliminary investigation into the occurrence of these topics in each textbook and consideration of the close relationships and/or overlapping meanings, related topics were grouped together for a total of four topic categories as opposed to eight individual topics. Domain and range comprised one category; zeros and intercepts another; maxima, minima and vertex yet another; and, slope, increasing behavior and decreasing behavior the last.

Types of connections. Essentially, this study aimed to uncover the number and types of conceptual connections related to key features of functions for each textbook. It was expected that connections from level to level, from topic to topic, among representations, across content areas or disciplines, and to the real-world would be found; however, two other types of connections surfaced during the investigation. One was the connection between solution procedures and the underlying concepts. The other was the connection between mathematical representations and the English language. The mathematics to English connection encompasses more than vocabulary and definitions. It also includes translation from English to mathematics and interpretation of mathematical ideas. All types of connections are summarized in Table 1.

The preliminary evaluation instrument. Another objective of the study was to describe connections based on perceived quality. Specifically the study employed a rating system to evaluate each observed connection. The preliminary framework for quality evaluation was modeled after Project 2061 (American Association for the Advancement of Science, 2000), EQuIP (2013), and Price (2012). Similar to Project 2061, three evaluation categories were developed: *clarity*, *promoting thinking*, and *teacher support*. Two descriptive criteria were decided for evaluating each category. It was planned that each criterion would be rated on a scale of 1 to 3. The intended rating tool is shown in Table 2.

Table 1

Types of Connections

Descriptor	Description
1. Level to Level	addresses the topic in relation to how it connects to previously learned concepts from a different mathematics course.
2. Topic to Topic	addresses the topic in relation to how it connects to previously learned topics within the same course.
3. Among Representations	addresses connections among different representations.
4. Across Content/ Interdisciplinary	addresses the topic in relation to how it connects to other academic subjects.
5. Real-World	addresses the topic in relation to how it connects to applications in life and career.
6. English to Math	connects the vocabulary/ language of mathematics to meanings in the English language.
7. Procedures to Concepts	connects a solution technique or procedure to the underlying concept.

Table 2

Preliminary Rating Tool

	Quality Criteria:	*Rating:* 1	2	3
Clarity	**Mode of Conveyance**	**Unstated** Completely implied	**Indirect** Ambiguous or vague reference	**Explicit** Directly addressed or clearly stated
	Prominence	**Obscure** Does not stand out/plain fonts buried in wordy text	**Visible** Is separate from lengthy body text but does not draw attention.	**Distinct** Stands out/draws attention, uses colors, bold fonts, or highlighting.
Promoting Thinking	**Relatable Context**	**Improbable** Context is likely to be unfamiliar or confusing to most students	**Possible** Context is understandable but not likely to interest many students	**Probable** Context is relevant to students and is likely to provoke interest.
	Level of Engagement Required	**Passive** Connections are explained in body text and simply need to be read.	**Moderate** Connections can be observed through following a worked example	**Active** Connections must be made in the solution process/ explanation
Teacher Support	**Guidance for Teachers**	**Deficient** No indication of how to help students build connections	**Minimal** Mention of connection but limited suggestions for implementation	**Sufficient** Ideas for reinforcing concept connections in the classroom
	Correlation to Course TEKS	**Lacking** Does not support the TEKS objectives for the course	**Partial** Addresses a portion of one or multiple TEKS	**Full** Completely fulfills one or more TEKS requirements

The preliminary rating system described in Table 2 was developed before any textbook items were reviewed. The author and the research mentor who is an expert in curriculum analysis research agreed upon criteria. The criteria were chosen based on measures that were believed to be significant and measures from other published studies. Once data collection began, it became apparent that the preliminary rating system was not sufficient.

Modification of ratings. The first issue was with the ratings. Ratings from one to three were challenging to work with. It was difficult to assign consistent ratings to similar items because the scale was too broad and often items seemed to fall between categories.

In an effort to fine tune ratings for more consistency, it was decided to develop a four-level rating system. This meant that all descriptors and descriptions for every level and all criteria needed to be revisited and revised. In most cases, the one and two ratings were considered to be low with a one rating being worse than a two. Similarly, the three and four ratings were considered high with four being better than three. This made the process of rating easier and more reliable because the rater could first determine whether the item had more quality or less quality and then subsequently decide if it was on the higher end or lower end of that block.

In a few cases, the rating scale was not necessarily arranged from low rating to high rating. For example, for the criterion called *acquisition*, formerly labeled as *level of engagement required*, the scale is not guaranteed to be in low-to-high order. Ratings 1 and 2 are reserved for items that require only observations like reading or looking at examples or diagrams in the textbook. Ratings 3 and 4 are dedicated to items that require some sort of action like working a simple exercise or doing more advanced problems. It is possible for ratings 2 and 3 to be out of order but not in most cases. Typically, it is easier to make observations than it is to perform an action independently, but there are a few instances in which it is more difficult to draw meaning from the observed facts than from a simple problem to be solved.

In some cases, a 5-point-scale was introduced when textbook items seemed to display a quality that could not be described by the existing four ratings. This rating was not included until it was deemed necessary. Three criteria were ranked by using a 5-point scale, these are: *context*, *acquisition*, and *TEKS correlation*.

Modification of criteria. After numerous items were rated, it also became apparent that the preliminary criteria were not describing the quality as completely as possible. The author and the research mentor agreed that a more thorough description may be obtained by adding three additional criteria: *role*, *effectiveness,* and *uniqueness*.

The *role* criterion was introduced to categorize items by purpose or function. The scale for *role* essentially divides items into two main groups, supporting or main focus, and further describes how they function within the broader category. This criterion's purpose was to identify a characteristic that can help describe the value of each connection and contribute to a determination of overall quality.

The *effectiveness* criterion was created to rate how well the information is conveyed. This category was thought to be necessary because at times the information in the textbook was found to be very misleading or inaccurate, while other times the information was satisfactory. This type of evidence may be significant in determining overall quality.

The *uniqueness* criterion was added to help identify repetitious and more novel items. If students are expected to eventually transfer knowledge, then it is imperative for them to learn to think mathematically instead of simply mimicking work done in examples and previous exercises. The number of redundant items is an indicator of movement toward or away from the goal of knowledge transfer. Repetitious items suggest that included procedures may be imitated and well-rehearsed for a particular problem type. While the repeated practice may lead to longer lasting memorization of a process, does it ensure mathematical understanding flexible enough for transfer? The assumption is that the ability to transfer knowledge is more likely enhanced by original items that require more independent thought and less imitation. With a higher number of unique items available in the textbook, the teacher is better supported in helping students improve knowledge transfer ability.

Relabeling, regrouping, and restructuring. With the addition of new criteria, editing and restructuring followed. First, the name of the overarching category *clarity* was changed to *characteristics* so that the new criterion *role* could be included with the criteria *prominence* and *mode of conveyance*.

It was decided that three of the criteria clearly addressed the content of textbook items; *effectiveness*, *context*, and *uniqueness* were grouped together under the heading *content*. The criterion was formerly included in a category named *promoting thinking* which was removed and replaced with the label *content* in attempt to describe more of the criteria in the group. While the title *promoting thinking* is catchy and seemed to encompass *context* and *uniqueness*, it did not describe enough of the criteria.

With the *promoting thinking* category removed and the *context* criterion absorbed into *content*, *level of engagement required* needed to be placed in a category. The criterion title *level of engagement required* also seemed very long so the decision was made to rename it to *acquisition*. The *acquisition* criterion was then grouped with *teacher guidance* and *TEKS correlation*, in a category labeled *other considerations*.

The final evaluation instrument. After all discussed modifications of the preliminary evaluation tool were complete, the final instrument emerged in four parts. It was divided into four separate parts for a convenient layout that was readable and easy to use during the investigation. Table 3, Table 4, and Table 5 show the main parts of the instrument. The fourth part, in Table 6, contains descriptions of the three 5 ratings that were added as needed. This resulting evaluation tool was used for the entire textbook investigation.

Table 3

Evaluation Tool Part 1: Characteristics Category

	Rating: 1	2	3	4
Mode of Conveyance	*Unwritten*		*Written*	
	Diagrams/Images Connections are implied through diagrams including graphs and tables. Written clues are limited.	**Symbolic Statements** Connections are implied through symbolic statements including equations and inequalities. Written clues are limited.	**Directions** Connections are directly called for in the question text or directions with no further explanation or support.	**Explanatory Text** Connections are explicitly mentioned or explained in the body of the text. Diagrams or symbolic representations may aid explanation.
Prominence	*Less Prominent*		*More Prominent*	
	Unremarkable Information does not stand out more than other items on the page. It may be written in plain fonts or contained within wordy text making it easily overlooked.	**Competing** Is in close proximity to colors, bold fonts, highlighting, or other marking that is likely to detract attention.	**Accentuated** Is in close proximity to colors, bold fonts, highlighting or other marking that is likely to draw attention.	**Distinct** Information stands out/draws attention. Colors, bold fonts, highlighting, or other markings work to ensure detection.
Role	*Supporting*		*Main Focus*	
	Off Task/Minor The information helps support the development of a separate concept in a small way.	**Major** The information is significant in developing a foundation for a pertinent concept or addresses a closely related concept.	**Specifics/Detail** The information focuses on a concept detail or a particular instance of a pertinent concept.	**Generalization/ Extension** The item focuses on overarching ideas of a pertinent concept. It extends beyond the detail a particular instance.

Table 4

Evaluation Tool Part 2: Content Category

	Rating: 1	2	3	4
Effectiveness	*Less Effective*		*More Effective*	
	Misleading Information is incorrect or misleading. It could lead to an inaccurate concept image.	**Ambiguous/ Vague** Information is unclear and/or confusing. It may lack precision, which could lead to an underdeveloped or inaccurate concept image.	**Incomplete** Information is clear but incomplete or lacking. It leaves unanswered questions but contributes to developing the concept image.	**Satisfactory** Information is as complete, accurate as can be expected. It is likely to be understood furthering the development of the concept image.
Context	*Less Enhancing*		*More Enhancing*	
	None No context included.	**Unnecessary** Context is likely to be overlooked, unfamiliar or confusing to most students. It is present for the sake of having context.	**Limited** Context is understandable. It may not be likely to interest many students. Contextual engagement and benefit are minimal.	**Valuable** Context is an important part of the information. It may be likely to provoke interest. Contextual engagement and benefit are maximal.
Uniqueness	*Less Unique*		*More Unique*	
	Redundant The item is a simple reiteration of previous statements, worked examples, or other problems. It requires no adjustment in thinking patterns.	**Altered** The item is a slightly modified version of previous statements, worked examples, or other problems. It requires minor adjustments in thinking patterns.	**Modified** The item includes significant modification of previous statements, examples, or problems building toward more advanced thinking.	**Novel** The item is new or different than previous statements, worked examples, or other problems. It demands activation of knowledge and flexible thinking.

Table 5

Evaluation Tool Part 3: Other Considerations Category

	Rating: 1	2	3	4
Acquisition	*Observations*		*Action*	
	Passive The connection can be discovered by simply reading a few sentences or looking at a diagram.	**Active** The connection can be observed through following a worked example or piecing together nearby information.	**Simple** The learner must use the connection in order to work through a simple exercise or activity.	**Advanced** The learner must think at a higher level, go back and find information, or "do math" instead of mimicking mathematics that has been done.
Teacher Guidance	*Less Guidance*		*More Guidance*	
	None No support is available for addressing connections. Support may be available for other purposes.	**Minimal** Support does not indicate the connection significance. Support is limited to solutions or solution locations and additional examples. (more of the same)	**Average** Support conveys the importance of the connection but includes limited suggestions for concept development and classroom implementation. Support is likely limited to scaffolding questions.	**Sufficient** Support recognizes the importance of the connection and includes ideas for reinforcing concept connections in the classroom.
TEKS Correlation	*Weak Correlation*		*Strong correlation*	
	Lacking The item does not support the targeted TEKS objectives for the lesson.	**Other** The item addresses the targeted TEKS objectives unrelated to the study topic.	**Partial** The item addresses a portion of the targeted TEKS objectives related to the study topic.	**Full** The item completely addresses the requirements of the targeted TEKS objectives related to the study topic.

Table 6

Addition to the Evaluation Tool: Description of 5 Ratings

Criterion	Rating 5 description
Context	No context is observed. Context is significant because it is to be created by the learner.
Acquisition	The learner is not likely to acquire the information through observation or action. Connection may be poorly developed or the item may be too advanced.
TEKS Correlation	The item might not address the targeted TEKS. It is a weak correlation but on the other end of the spectrum from a 1 rating. This can be thought of as above and beyond the TEKS requirements.

Data Collection Procedure

Data collection began April 1, 2016 and continued through May 25, 2016. Two raters coded the items. Altogether, 1340 items were rated by two high school mathematics teachers. Rater A evaluated a total of 1228 items from all of the textbooks, while Rater B evaluated 112 items from the Algebra 2 textbook only. Before the second rater began coding independently, he was asked to review the rating criteria and practice applying it to ten different items from the Algebra 1 textbook that Rater A had previously evaluated. After each of the ten practice items was evaluated, raters discussed any difference for further clarification. On the last five items, Rater B matched Rater A on 88% or more of the criteria for each item. Because of the high percentage of agreement, it was determined that he could begin examining items independently.

