MATHEMATICAL VOCABULARY: A LOOK BACK TO MAKE MEANING OF THE PRESENT AND THE FUTURE

A Dissertation

by

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This dissertation meets the standards for scope and quality of
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ABSTRACT

This dissertation researched the body of scholarly work that addresses mathematical vocabulary in the United States in order to identify trends, major topics, and gaps in the research that have not been previously explored, which define the field. The body of work extends from the seminal book by Davies and Peck (1855), Mathematical Dictionary and Cyclopedia of Mathematical Science, to the present day.

The research methodology included an historical analysis combined with a case study. The historical analysis examined the 126 scholarly works found as a result of an exhaustive search of all works addressing mathematical vocabulary in the United States. The case study involved four participants who were active mathematics education professors at the time of the study. The case study determined the views of current mathematics education vocabulary experts, how they interpreted and made meaning from the history of the field, and how they situated their works into the field of mathematical vocabulary. This study examined the historical analysis and the case study separately, and then in combination.

Several major themes emerged from this research, including mathematical vocabulary implications that address instructional strategies, issues with English language learners, and discourse. These dominant themes arose from both the historical analysis and the case study.

The researcher defined the field of mathematical vocabulary, including the major categories and themes of scholarly works, and has thus created a foundation for future scholarly endeavors. The results of this study provide a clear map of the gaps in the research and offer future researchers a treasure of research opportunities. This study also serves as a rich source of information for practitioners seeking insight into the teaching of mathematical vocabulary. An

abundance of works exist that pertain to instructional strategies for addressing the needs of English language learners.

DEDICATION

I dedicate this dissertation to my husband, Abelardo, my children, Christine and Jere, and my grandchildren, as well as my parents and siblings. I am forever grateful for your unconditional love and support during the entire doctoral process. There were many sacrifices made which allowed me to pursue my degree, the greatest of which was time spent with family. Thank you Mom and Dad; you always encouraged me to pursue my dreams in all avenues of my life. Abelardo, I cannot thank you enough for loving me and listening to me both when I struggled and when I celebrated. It is a certainty that without your support I could not have undertaken or completed the doctoral journey. ¡Tu es mi gran amor!

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CHAPTER I: INTRODUCTION

For more than 100 years, there has been scholarly work devoted to mathematical vocabulary, as evidenced by the publication of *Mathematical Dictionary and Cyclopedia of Mathematical Science* by Davies and Peck in 1855. These writings have focused on a variety of different topics and have caused controversy over the understanding of mathematical vocabulary instruction and its effects. The amount of writing related to mathematical vocabulary is limited in comparison to other aspects of mathematics teaching and learning; to date, not one has identified the major topics and trends that have emerged over time. There is a need to bring clarity to the understanding of the field of mathematical vocabulary.

Vocabulary in English language arts has been studied extensively and has led to an understanding of vocabulary in general but not specific to the content area of mathematics. Petty, Herold, and Stoll (1967) noted the overall importance of vocabulary:

> The importance of vocabulary is daily demonstrated in schools and out. In the classroom, the achieving students possess the most adequate vocabularies. Because of the verbal nature of most classroom activities, knowledge of words and ability to use language are essential to success in these activities. After schooling has ended, adequacy of vocabulary is almost equally essential for achievement in vocations and in society. (p. 7)

Essentially, all experts on literacy education agree with the stance that vocabulary knowledge is key for success not only in reading, but also in overall literacy and school achievement, as well as the world beyond schooling. Vocabulary knowledge in the initial years of schooling is a significant predictor of a child's reading comprehension in later years of schooling (Cunningham & Stanovich, 1997; Scarborough, 1998). Vocabulary knowledge is one of the best gauges of verbal abilities (Sternberg, 1987; Terman, 1916). The difficulty of vocabulary strongly influences the measured readability of text (Chall & Dale, 1995). Acquiring English vocabulary is one of the most critical tasks for English language learners (ELLs) (Folse, 2004; Nation, 2001). The lack of vocabulary can be a vital contributing factor to school failure for students who are identified as disadvantaged (Becker, 1977; Biemiller, 1999). Educators have long assumed that the study of general vocabulary skills and knowledge is sufficient to apply to all content areas. However, the vocabulary of mathematics is unique as compared to the vocabulary of other disciplines.

One of the characteristics that sets mathematical vocabulary apart from other content areas is the mixture of words and symbols that are used to communicate ideas. Monroe and Panchyshyn (1995) divided mathematical vocabulary into four categories: technical, subtechnical, general, and