As was done in the textbook study by Dole and Shield (2013), the three textbooks from the McGraw-Hill series were reviewed sequentially in order to provide more insight into concept development from level to level. Notes were recorded for each item and for each section as a whole. Only certain sections, pages, and items from each textbook were reviewed. Details about selecting pages and items are outlined in the following subsections.

Identifying pages to review. Only pages containing information related to analyzing key features of graphs of functions were selected for review. To identify the pages, first the TEKS were examined. The TEKS were electronically scanned for the key words domain, range, minimum or minima, maxima or maximum, vertex, intercept, slope, zero, root, and solution. Then, the TEKS were read and carefully examined for related concepts that may not include the key words exactly as searched. Through this process, relevant TEKS were identified. A maximum of ten different TEKS were selected for each subject as shown in Table 7 and Table 8. Only one objective was selected for Precalculus, TEKS P.2(I) which is written as follows:

> Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing. (SBOE Rules - TAC, 2012)

It is a very all-encompassing standard that addresses all of the study topics of interest.

Once the TEKS were identified, the textbook sections and pages associated with each of TEKS were found. Near the front of each of the McGraw-Hill textbooks under review, there is a correlation table matching each of the TEKS with corresponding lessons and locations where the standard are addressed. This correlation chart was used as a guiding list for finding items to evaluate. Although the pages on the guiding list were scrutinized more carefully than others, most pages in the textbook were quickly scanned for relevant content. If relevant content was found elsewhere, it was added to the list, and if pages on the guiding list lacked relevant content, they were eliminated.

Table 7

Algebra 1 TEKS Selected for the Study

A1.2(A)	determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities
A1.2(G)	write an equation of a line that is parallel or perpendicular to the X or Y axis and determine whether the slope of the line is zero or undefined
A1.3(A)	determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$
A1.3(B)	calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems
A1.3(C)	graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems
A1.6(A)	determine the domain and range of quadratic functions and represent the domain and range using inequalities
A1.7(A)	graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry
A1.9(A)	determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities
A1.9(B)	interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems
A1.9(D)	graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems

Table 8

Algebra 2 TEKS Selected for the Study

A2.2(A)	graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval
A2.2(C)	describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range
A2.2(D)	use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other
A2.4(D)	transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ to identify the different attributes of $f(x)$;
A2.5(A)	determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative real values of a, c, and d
A2.6(A)	analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d
A2.6(C)	analyze the effect on the graphs of $f(x) = \lvert x \rvert$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x-c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d
A2.6(G)	analyze the effect on the graphs of $f(x) = 1/x$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x-c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d
A2.6(K)	determine the asymptotic restrictions on the domain of a rational function and represent domain and range using interval notation, inequalities, and set notation
A2.7(I)	write the domain and range of a function in interval notation, inequalities, and set notation

Selecting items to evaluate. Choosing which items to evaluate and which items to exclude from each page was at times the most difficult part of the process. To be clear, an item is any element on the page that can stand-alone without losing meaning. Items are usually composed of some sort of cluster or grouping, like a paragraph explanation of a solution technique, a collection of multiple representations, or a boxed-in concept summary. Example problems, practice problems, definitions, call out boxes, and margin notes are all also examples of items. Table 9 includes all of the item types and the codes used to identify the item type during data collection.

Table 9

Item Type Codes

A. Definition/Vocab	B. Key Concept	C. Concept Summary
D. Explanation	E. Example	F. Practice Problems
G. Guided Practice	H. H.O.T. Problem	I. Prep for Assessment
J. Launch	K. Error Alert	L. Study Tip
M. Scaffolding Question	N. Answer/ Sample Answer	O. Step-by-Step solution
P. Additional Example	Q. Other Margin Note	R. Planning Guide/ Skills Trace
S. Mathematics Background	T. Differentiation/ Extension	

For consistency in selecting items to evaluate, some guidelines were loosely followed. First, each page was visually scanned for occurrences of the key words domain, range, minimum or minima, maxima or maximum, vertex, intercept, slope, zero, root, and solution. When instances of key words were found in items that displayed or implied one of the connection types from Table 1, the item was chosen for evaluation. Next, items on the page that did not contain key words were reviewed and selected for evaluation as appropriate. Items that did not contain key words were only ignored if they were not related to the chosen study topics or did not display or imply one of the connection types. Otherwise the items were chosen for evaluation. For example, Figure 1 shows a word problem similar to one found in the textbook series. The problem is asking for the time a ball was in the air does not directly ask for *x*-intercept, but it is expected that *x*-intercept will be found. Because it is related to the study topic *x*-intercept and it contains real-world connection, it would be chosen for evaluation.

BASEBALL The equation $h = -16t^2 + 48t + 4$ models the height h, in feet, of a ball that Sandy hits after t seconds. How long is the ball in the air?

Figure 1. Word problem with real-world connection related to *x*-intercept.

Using the evaluation tool. Once items were selected for evaluation, then the evaluation tool was used to rate the item on nine criteria. The evaluation process was straightforward but did involve subjectivity. How each criterion was considered is explained in the following subsections.

Mode of conveyance. This criterion was generally thought of as the method for communicating the connection. For this criterion, many items were multi-coded with ratings 1, 2 and 4 because they contained tables, graphs, and verbal descriptions that all contributed to developing the connection. Exercises, on the other hand, were typically only coded with a 3 to indicate that the connection was called for in the directions of the problem.

Prominence. This was the most difficult criterion to rate. The chosen series is filled with color and markings that are intended to draw attention. Because there are so many colorful markings on a page, it is difficult to determine what stands out the most. Additionally, this is a very subjective category because what stands out more to one person could be less noticeable to another person. It seemed that most items could be rated as a 2 because so many of them seemed to compete for attention. Although this was the case, the 2 rating was reserved for items that were truly likely to be overlooked due to nearby markings that detracted attention.

Role. This criterion identifies the importance of evaluated items by categorizing them by purpose or function. First, an item was categorized as either supporting or main focus. Main focus items are those that tend to relate directly to the concepts or skills called for in the TEKS. Supporting items are those that may be related to the main focus items, but are at a very basic level or serve as a review of previously covered material. For example, as needed, textbooks may review concepts that have previously been learned. The review concepts might be related to the TEKS but they are not the main focus; therefore, they would be categorized as supporting.

Once an item was categorized, then the rating values were assigned. Ratings 1 and 2 were used to describe the significance of the supporting items, while ratings 3 and 4 were used solely for main focus items. When a main focus item contained specific numbers that were unique to that item, it was coded as a 3. If it contained broad statements or information that extended beyond expectation, then it was coded as a 4.

Effectiveness. As previously stated, this criterion was necessary because at times information in the textbook was found to be very misleading or inaccurate. While necessary, the effectiveness rating was also very subjective and challenging to think about. It attempts to rate how well information is conveyed, but depends largely on opinion and experience of the rater. For example, the rater may not realize information is missing or might feel that missing information is not appropriate to include in the circumstance. Also, a rater might not realize that information is misleading or inaccurate.

For consistency, every item was initially viewed as a satisfactory 4 rating and adjusted as needed. If some information seemed to be lacking but clarity was not significantly compromised, the rating was reduced to a 3. If information was confusing or imprecise, a 2 rating was assigned. And if the information was misleading or incorrect, it was given a 1 rating. Because all items started with a rating of 4 and were only reduced as needed, most exercises were coded as a 4. It was very difficult to consider assigning any other rating because the solver plays an important role in determining effectiveness.

Context. The *context* criterion was used to evaluate the presence and quality of real-world and interdisciplinary context. If context was not present, the item was rated as a 1. If context was present, its value was determined. A 2 rating was assigned if the context could be completely removed and without affecting the outcome as in the example problem shown in Figure 2. A 3 rating was used for items that forced interaction with the context but the context was not otherwise significant. For example, many exercises like the one in Figure 3 contain context that could be removed without consequence if the question did not ask for meaning to be interpreted for the given context. The 4 rating was reserved for items where context was more meaningful and interaction was required beyond simple interpretation. An example similar to items found in the textbook is shown in Figure 4. A fifth rating was also added to identify context that had to be self-generated by the reader. Some exercises and problems asked for the reader to describe a real-world situation for the concept at hand. A 5 rating does not necessarily indicate that an item is better than one rated as a 4. Items rated as a 5 are wildcard items in which quality depends largely on the individual solver.

SALARY The table shows the average income for teachers for various years of experience.

a. Assume that the years of experience are the domain. Identify the domain and range.
b. Write a relation of ordered pairs for the data

Years of Experience	Average Salary
1	44,000
2	44,556
3	45,214
4	46,032

Figure 2. An example of a level-2 rating for *context* criterion.
In this problem context is unnecessary. Both tasks could be accomplished without noticing or considering the context.

First determine if the graphed function is linear or nonlinear then do the following:

a. Identify any symmetry.
b. Determine where the function is positive, negative, increasing, and decreasing.
c. Fine the x-coordinate of any relative extrema.
d. Describe the end behavior of the graph.
e. Estimate and interpret the intercepts of the graph.

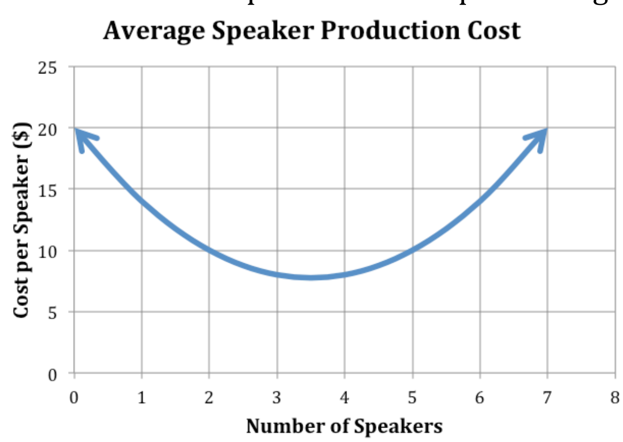

Figure 3. An example of a level-3 rating for *context* criterion.
Context is only important because the question asks for an interpretation (level 3 rating). If an interpretation were not requested, then the context would not be important. All other parts can be answered without considering the context.

PACKING MATERIALS Barbara needs to design a shipping box with a square base. The volume must be 4025 cubic inches. What dimensions would minimize the surface area of the box? Explain.

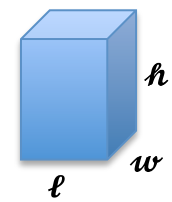

Figure 4. An example of a level-4 rating for *context* criterion.
Interaction with context is required in this problem. Consideration and focus on context is beyond simple interpretation

Uniqueness. As discussed, the *uniqueness* criterion was added to help identify repetitious and more novel items. The 1 and 4 ratings were the easiest to assign; either an item was a near duplicate of a previous and earned a 1 rating or an item was significantly different than other items and earned a 4 rating. The 2 and 3 ratings were slightly more difficult to distinguish. A 2 rating was generally thought to be something that was not quite a duplicate, but it was fairly close to being a duplicate. For example, a word problem that requires solution procedures identical to a previous word problem but employs a different scenario would be rated as a 2. A good way to describe a 2 rating would be as follows: Close similarities exist but they may be slightly disguised. A rating of 3 was assigned to items that were more unique than 2-rated items but not quite novel.

Acquisition. This criterion was formerly called level of engagement required and was renamed to be more concise. In short, *acquisition* refers to intended method for taking in the information. Similar to the *role* criterion, this rating scale is broken into two main categories, observation and action.

Observation means that the reader is not doing anything other than looking and thinking, and this category is further subdivided into types of observation required. A rating of 1 means the information is simply meant to be read while a rating of 2 indicates some active observations that require comparison are necessary. Reading a definition would be rated as a level 1 observation while following a worked example would be rated as a 2.

Action means that the reader will have to do some work, and this category is subdivided into ratings that address the difficulty of the task. A 3 means it is a relatively simple task while a 4 indicates a more challenging task.

Additionally, a 5 rating was created to account for information that was not likely to be acquired through observation or problems that were not likely to be solved even if attempted. This rating was used sparingly to identify items that were not adequately organized or supported and would be more difficult to acquire. Figure 5 shows an item similar to one that was rated with a 5 because of the difficulty in observing the part of the graph with the steepest slope or greatest rate of change. The graph appears to be almost linear making it very difficult to acquire.

Refer to the graph shown. Without calculating, find the 4-year period that has the greatest rate of change. Then calculate to verify your answer.

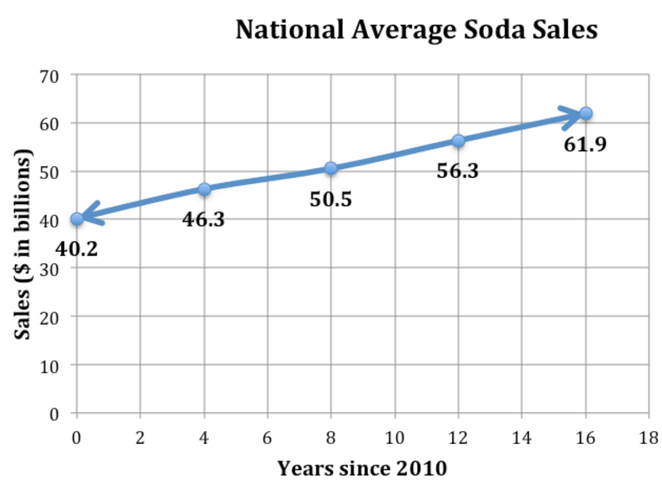

Figure 5. Item that would be rated as a level 5 for the *acquisition* criterion.
It is considered difficult to acquire because the solver is expected to observe the section of the graph with the greatest rate of change. It is very difficult to see the slight change in the slopes.

Teacher guidance. This category addresses the teacher support pieces included in the margins of each page and in the chapter appendix that are directly associated with an item on the main page. If an item has no associated support item, it is rated as a 1. If there is an associated teacher support item, then the level of support is rated. Teacher support elements above and beyond those associated with particular items on the page were individually selected for rating if they qualify as an item that should be rated. For example, some pages include teacher support elements that recommend in-class activities. This type of teacher guidance is not associated with a particular item on the page but should be rated as an item itself.

TEKS correlation. The criterion for *TEKS correlation* is a measure of how well the item addresses the TEKS. If the entire standard was addressed, it was rated as a 4. If only part or parts of the standard were addressed then it was rated as a 3. The 2 rating is a little more confusing at first glance. The rating 2 was assigned to items that addressed a standard targeted by the chapter or section of the textbook but not included as part of this study. A 1 was assigned to items that did not correlate with the standards. The item might be related to the standard or be serving in a supporting role but does not address the rigor and level of the standard. A 5 rating was created to account for items that seemed to exceed the TEKS requirements. A rating of 5 and 1 are similar in that they may not address the standard. The difference in the two ratings is where they fall on the spectrum. A rating of 1 can also be thought of as below level and rating 5 can be thought of as above.

IV. Results

Overview

The Algebra 1 results are derived from evaluation of 628 items selected from 108 pages of text contained in 15 sections of the textbook. The Algebra 2 results are derived from evaluation of 345 items selected from 80 pages of text contained in 18 sections of the textbook. The Precalculus results are derived from evaluation of 367 items selected from 58 pages of text contained in 10 sections of the textbook.

The selected sections, pages, and items were chosen because they are tagged with the main TEKS related to the study topics. The chosen TEKS for Algebra 1 are summarized in Table 7. The chosen TEKS for Algebra 2 are summarized in Table 8. For Precalculus, the only chosen standard was TEKS P.2(I) which encompasses all the study topics and more. During the evaluation process, some additional items of other closely related TEKS may have been evaluated due to relatedness and/or inclusion in reviewed sections of the textbooks.

Number and Types of Connections

A summary of the number of each connection types for each course is shown in Table 10. The three most frequent connection categories for each course are marked with an asterisk. The *real-world* and *across content* categories that are more closely related to applications and knowledge transfer outside of mathematics only account for 152 of the 1340 items or about 11%. When examining for *real-world* and *interdisciplinary* items for Precalculus and Algebra 2 individually, the percentage drops to about 6.5% and 5.8% respectively.

Table 10

Count of Connections by Subject

Connection Category	*Number of Items* **Algebra 1** out of 628	**Algebra 2** out of 345	**Precalculus** out of 367	**Overall** out of 1340
Among Representations	376* *59.9%*	245* *71%*	278* *75.7%*	899* *67.1%*
Procedures to concepts	181* *28.8%*	80* *23.2%*	109* *29.7%*	370* *27.6%*
English to Math	200* *31.8%*	79 *22.9%*	75* *20.4%*	354* *26.4%*
Topic to Topic	39 *6.2%*	87* *25.2%*	40 *10.9%*	166 *12.4%*
Level to Level	22 *3.5%*	10 *2.9%*	6 *1.6%*	38 *2.8%*
Real-World	94 *15.0%*	17 *4.9%*	20 *5.4%*	131 *9.8%*
Across Content/Interdisciplinary	14 *2.2%*	3 *0.9%*	4 *1.1%*	21 *1.6%*

** Among the top three connection types for the course.*

Quality Criteria

The result of the quality ratings show strong patterns in data from among rated subjects. Figure 6 is a graphical summary depicting the patterns for each criterion. Enlargements of each graph are included in the Appendix. Actual values for the quality criteria ratings are summarized for Algebra 1 in Table 11, Algebra 2 in Table 12, and Precalculus in Table 13. All quality criteria are included except for *mode of conveyance*.

Most rated items had multiple modes of conveyance making this data problematic and less meaningful. Because it is not as effective as expected for determining quality, the *mode of conveyance* data has been excluded from consideration.

In Table 11, Table 12, and Table 13 the values represent the total number of items counted for each rating and the percentage of total items for each individual course. All rows, except for the seventh, sum to 100% of the total number of items evaluated for each subject. The seventh row, *teacher guidance*, is reduced by the number of teacher support items that were rated. Teacher support items were not rated for the *teacher guidance* criterion because they are the teacher guidance and would not have additional support structure to rate. Further discussion of results for the individual criteria is in the subsections that follow.

Figure 6. Summary of patterns in data for each criterion and all three subjects.

Table 11

Overall Count and Percentage of Ratings for Quality Criteria in Algebra 1

Criteria	*Rating* **1**	**2**	**3**	**4**	**5**	**Weighted Average Rating**
1. Prominence	385 *61%*	27 *4%*	174 *28%*	42 *7%*	-	1.8
2. Role	1 *0%*	43 *7%*	443 *71%*	141 *22%*	-	3.2
3. Effectiveness	10 *2%*	59 *9%*	95 *15%*	464 *74%*	-	3.6
4. Context	514 *82%*	20 *3%*	45 *7%*	44 *7%*	5 *1%*	1.4
5. Uniqueness	337 *54%*	105 *17%*	84 *13%*	102 *16%*	-	1.9
6. Acquisition	44 *7%*	81 *13%*	393 *63%*	102 *16%*	8 *1%*	2.9
7. Teacher Guidance	72 *11%*	479 *76%*	60 *10%*	1 *0%*	-	2.0
8. TEKS Correlation	20 *3%*	21 *3%*	481 *77%*	20 *3%*	86 *14%*	3.2

Table 12

Overall Count and Percentage of Ratings for Quality Criteria in Algebra 2

Criteria	*Rating* 1	2	3	4	5	**Weighted Average Rating**
1. Prominence	163 *47%*	35 *10%*	89 *26%*	58 *17%*	-	2.1
2. Role	11 *3%*	53 *15%*	248 *72%*	33 *10%*	-	2.9
3. Effectiveness	4 *1%*	17 *5%*	167 *48%*	157 *46%*	-	3.4
4. Context	198 *57%*	25 *7%*	106 *31%*	16 *5%*	0 *0%*	1.8
5. Uniqueness	158 *46%*	91 *26%*	25 *7%*	71 *21%*	-	2.0
6. Acquisition	24 *7%*	74 *21%*	194 *56%*	51 *15%*	2 *1%*	2.8
7. Teacher Guidance	62 *18%*	223 *65%*	47 *14%*	6 *2%*	-	2.0
8. TEKS Correlation	2 *1%*	46 *13%*	280 *81%*	8 *2%*	9 *3%*	2.9

Table 13

Overall Count and Percentage of Ratings for Quality Criteria in Precalculus

Criteria	Rating 1	2	3	4	5	Weighted Average Rating
1. **Prominence**	297 *81%*	7 *2%*	52 *14%*	11 *3%*	-	1.4
2. **Role**	1 *0%*	10 *3%*	298 *81%*	58 *16%*	-	3.1
3. **Effectiveness**	0 *0%*	10 *3%*	64 *17%*	293 *80%*	-	3.8
4. **Context**	341 *93%*	6 *2%*	9 *2%*	11 *3%*	0 *0%*	1.2
5. **Uniqueness**	230 *63%*	37 *10%*	40 *11%*	60 *16%*	-	1.8
6. **Acquisition**	32 *9%*	39 *11%*	250 *68%*	44 *12%*	2 *1%*	2.9
7. **Teacher Guidance**	30 *8%*	317 *86%*	6 *2%*	0 *0%*	-	1.9
8. **TEKS Correlation**	10 *3%*	3 *1%*	327 *89%*	0 *0%*	27 *7%*	3.1

Table 14

Side-by-Side Comparison of Weighted Average Ratings

Criteria	*Course* Algebra 1	Algebra 2	Precalculus
1. Prominence	1.8	2.1	1.4
2. Role	3.2	2.9	3.1
3. Effectiveness	3.6	3.4	3.8
4. Context	1.4	1.8	1.2
5. Uniqueness	1.9	2.0	1.8
6. Acquisition	2.9	2.8	2.9
7. Teacher Guidance	2.0	2.0	1.9
8. TEKS Correlation	3.2	2.9	3.1

Prominence. Overall, *prominence* earned a weighted-average rating between 1.4 and 2.1, indicating less prominence in general. The data for *prominence* shows that most items are *unremarkable* in appearance. Further evaluation shows that 719 of the 845 total *unremarkable* items are student practice problems of some sort. The other *unremarkable* items are explanations buried in wordy text and margin notes that are not very pronounced. This leaves all of the examples, key concepts, and warnings on the upper half of the rating scale. Most of the examples, key concepts, and warnings earned a 3 rating because they were enclosed in colorful boxes that had bold and colored text. Although ratings were assigned as they were, it was truly difficult to determine prominence with so many accentuated items on a page. Many items could have easily been considered as a 2 rating for *competing* prominence.

Role. With a weighted-average rating of about 3 for all subjects, most items are main focus items centered on specific details of a concept or a particular instance of the concept. Overall, 9% of the items are supporting or off-task. Around 17% of the items are generalizations or extensions occurring most frequently in the key concept boxes, paragraph explanations, and higher order thinking (H.O.T.) problems.

Effectiveness. The *effectiveness* rating is an indicator of clarity and identifies items with misleading or inaccurate information. As evidenced by the weighted-average ratings near 3.5, the effectiveness of a majority of the items is satisfactory. Almost 70% of items are rated as satisfactory while only 1% is identified as misleading. Overall, there are only 100 items on the lower half of the rating scale, leaving the remaining 1240 items on the upper half of the rating scale.

Context. Only about 21% of all evaluated items have context beyond the mathematics of the study topics. Of the items with context, 18% were assigned a 2 rating. A 2 rating means that the context seems to be present for the sake of having context, but is otherwise unnecessary. Eliminating the 2 rated items, reduces the percent of higher-rated *context* items to about 18% or 236 of the 1340 total items. Algebra 2 had the most higher-rated *context* items contributing 122 or about 52% of the 3 and 4 rated items. Precalculus only contributed abut 8% or 20 of the 236 *context* items. The weighted average of the *context* items shows that meaningful context was low across all textbooks but Precalculus has the lowest weighted-average rating of 1.2.

Uniqueness. General *uniqueness* is summarized by a fairly consistent weighted-average rating at 2 or slightly below. This weighted-average rating indicates less *uniqueness* overall. Fifty-four percent or 725 of the 1340 evaluated items are simple reiterations of one of the 233 items rated as novel. At least 631 or 87% of the repetitious items are intended for student practice. An additional 17% of the total number examined items are one level above the lowest rating signifying that almost three quarters of the rated items are near duplicates of one another. Of the 382 higher rated items for *uniqueness*, 140 are also rated as advanced or level-4 for the *acquisition* criterion.

Acquisition. *Acquisition* is an indicator of difficulty level. Approximately 77% of the items are categorized as action items while the remaining 23% are observation items. Of the action items, most are simple exercises that do not require much advanced reasoning.

Teacher guidance. Only thirty-seven or about 2.8% of the rated items are stand-alone teacher support items unique to the teacher's edition of the textbook and directly related to the study topics. The other 1,303 or 97.2% of the rated items are common items that are included in both teacher and student editions of the textbook. About 76% of the evaluated items were given a 2 rating for minimal teacher guidance and about 15% were given a 1 rating for no teacher guidance. Minimal teacher guidance means that related teacher support does not indicate the connection significance and is likely limited to solutions or additional similar examples while no teacher guidance means that no support addressing the connection is available. Combining the items rated as 1 or 2 on *teacher guidance* gives a total of about 91% of items that have little to no guidance that helps develop the connection significance.

TEKS correlation. Overall, the data shows that 81% of the items address at least a portion of a chosen TEKS standard. Few are able to completely address one of the standards due to the dense nature of the TEKS. For example, Precalculus TEKS P.2(I) it is very lengthy and covers several different features of many different functions, it is difficult for any single item to fully address this standard. Other standards in Algebra 1 and Algebra 2 are similar.

Although it is not common for items to fully address one of the TEKS, about 9% of items reach beyond the target standard. For example, the items rated as being beyond the Precalculus TEKS are items that are intended for developing a foundation for learning calculus concepts. For example, the item in the Precalculus textbook that covers extrema also includes calculus related vocabulary like critical points and points of inflection. Along with this vocabulary are depictions of horizontal and vertical tangent lines which are beyond the TEKS requirements. Figure 7 is another problem similar to what can be found in the Precalculus textbook. The problem demonstrates an item that would be considered to extend beyond the standard.

REASONING If the average rate of change of $g(x)$ $on\ the\ interval$ (c, d) is negative, is $g(x)$ *sometimes, always*, or *never* increasing on the interval? Justify your answer.

Figure 7. Example item that reaches beyond the TEKS standards.

V. Discussion

The overarching theme of this study is the textbook potential. In general, textbooks affect teacher decisions about content covered in mathematics classrooms, teachers control method for delivery of the content, and students are affected by the chain of events preceding their instruction. The main goal of this study is to examine the extent to which a chosen textbook series can support teaching and learning of function-related concepts associated with STEM majors and careers.

The first step in undertaking the study of the McGraw-Hill series of Algebra 1, Algebra 2, and Precalculus textbooks was to addresses the number and types of connections related to understanding key features of graphs of functions that can be found in the textbooks. Data about the number and types of connections is in Table 10 of the results section. The significance of the data and its contribution to the evaluation of the textbook series is discussed in this section.

The second guiding question of this research directly addresses the potential of the series to support teachers in emphasizing connections in teaching and learning of function related topics. In order to measure potential, multiple criteria were used to evaluate function-related items in each textbook. A weighted-average rating was calculated for each of the quality criteria in an effort to summarize findings into a single representative value. Further review and discussion of the rating instrument and this data is also included in this section.

As a result of close scrutiny of the items in the textbook, several striking instances and patterns were revealed. These findings are both related and unrelated to the study topics, and they include discoveries of misleading and incorrect information, remarkable trends in concept development, and observed deficiencies of each textbook. The final portion of this section is dedicated to examining the notable findings not otherwise exposed by the rating tool.

Discussion Rating Instrument

In order to address the research questions, a rating instrument was developed and employed. The design of the tool was previously explained in the Design and Methodology section. Further insights of the rating system were gleaned from the evaluation process.

The rating system is effective but not perfect. During the evaluation process, each selected item was assigned to a connection category and rated for quality based on criteria of the instrument. In effort to gain a complete description of each item, the criterions *role*, *effectiveness* and *uniqueness* were added to the planned instrument for a total of 9 criteria. After extensive use and reflection on the criteria that initially seemed important, not all of them are deemed necessary. The four most significant quality criteria turned out to be *context*, *uniqueness*, *teacher guidance*, and *acquisition* because of their high potential for impact on knowledge transfer. *Role*, *effectiveness*, and *TEKS correlation* were important but could be improved upon for future studies. *Prominence* and *mode of conveyance* ultimately did not contribute to a conclusion of the textbook series' quality, and the two criteria can be removed for future curriculum investigations.

Role was a good addition but could have been more effectively implemented. Most of the items were included on the upper half of the *role* scale because items that would fall on the lower half of the scale were not typically selected for review. This fact indicates that the scale could have been more effectively used to further distinguish items that fell into the upper ratings. For example, it would have been better to separate level 4, *generalization/extension*, into two separate categories, *generalization* and *extension*. *Generalization* could have further been divided into separate ratings for *imposed generalizations* and *arrived generalizations*. The *imposed generalization* category would consist of items like key concept boxes and explanatory text that precedes most observations. *Arrived generalizations* would likely consist of items like H.O.T. problems that ask for conclusions to be made from a series of observations or experience with simpler tasks. If *role* had been implemented in this way, it would be more effective than it is.

Effectiveness was introduced as a way to capture inaccurate and confusing information that could interfere with concept understanding and knowledge transfer. While there were some low ratings for *effectiveness*, it was not the case for a majority of items. Additionally, since the highest rating on the *effectiveness* scale was considered average, the *effectiveness* criterion did not adequately distinguish items on the upper end of the scale. Essentially, this criterion can reveal shortcomings that may negatively affect transfer but does not equally highlight qualities that promote transfer. Hindsight suggests that for this study, *effectiveness* could have been dropped from the rating instrument with little consequence; however, if the data had reflected more low ratings, the category would be more significance. Future studies might choose to reduce the number of *effectiveness* ratings to include only three levels, incorrect, misleading, and acceptable in effort to simplify and clarify. Furthermore, collected data for the *effectiveness* category is potentially useful for comparison with results from possible future teaching experiments that explore student or teacher interactions with the items in the textbook.

TEKS correlation indicates that TEKS were addressed but does not provide a complete picture of how well each standard was addressed in its entirety. Each standard is comprised of many parts that are not separately considered in this rating system. An improvement would be to separately list every subcategory of each of the chosen TEKS and identify the textbook items that address each individual part. The *context* category helps identify items that require the use of mathematics in everyday life, society, and the workplace as outlined by the process TEKS of each course.

The *prominence* and *mode of conveyance* criteria were not as significant as envisioned. The pages were very colorful and filled with bolded boxes, marking, and highlights making it difficult to conform to the structure of the *prominence* scale. Additionally, most of the items had multiple ways of conveying information. These two categories did not lead to noteworthy conclusions and could be left out of future studies altogether.

Overall, the rating system could use adjustments, but appeared to be sound. As previously mentioned, the collected data appears to maintain a consistent pattern from textbook to textbook. This could indicate an effective rating system, and it could also be a result of consistency of quality of each textbook in the series. Additionally, although ratings were subject to bias of two different raters, there appears to be a high level of inter-rater reliability. Of the 1340 evaluated items, 1228 were examined by the Rater A. The other 112 items were examined by a second rater, Rater B. Of Rater B's 112 items, 25 were randomly selected and re-evaluated by the Rater A to account for inter-rater reliability. The ratings of each rater and number of matches for each of the quality criteria are summarized in the tables of Appendix B. Overall, for the 25 randomly selected items, 93% of codes by the two raters matched. With the match percentage above 90%, the reliability in data collected from the two raters was acceptable based on similar guidelines by Cohen (1960).

Discussion of Research Question 1

For connection type, the general patterns in data from each course appear to be similar as previously shown in Figure 6. The most frequent connection type is *among representations* with over half of the items displaying this type of connection. It appears that the number of occurrences of the *among representations* connection type increased from course to course. About 60% of items in Algebra 1 include multiple representations. The percentage increases to about 70% for Algebra 2, and increases further to around 75% for Precalculus. This raises a question of intention. Did the publishers intentionally increase the proportion of items that connect various representations?

It is generally expected that in AP calculus, students will learn solution methods that they can apply to real-world problems. It is evident that College Board stresses connections among representations in AP Calculus curriculum by their statement that:

> AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Although computational competence is an important outcome, the main emphasis is on a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally.

The connections among these representations are important. (College Board, 2016)

In addition to multiple representations, there are other aspects that College Board addresses. The course overview delineates six Mathematical Practices for AP Calculus (MPACs) that “should be used frequently and in diverse contexts to support conceptual understanding of calculus,” (College Board, 2016). The MPACs as outlined by College Board are as follows:

1. Reasoning with definitions and theorems
2. Connecting concepts
3. Implementing algebraic/computational processes
4. Connecting multiple representations
5. Building notational fluency
6. Communicating

If McGraw-Hill publishers aim toward addressing the recommendations for AP Calculus, then they have at least partially addressed some of the Mathematical Practices. Data from this study shows a high occurrence of items connecting procedures to concepts and English to Mathematics in addition to multiple representations. Although the MPACs are not well defined in the AP Calculus overview, it is likely that the procedures to concepts and English to Mathematics connection types would correspond to MPAC 3 and MPAC 6 respectively. More obviously, the among representations connection type would pair with MPAC 4. Even with a generous assumption that MPACs 3, 4, and 6 have been well developed in the reviewed sections of the McGraw-Hill textbooks, there are still three remaining practices that have not been addressed as well. Although a later portion of this discussion includes more detailed observations about notations and notational fluency, in general many of the notations in the McGraw-Hill series were not well developed or consistent. Additionally, there is not much evidence to support a claim that MPACs 1 or 2 are adequately developed in the examined sections of the McGraw-Hill textbooks. In the textbook series examined, there are few level-to-level and topic-to-topic connections, and reasoning with definitions seemed to be reserved only for an occasional H.O.T. problem. Overall, McGraw-Hill textbooks appear to build toward 50% of the practices outlined by College Board. If AP Calculus is the end goal, then the means are at most halfway aligned.

Although it is likely that McGraw-Hill deliberately chose to emphasize multiple representations above other types of connections in effort to build a foundation for the next course, it is not the purpose of this study to speculate about the intentions of the publishers or quality of materials for Calculus preparation. McGraw-Hill did very little with regard to the motivating factor of this study, knowledge transfer. Opportunities to transfer knowledge to real-world and interdisciplinary applications occur less frequently than opportunities for any other connection type examined. There are a total of 152 real-world or interdisciplinary items in all sections of the examined textbook series. Of these items, only 118 had meaningful context with as few as 55 out of 1340 or about 4% rated as valuable context. With so few items to choose from spread out over three years of instruction, it is unlikely that the McGraw-Hill series can support the teacher in developing student confidence and capability to transfer knowledge when it comes to analyzing key features of graphs of functions.

Discussion of Research Question 2

Evidence from the quality criteria supports the claim that the chosen series is insufficient for developing connections that will likely enhance student ability to transfer knowledge. The quality criteria, as summarized by weighted-average ratings in Table 14, show low ratings on some of the most significant criteria, like *context*, *uniqueness*, and *teacher guidance*. *Context* is important because it provides a glimpse of reality and applications of the mathematics. Exposure to context is a step toward knowledge transfer. *Teacher guidance* is essential because this study is founded on the idea that the textbook is a major influencing factor on the teachers and their instructional practices. More useful guidance items means more potential to positively impact teaching practice. *Uniqueness* is also extremely important in making a conclusion about the textbook series' potential. Transferring mathematics knowledge means that a person recognizes conceptual similarities in problems that may be superficially dissimilar and correctly identifies and applies appropriate mathematics. Transfer implies familiar mathematics procedures will be used for novel situations. If almost all items are rated as near repetitions of other items, then opportunities to practice transfer will not occur enough. *Prominence* is also rated low, but understandably so. It is unreasonable and counterproductive for a majority of items to be prominent. Everything cannot stand out, so most things will be fairly unremarkable even if the intent is to make all of them prominent.

Four of the criteria appear to be rated closer to the upper half of the scale. *Role* is close to 3 for all subjects. This simply means that most items were on topic, however, they were not necessarily high quality. A 3 rating for *role* indicates that most items focused on a concept detail or a particular instance of the concept. *Effectiveness* is between a 3 and a 4 rating. The *effectiveness* criterion is not descriptive about high quality because the scale is skewed. A rating of 4 is simply *average* not *excellent*. Additionally, it is difficult to legitimately rate effectiveness in the absence of an actual user, authentic interactions, and additional data. Items were rated as levels 3 and 4 unless they were very confusing, misleading, or incorrect. There were some instances of incorrect and misleading information, but the majority of items were understandable.

Acquisition is just below 3 for all subjects. Although a 3 appears to be quite a high rating, the scale reveals that is not the case. A rating of 3 for *acquisition* means that most items were simple exercises that did not require much advanced thought. Knowledge transfer likely requires more advanced capability.

Lastly, *TEKS correlation* is rated near a 3 for all subjects. This is a high rating. It was near impossible for items to earn a 4 rating due to the density of each of the TEKS. Because a rating of 4 was rare for this criterion, a rating of 3 is closer to the top of the scale than it may seem. However, simply covering components of the TEKS does not mean that depth was achieved. It also does not account for holistic coverage of the TEKS. Maybe it is difficult for one item to cover an entire TEKS objective, but several items can wholly cover an individual objective. That aspect was not measured. More data collection would be necessary to better address TEKS coverage.

Overall, the McGraw-Hill textbooks were rated on the higher side for *role*, *effectiveness*, *acquisition*, and *TEKS correlation*. The four higher-rated qualities are either less informative or have a skewed scale causing them to be less favorable than they seem. A low overall rating for *prominence* is also not necessarily a negative quality due to the necessary balance between more and less distinct items. The three most revealing quality criteria are *context*, *uniqueness*, and *teacher guidance*. The overall rating for each of these categories is low indicating that the McGraw-Hill textbooks are lacking with regard to supporting the teacher in developing student confidence and capability to transfer knowledge related to analyzing key features of graphs of functions.

Other Notable Findings

During the review of the three textbooks, several observations were noted. Some notes directly related to study topics and rating criteria, but others did not. Any notes that seemed to recur are discussed in this section. Although some findings are beyond the scope of this study, they might provide direction for future curriculum investigation.

Lab activities. The calculator activities seem to be a way to help students build connections, especially among representations. One complication with the calculator labs is the inconsistency in calculator model used. In all the three textbooks, the type of calculator used in the labs and in some of the examples changes from one lesson to the next. For example, on page 55 of the Algebra 1 textbook, a lesson is designed for the TI N-spire calculator. Later in the same textbook on page 169 the calculator is switched to the TI 83/84 Plus. Similarly, on page 263 of the Algebra 2 textbook, the TI-Nspire CAS calculator is used. Ten pages later on page 273, a lesson is built around the TI 83/84 Plus calculator without mention of the change. This pattern continues in the Precalculus textbook as well. All the labs were not examined, but at least three different calculator models were observed in the few labs that were reviewed. This is problematic because different calculators have different keystroke and screen images for similar actions. It can be confusing for new users to adapt directions for a model calculator different than the one being used in class. Because of the inconsistency in of type of calculator used in the textbook activities and the difficulty adapting lessons and directions, it is also likely that teachers will choose to ignore the calculator labs altogether.

In line with the topic of labs, is the observation that labs can be critical in teaching and learning of the TEKS. For example, interval notation is an explicit part of the Algebra 2 TEKS. Although it is sporadically used in examples, key concepts, and solutions it is only partially taught in one location, an Algebra Lab lesson. Although it is not a calculator lab, it does have the word lab in the title. If teacher views labs as optional, it is quite likely that interval notation will not be taught. Additionally, even if the lab is used, it only aims to develop interval notation for describing the solution set for simple and compound inequalities. Aside from one standardized test practice item after the interval notation lab, occurrence of the notation is missing until it suddenly appears again 63 pages later in a key concept box describing the range of an absolute value function. There is no explicit mention of the notation or its connection to other notations or the function itself. Notational fluency is one of the mathematical practices College Board recommends for AP Calculus. By not adequately addressing interval notation in the Algebra 2 textbook, McGraw-Hill is not fostering this practice.

Other inconsistencies. Similar unsystematic occurrences of mathematical notation appear throughout the series of textbooks, although the Precalculus textbook appears to make a more concerted effort to develop understanding of notation and maintain consistency. In the Algebra 1 textbook, domain is first introduced and consistently presented as discrete set of values that can easily be individually listed. Without warning or explanation in Chapter 3, domain and range are suddenly represented using inequality notation. Additionally, the inequality notation is inconsistent. For example, in some cases something similar to $4 < x < \infty$ is used when other times it is simply written as $x > 4$. While it can be argued that students must learn to understand all notations, the inconsistencies and lack of explanation make meeting that expectation more difficult.

Furthermore, in Algebra 1, students are initially presented with a fixed table of values and asked for the domain and range. The answer for domain is simply a list of the x-values from the table, while the answer for range is a list of the y-values. When lines are introduced, there is a subtle unstated shift that is critical to student understanding. The line is not drawn between two points simply because that is the procedure. McGraw-Hill implies a procedural approach to lines and does not work to sufficiently develop the idea that a line is a collection of infinitely many points. When a table is generated from a linear equation, it is not fixed. There are an infinite number of points that the table could contain and this affects the domain and range. Because of this noted lack of development, it is sudden and confusing when it no longer suffices to simply list the numbers from a table when asked for domain and range.

Misleading and misinformation. Also on the topic of domain and range is a repeated error throughout the Algebra 2 textbook. In numerous places, the textbook attempts to write domain in range using multiple notations. In most of these instances, the variable x is used for range. This may seem to be a small error, but it occurs on a large scale throughout the textbook and can be confusing for inexperienced readers.

Other misconceptions seem to be carelessly propagated throughout the series. It seems that a lot of repetition was used to fill the space without purposefully including variety that might dispel misconceptions. For example, when finding domain and range of linear functions, almost every item resulted in a domain between zero and the x-intercept and a range between zero and the y-intercept. For identifying x and y-intercepts in a table of values, every item including the additional examples, had the zeros positioned in near identical locations in a similarly sized and formatted table. This kind of haphazard collection of material is likely to be counterproductive to the development of critical thinking skills required for problem solving and knowledge transfer.

Missed opportunities. Although many problems seemed to miss opportunities for encouraging deeper level of thinking, a couple of items stand out above the others. The TEKS explicitly state that students should determine reasonable domain for real-world situations. Several problems and examples include real-world context but do not ask for a reasonable determination of domain or range. Instead the domain is given and the corresponding range is expected. This type of question fails to provide opportunity for making meaningful connections.

Another notable omission is a particular mathematics and physics connection related to projectile motion. The textbook simply states that the coefficient of the x^2 term is always -16 for projectile motion instead of exploring the meaning of the value. Furthermore, -16 is used mostly in mathematics classrooms for projectile motion; however, science classes typically work in SI units and use -9.8. Because of this difference in instruction, it is even more imperative that students and teachers be alerted to the connection, otherwise, they may never independently make the connection.

Continuous and discrete. Continuity was not a focus of this study but it was observed to be a possible area of developmental deficiency. The difference between the terms continuous and discrete was first addressed in the Algebra 1 textbook as follows:

> A graph that consists of points that are not connected is a discrete function.
>
> A function graphed with a line or smooth curve is a continuous function. (Carter, Cuevas, Day, & Malloy, Algebra 1, 2016)

The imprecise description may be acceptable as a simple introductory statement, but it leaves room for misconception. First, graphs of disconnected points do not have to be discrete functions; they could be discrete relations. Also, a function graphed with a line or smooth curve is continuous, but it is not the only type of continuity. Continuous graphs could have sharp turns; they do not have to be smooth curves. Additionally, there is no image that follows the description to further clarify the concept, leaving the reader to envision meaning of the new concept.

Immediately following the descriptions for continuous and discrete in the Algebra 1 textbook, there is a real-world example that attempts to address continuity. It is about an ice-sculpting contest and provides a table of five pairs of values for team number and height of sculpture. Part d of the question asks if it is continuous or discrete, and provides the answer that it is discrete because the points are not connected. In part d, there is no justification or explanation as to why the points are not connected. The explanation is included in a more lengthy block of text one line above part d where it is less likely to be found and connected with the answer to part d.

In the Algebra 1 textbook, there is another example problem that confuses the concepts of discrete and continuous. The real-world problem is about a girl attending a local carnival. An equation that represents the amount of money the girl has remaining after each ride is given. The problem asks for the solver to find the zeros, domain, and range. Additionally, the problem asks for an interpretation of meaning and significance of the answers. This example provides a continuous graph as part of the solution process while simultaneously stating that the number of rides is limited to only particular values because partial rides are not possible. Immediately after making the statement about partial rides, the solution for domain appears to be given as $\{0 \leq r \leq 26\}$. There is a note that follows stating that *r* must be an integer, but the note is not emphasized and is easily overlooked, especially considering the proximity to the continuous graph. Money is not discussed, but partial amounts of pennies are also not possible. This limitation is not noted for the range.

Further carelessness results in unexpected and wrong information. In an effort to address the difference between continuous and discrete domains, an item in the Algebra 1 textbook states, "the domain can be any real number in the range," (Carter, Cuevas, Day, & Malloy, Algebra 1, 2016). The context of range in this case is not clear and is not expounded on further. The Algebra 1 textbook repeatedly has issues with clarity on topics of discrete and continuous.

In the Algebra 2 textbook, the development of continuity is improved. The description for discrete and continuous is accompanied by a graph of each and is less vague:

> A relation in which the domain is a set of individual points, like the relation in Graph A, is said to be a discrete relation. Notice that its graph consists of points that are not connected. When the domain of a relation has an infinite number of

> elements and the relation can be graphed with a line or smooth curve, the relation is a continuous relation. (Carter, Cuevas, Day, & Malloy, Algebra 2, 2016)

The items that follow the description of discrete and continuous are a mixture of algebraic equations and contextual situations that ask the solver to determine if the items are continuous or discrete. Most of the items convey the concept well, but there are a couple that could offer more discussion or explanation. For example, there is an exercise asking for domain and range of a realistic situation involving the weight of a bag of apples and the cost. Although cost is an inherently discrete variable, the textbook provides an answer that suggests it is continuous. Because it is not expected to cause confusion for this exercise or other similar items, it may be acceptable to present cost as continuous at the Algebra 2 level. While it may momentarily be simpler, it does not help advance toward a complete understanding of continuity or a major concept calculus, infinitesimally small increments.

Following the extremely ambiguous Algebra 1 and very basic Algebra 2 concept development, the Precalculus textbook assumes a solid understanding of the term discrete. There is one simple reminder in a callout box in the margin that warns teachers to remember that the domains of discrete functions cannot be described by an interval and must be composed of individual points, but there is nothing for students. A couple of pages after the warning, the textbook simultaneously drops limits, various types of discontinuity, and the continuity test within a page and half. The concepts are weighty and presumably need more careful development; however, immediate use of the limit concept for determining continuity is expected in the first example with minimal explanation. Additionally, no explanation is provided to justify why visual inspection of the graph is not the chosen method for determining continuity.

The development of concepts related to continuity, seems to be weak and fairly basic in the in the two Algebra textbooks. Without warning the Precalculus textbook suddenly increases expectation and rushes through advanced concepts. Furthermore, the TEKS do not require that limits be introduced and imply a more graphical approach supported by calculator use for understanding the various types of discontinuities.

Sections in isolation. Although there were some connections made from topic to topic, the impression is that it was not done frequently enough to mend the individual sections of the textbook into a cohesive unit. Overall, the sections of the textbook were presented in such a way that each task appeared to be something completely new – a new set of rules to learn – instead of another method for flexible consideration of the same concepts. For example, there is a section of the Algebra 1 textbook that begins with a definition of slope intercept form of a line. Although direct variation and the linear function written in $y = mx + b$ form were developed in different ways in previous sections, absolutely no connection is made between the prior lessons and the representation of interest. Instead of noting that equations were previously found by observing patterns and that slope intercept form is a way to generalize that process, the textbook ignores it. Additionally, the textbook developed the direct variation equation using examples that emphasized building equations in specific systematic ways. For example, distance is found by multiplying rate and time. This idea could have more explicitly served as the foundation for slope-intercept form, but it was not mentioned again after the section in which it was presented.

Methods for graphing lines were similarly segregated. One section of the textbook states that a line can be graphed by plotting the x-intercept and the y-intercept, then connecting the two points. Not only does this ignore the development of continuity, it also is never connected to other methods of graphing lines. When graphing using slope and y-intercept is discussed, the textbook presents it as the singular method for graphing. Equations that were previously graphed with intercepts suddenly must be put into slope intercept form. Because no explicit connection is made between the methods, there is also no mention of how to determine which method would be best for various circumstances.

Another example of this type of sterile isolation of topics can be seen in the Algebra 2 textbook. For example, Algebra 2 covers various ways of finding solutions to quadratic equations without clarifying the connection. In section 4-2 of the Algebra 2 textbook, the lesson covers solving quadratic equations by graphing. In section 4-3 of the same textbook, the lesson covers solving quadratic equations by factoring. The examples for the factoring lesson do not indicate that there is any connection to the previous methods or why one method might be preferred or chosen over another.

Favorable findings. Although there are numerous issues identified with the McGraw-Hill series, there are also a few positive remarks. For Algebra 1, the items frequently asked for interpretation, verbalization, or explanation of mathematical concepts. These kinds of items are significant in developing connections between mathematics and meaning in the surrounding world. Unfortunately, the frequency of this type of item diminished in Algebra 2 and almost disappeared in Precalculus. This was surprising, since it seems that interpretation would be more frequently expected in Precalculus.

The Precalculus textbook excelled at providing more consistency and providing notes for new teachers. The notes for new teachers include valuable tidbits that might not be known or might be overlooked by inexperienced teachers. For example, there is a note reminding new teachers that there is a possibility of extraneous solutions when solving radical equations. This feature is unique to the Precalculus textbook and was not reflected in the ratings of this study because many of the tips for new teachers were off topic for the study. Nonetheless, they are a positive feature of the Precalculus textbook. Another part done well in the Precalculus textbook is the chapter planner pages at the onset of each chapter. They provide a lesson planning guide, an intervention guide, a list of differentiation ideas with in depth descriptions, and a chapter overview. The contents of the preliminary chapter pages in the Algebra 1 and Algebra 2 textbooks are more limited. For the two lower level textbooks, the useful information consists of a skills trace and a pacing suggestion. The skills trace is a list of TEKS identifying what was previously learned, what will be targeted, and what will happen next. Very frequently, especially in Algebra 2, the TEKS listed in the skills trace are completely inaccurate. Many of the TEKS in the Algebra 2 skills trace are not even recognizable as Algebra 2 TEKS. The preliminary chapter pages for Precalculus are much more useful.

Limitations, Implications, and Suggestions

This study involves a first-level analysis of specific contents of a new series of mathematics textbooks. It was limited to textbooks published by a single publisher and only included particular topics from each textbook. About a 100 or fewer pages from each textbook were reviewed using an instrument created for the study, and personal bias could play a role in evaluation. The rating instrument employed in this study was developed by the author and the research mentor within a limited timeframe that did not allow for a rigorous validation process; however, the instrument was considered to be suitable for the purpose of this study.

The study was further limited to topics relating to analyzing graphs of functions that are highly likely to be used in other disciplines and STEM programs. The study topics are specifically domain and range; minima, maxima, and vertex; slope and increasing or decreasing intervals; and, intercepts and zeros. The topics of end behavior and asymptotes are excluded. Additionally, this study did not analyze topics associated with trigonometric functions or topics more closely related to statistics like curve fitting. This decision was made because of time constraints and/or lack of occurrence in some of the chosen course TEKS.

Future investigations can provide additional useful information on factors like completeness of TEKS coverage and cognitive demand of tasks. Also, future studies might like to evaluate and to compare other traditional and non-traditional textbooks with the McGraw-Hill textbooks. An experimental study on human subjects can also give more insight into the effectiveness of the textbook items and nature of teacher interactions with the textbooks. Although much more work can be done to gain further detail and information, this initial review provides sufficient evidence to justify the conclusions of this study.

When it comes to analyzing key features of functions and preparedness for knowledge transfer, it is unlikely that McGraw-Hill can independently provide enough support for teachers. Although there is a significant emphasis on multiple representations to help develop understanding within the context of mathematics, there is not a sufficient number of meaningful real-world and interdisciplinary connections in the chosen series. Additionally, other observations and measures of quality indicate the series is deficient.

The results suggest that school districts using the McGraw-Hill series of textbooks as a primary teacher resource provide additional supplemental resources that will more purposefully develop thinking and connections to the real-world. Additionally, districts may also offer professional development opportunities focused on adapting curriculum to emphasize connections and deeper learning.

Furthermore, teachers are cautioned to be aware of misleading and misrepresented information in the textbooks. When it comes to developing notational fluency, it is advised that teachers work to find or develop a plan for filling the developmental gaps. Students are not likely to understand the notational idiosyncrasies and effectively use symbolic representations without a more intentional focus on the symbols and their meanings.

VI. Summary and Conclusions

The intention of this study was multifaceted bringing together separate ideas from the research literature. Research findings about the role of textbooks in mathematics classrooms, student lack of college and career readiness, low attraction to and retention in STEM programs in the United States, and student difficulty with knowledge transfer, influenced the design of the textbook study. The study had two main goals. It aimed to identify quantity and types of connections found in a particular high school series of math textbooks and to evaluate the extent to which the series can support the teacher in emphasizing connections related to analyzing graphs of functions.

The 2016 Texas versions of Algebra 1, Algebra 2, and Precalculus by McGraw-Hill publishing were chosen for evaluation. All textbooks were reviewed sequentially for types of connections and quality of content relating to specific topics involved in the analysis of key features of functions. In particular, domain and range; minima, maxima, and vertex; slope and increasing or decreasing intervals; and, intercepts and zeros were the only topics examined.

The design decisions for this study were directly influenced by the research literature. A textbook study was selected because of related work indicating that the textbook is a key factor in determining content taught in the mathematics classes. If teachers influence student learning, then the contents of the textbooks that influence teachers will have an impact on classroom instruction. Topics related to analysis of key features of functions were selected due to the research literature pointing to student difficulty understanding functions and literature suggesting student inability to transferring their function related concept knowledge to other disciplines. In particular, domain and range; minima, maxima, and vertex; slope and increasing or decreasing intervals; and, intercepts and zeros were the only topics examined. Not only are these topics related to the concept of function but also they are likely to be used in STEM applications and would require transferring mathematics knowledge to a different context. Usefulness in STEM applications was considered in topic selection because findings in the research literature indicate that there may be a shortage of STEM workers in the future. The research literature implies that a weak mathematics foundation that does not emphasize knowledge transfer is part of the reason many students are discouraged from STEM programs. Examining connections seemed to be a way to predict the textbook series' ability to support teaching and learning practices that are likely to promote improved knowledge transfer. Examining topics related to analyzing function seemed to be a way to incorporate concept knowledge that would impact STEM involvement.

In addressing the first research question about the number and types of connections that can be found in each textbook of the McGraw-Hill series, all types of connections including, level-to-level, topic-to-topic, among representations, real-world, interdisciplinary, procedures-to-concepts, and language connections were considered. Although knowledge transfer implies use of mathematics in other disciplines, connections within math and to language were deemed important for building useful concept knowledge. Sections of the textbooks were selected for evaluation based on the TEKS standards relating to the chosen study topics. Items from each section were categorized and tallied by category type. Results show that there are many connections among various representations of functions but meaningful connections to the real-world and across disciplines are minimal, seeming to decrease in number as the series progresses.

Quality of items was judged based on a rating tool developed for this study. The rating instrument was originally modeled after other textbook studies found in research literature, but evolved and changed as needed during the study. The final rating instrument was composed of nine criteria that were rated on either a 4-level or 5-level scale. The nine criteria are as follows: *mode of conveyance*, *prominence*, *role*, *effectiveness*, *context*, *uniqueness*, *acquisition*, *teacher guidance*, and *TEKS correlation*. Although data was collected for all nine criteria, reflection on the data and the evaluation instrument indicates that the data from three of the criteria, *context*, *uniqueness*, and *teacher guidance*, is more meaningful in determining potential to support teachers in emphasizing function related connections. The results of the quality evaluation indicate that the series has low potential for helping teachers provide necessary opportunities to transfer knowledge. In addition to the data, other observations also identify weaknesses in the McGraw-Hill series.

Overall, this series is not likely to independently advance students to a higher level. It is unknown how the potential of this series compares to other series. Knowledge of the relative quality of the series would provide further insight.

Textbooks are often used as a guide for many classroom activities, including teacher preparation, planning, lesson content, and even evaluation. When knowledge for teaching mathematics falls short or questions and disagreements arise in mathematics classrooms or among teachers, the information from the textbook is likely accepted as accurate and appropriate. When organizing lessons, it is often taken for granted that the topics in the textbook will effectively build to create a well-connected and well-developed concept image. This curriculum study casts doubt on the sense of comfort provided by the textbook.

Not only was some information observed to be incorrect and misleading, it was also noted that there were significant gaps in concept development of particular topics. The McGraw-Hill textbooks were also rated at a low level for most of the quality criteria of this study indicating deficiencies in all textbooks of the series. Teachers must be aware and cautious of these disadvantages of the textbook. Although this study examined only one series of textbooks, deficiencies can occur in any textbooks.

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Appendix A:
Full Size Graphs Showing Patterns in Collected Quality Data

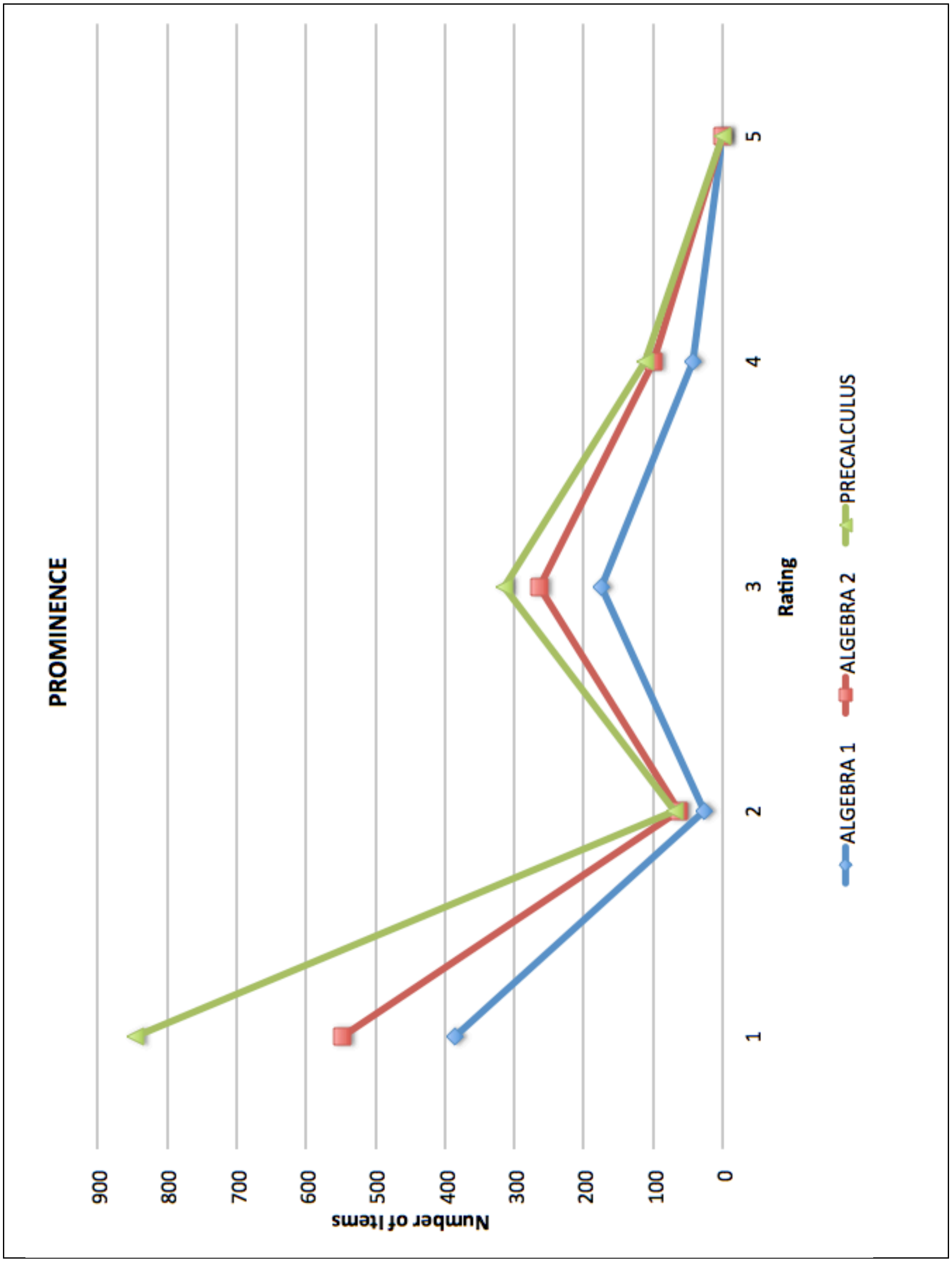

Figure A1. Patterns in *prominence* criterion data for Algebra 1, Algebra 2, and Precalculus.

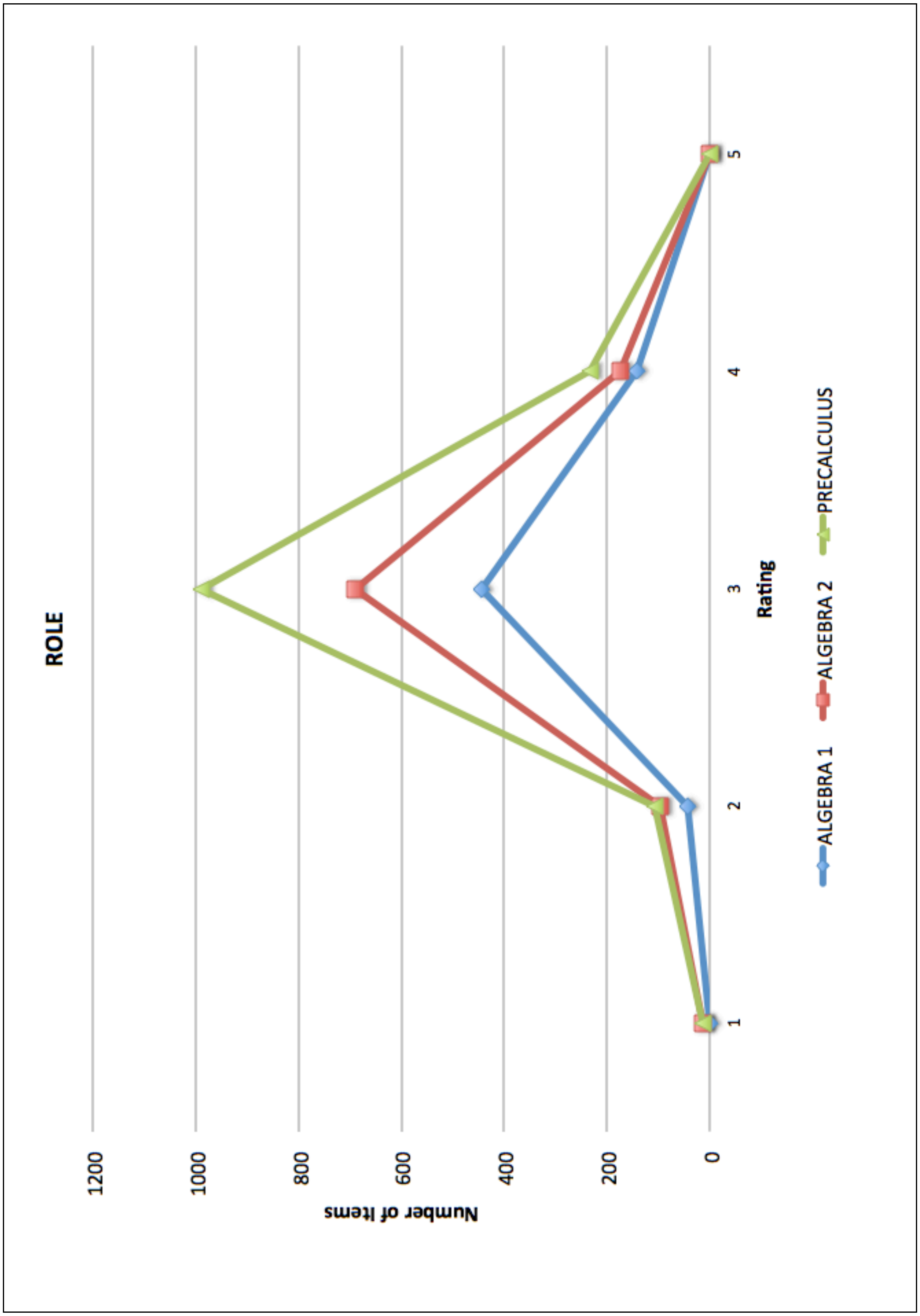

Figure A2. Patterns in *role* criterion data for Algebra 1, Algebra 2, and Precalculus.

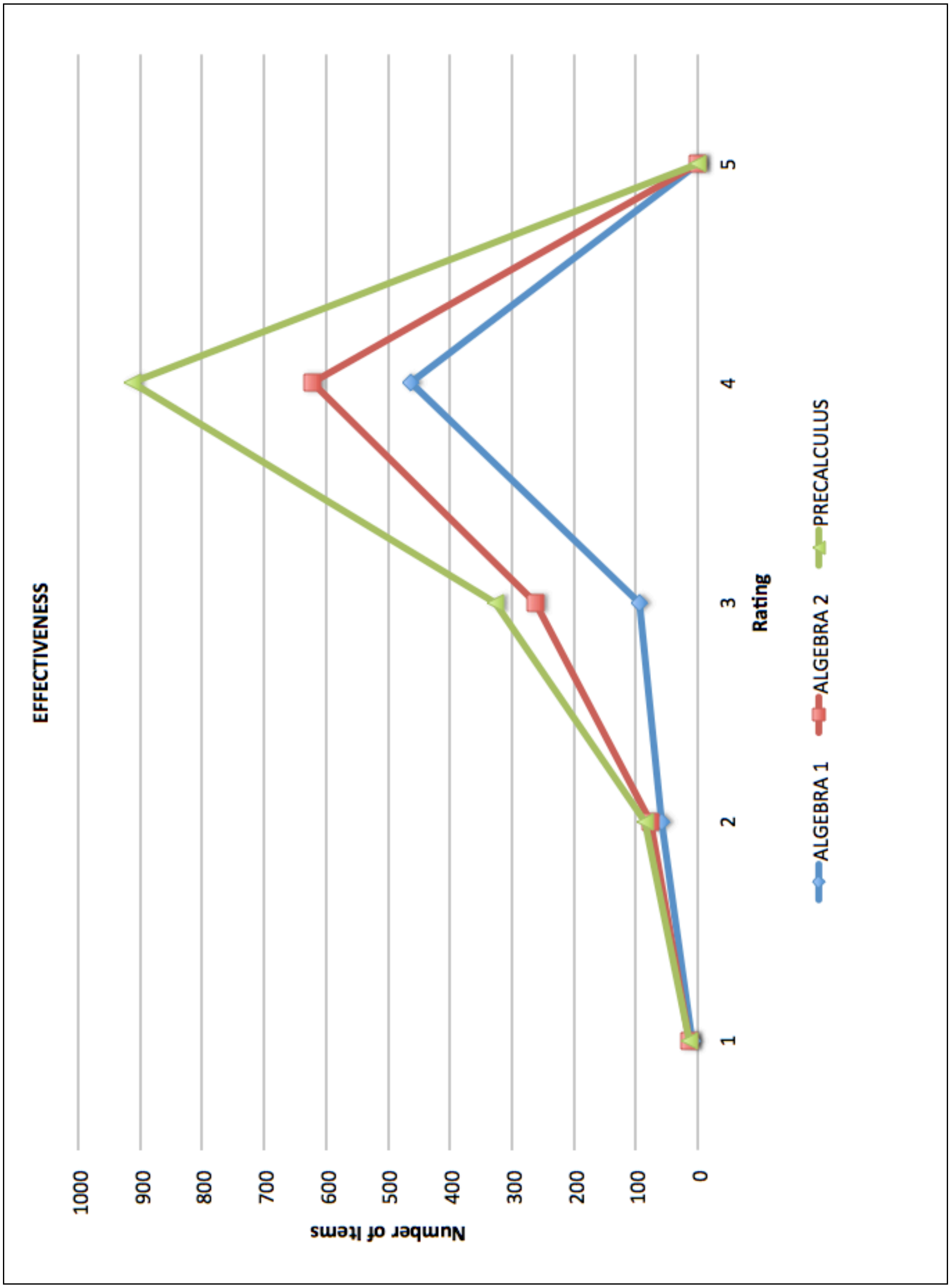

Figure A3. Patterns in *effectiveness* criterion data for Algebra 1, Algebra 2, and Precalculus.

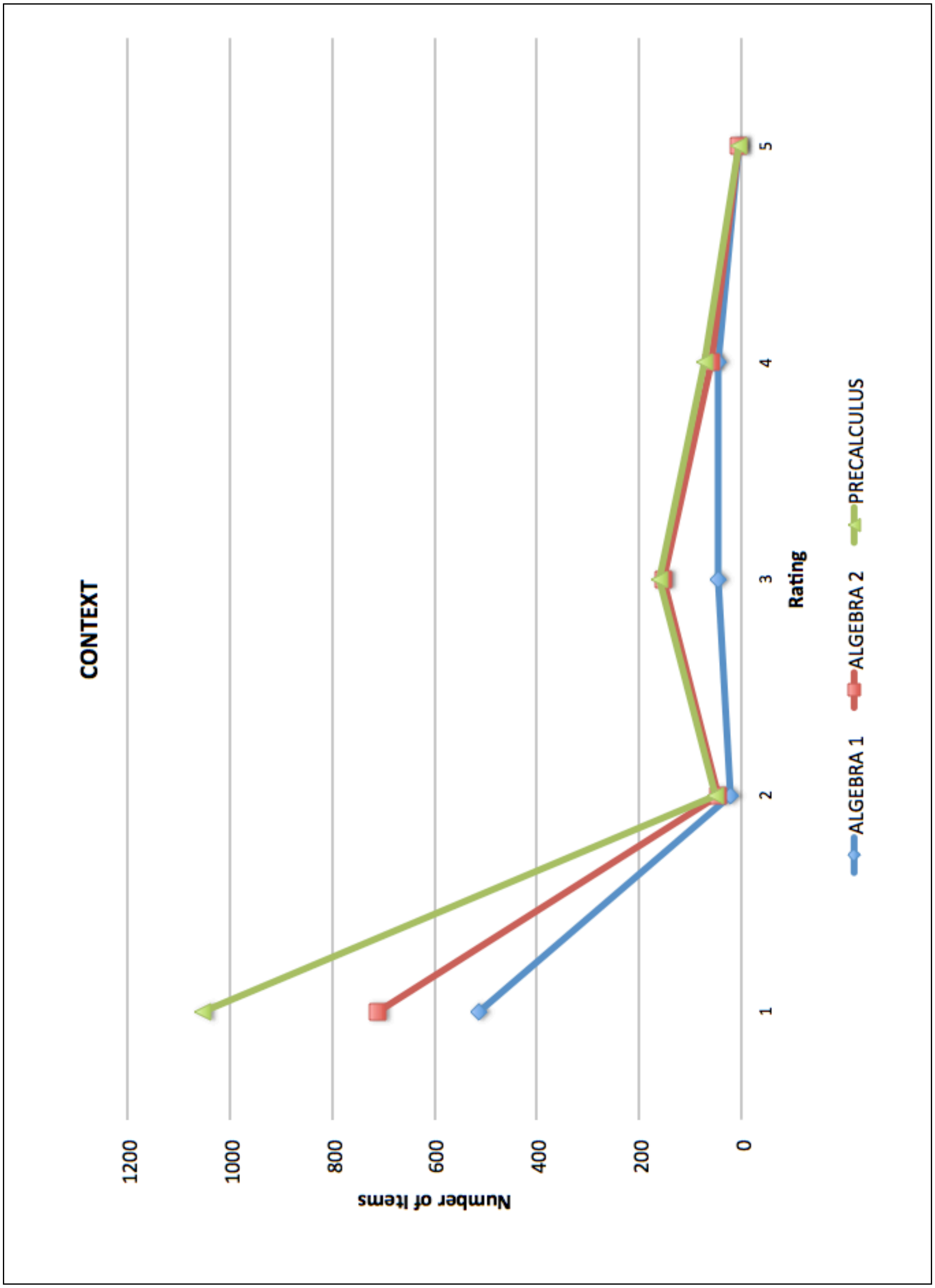

Figure A4. Patterns in *context* criterion data for Algebra 1, Algebra 2, and Precalculus.

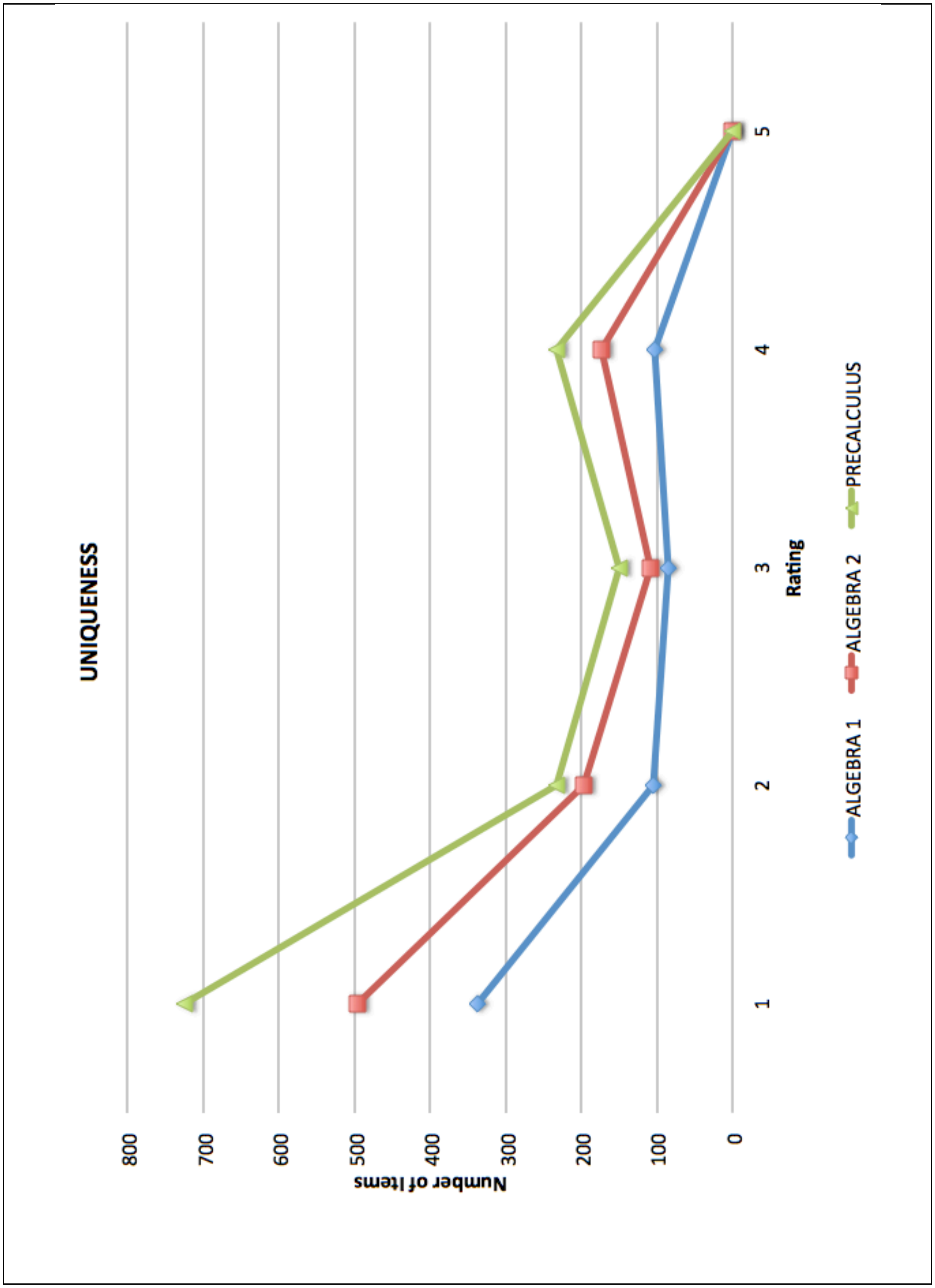

Figure A5. Patterns in *uniqueness* criterion data for Algebra 1, Algebra 2, and Precalculus.

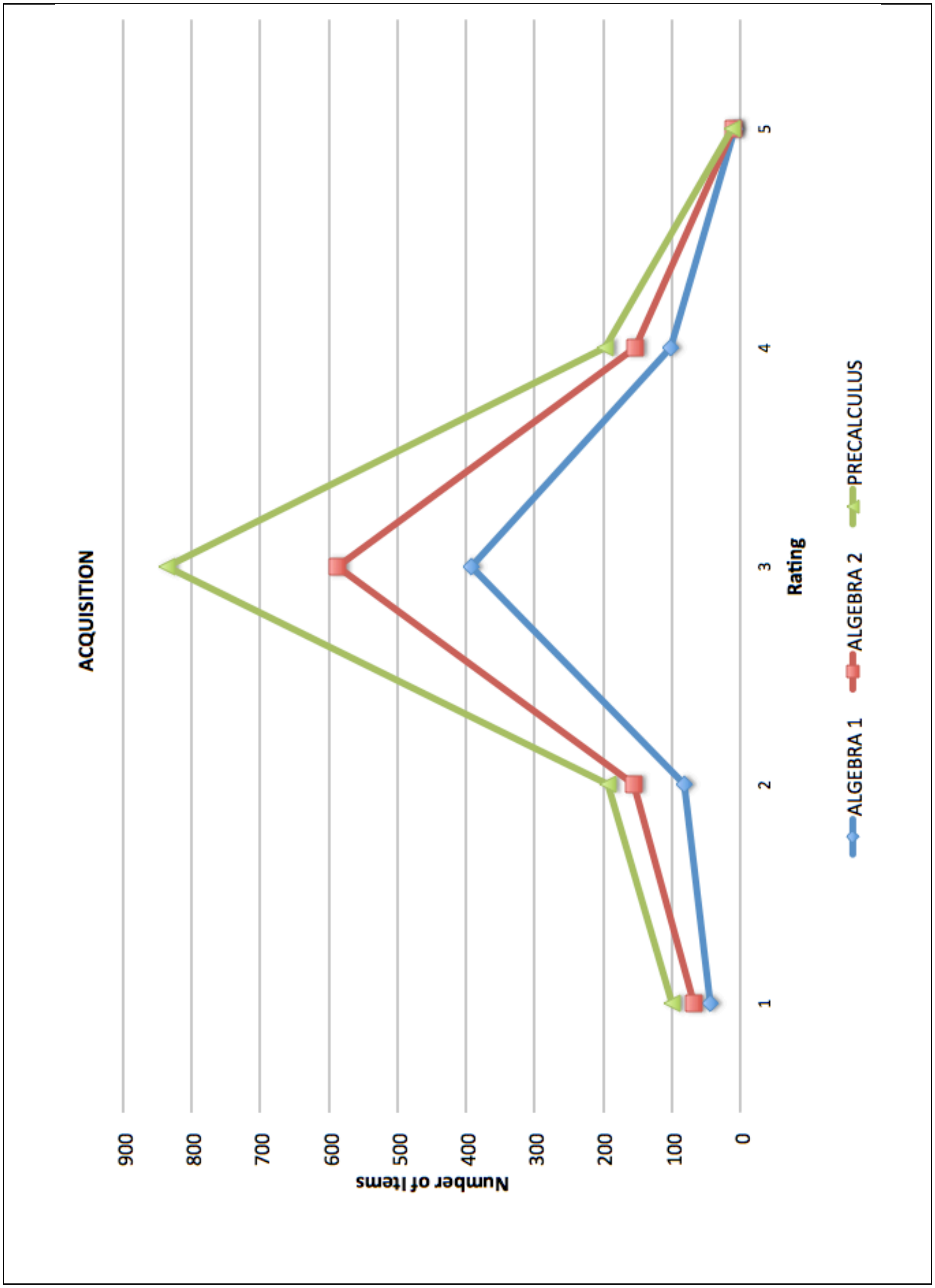

Figure A6. Patterns in *acquisition* criterion data for Algebra 1, Algebra 2, and Precalculus.

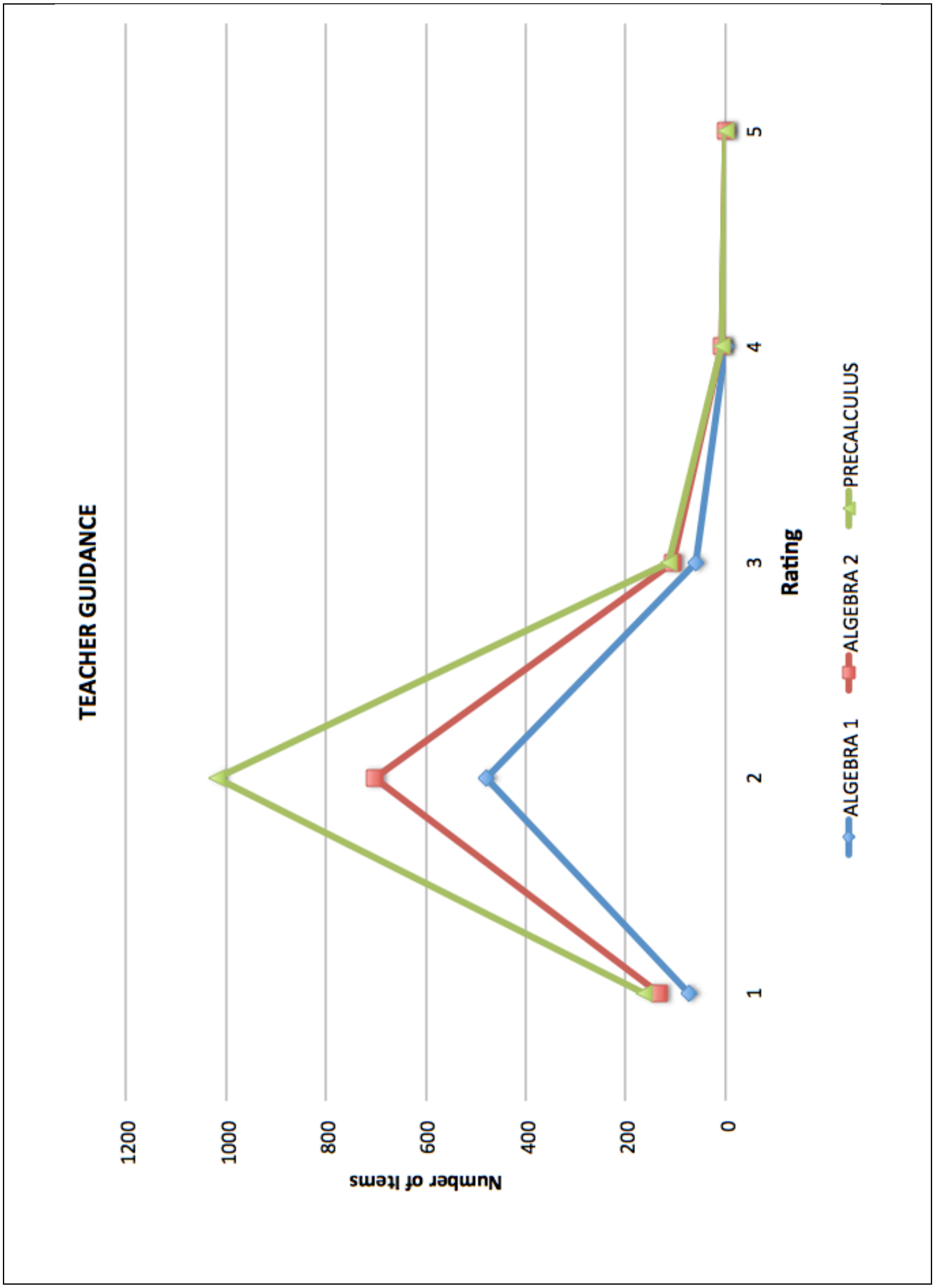

Figure A7. Patterns in *teacher guidance* criterion data for Algebra 1, Algebra 2, and Precalculus.

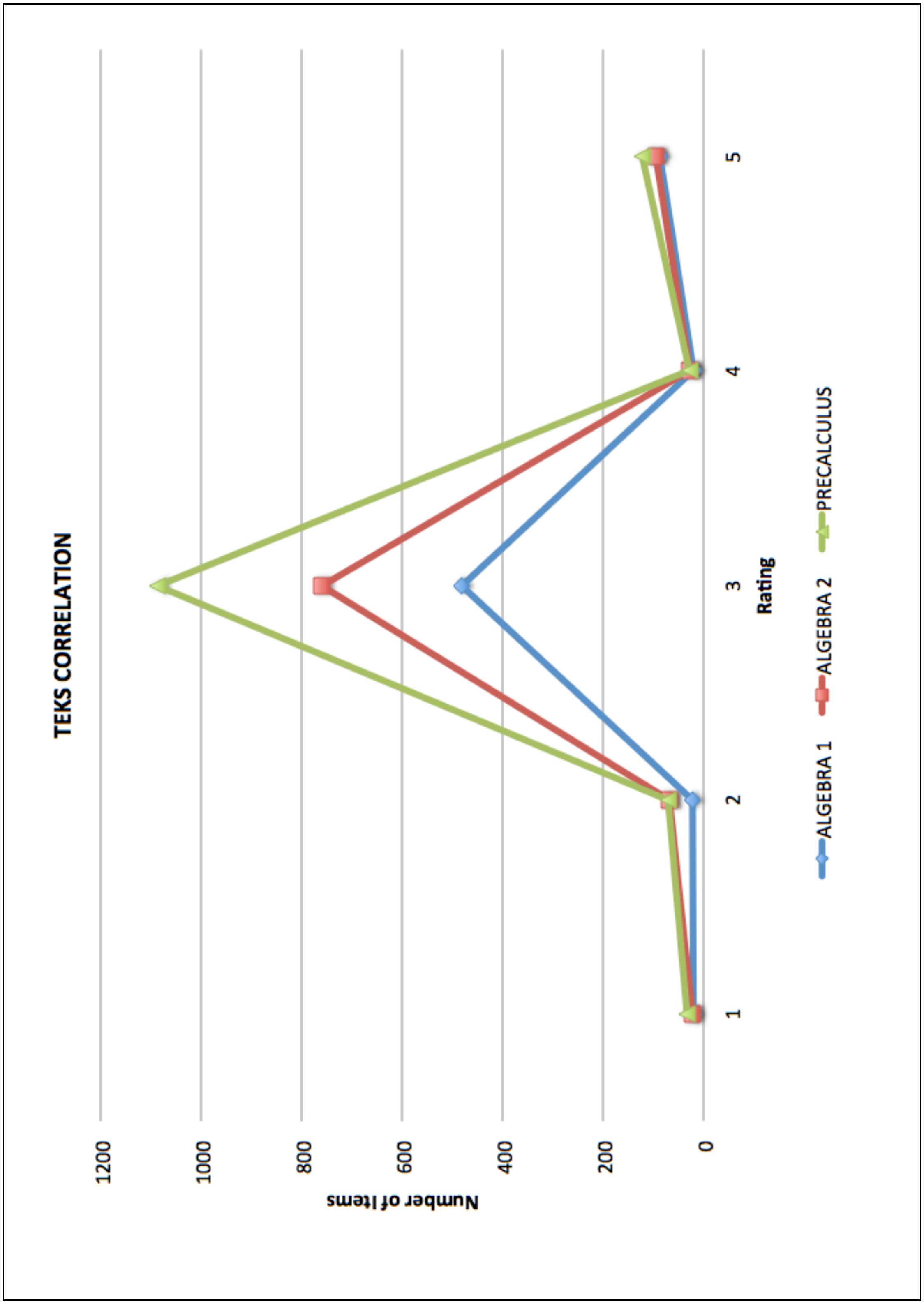

Figure A8. Patterns in *TEKS correlation* criterion data for Algebra 1, Algebra 2, and Precalculus.

Appendix B:
Comparison of Ratings by Rater A and Rater B

Table B1

Ratings and percent of matches for 25 items evaluated by both raters for four criteria: context, uniqueness, teacher guidance, and acquisition

Item	Context			Uniqueness			Teacher Guidance			Acquisition		
	Rater A	Rater B	Match	Rater A	Rater B	Match	Rater A	Rater B	Match	Rater A	Rater B	Match
1	1	1	1	4	4	1	3	3	1	2	2	1
2	1	1	1	2	2	1	1	1	1	2	2	1
3	1	1	1	3	4	0	2	1	0	3	3	1
4	1	1	1	4	4	1	2	2	1	4	4	1
5	1	1	1	4	4	1	2	2	1	4	4	1
6	1	1	1	2	2	1	2	2	1	3	3	1
7	1	1	1	2	3	0	2	2	1	3	3	1
8	1	1	1	4	4	1	2	2	1	3	3	1
9	1	1	1	2	2	1	3	3	1	4	4	1
10	1	1	1	4	4	1	2	2	1	4	4	1
11	1	1	1	4	4	1	2	2	1	2	2	1
12	1	1	1	4	4	1	2	2	1	3	2	0
13	1	1	1	2	2	1	2	2	1	3	3	1
14	1	1	1	3	3	1	1	1	1	2	2	1
15	1	1	1	4	4	1	2	2	1	2	2	1
16	1	1	1	2	2	1	2	2	1	3	3	1
17	1	1	1	4	4	1	2	2	1	4	4	1
18	1	1	1	2	2	1	2	2	1	3	4	0
19	1	1	1	2	2	1	1	1	1	2	2	1
20	1	1	1	2	2	1	2	2	1	3	3	1
21	1	1	1	2	2	1	2	2	1	3	3	1
22	1	1	1	2	2	1	2	2	1	3	3	1
23	1	1	1	2	2	1	2	2	1	3	3	1
24	1	1	1	2	2	1	2	2	1	3	3	1
25	1	1	1	2	2	1	2	2	1	3	3	1
	Number of matches		25	Number of matches		23	Number of matches		24	Number of matches		23
	Percent Matching		100%	Percent Matching		92%	Percent Matching		96%	Percent Matching		92%

Table B2

Ratings and percent of matches for 25 items evaluated by both raters for four criteria: role, effectiveness, prominence, and TEKS correlation.

Item	Role			Effectiveness			Prominence			TEKS Correlation		
	Rater A	Rater B	Match	Rater A	Rater B	Match	Rater A	Rater B	Match	Rater A	Rater B	Match
1	4	4	1	3	3	1	3	3	1	3	3	1
2	4	4	1	3	3	1	3	3	1	3	3	1
3	3	3	1	2	2	1	1	1	1	3	3	1
4	4	3	0	4	4	1	3	3	1	3	3	1
5	4	4	1	4	4	1	1	1	1	3	3	1
6	4	4	1	4	4	1	1	1	1	3	3	1
7	3	3	1	4	4	1	3	3	1	3	3	1
8	3	3	1	3	3	1	1	1	1	3	3	1
9	4	4	1	4	4	1	4	4	1	4	4	1
10	3	3	1	4	3	0	3	3	1	3	3	1
11	3	3	1	3	3	1	3	3	1	3	3	1
12	3	3	1	3	4	0	3	4	0	3	3	1
13	3	3	1	4	3	0	1	1	1	3	3	1
14	4	3	0	4	3	0	3	3	1	3	3	1
15	3	3	1	3	3	1	3	4	0	3	3	1
16	3	3	1	4	4	1	1	2	0	3	3	1
17	4	3	0	4	4	1	3	3	1	3	3	1
18	3	3	1	4	4	1	1	1	1	3	3	1
19	3	3	1	3	3	1	3	3	1	3	3	1
20	3	3	1	4	4	1	4	4	1	3	3	1
21	3	3	1	4	4	1	2	2	1	3	3	1
22	3	3	1	4	4	1	1	1	1	3	3	1
23	3	3	1	4	4	1	1	1	1	3	3	1
24	3	3	1	4	4	1	1	1	1	3	3	1
25	3	3	1	4	4	1	1	1	1	3	3	1
	Number of matches		22	Number of matches		21	Number of matches		22	Number of matches		25
	Percent Matching		88%	Percent Matching		84%	Percent Matching		98%	Percent Matching		100%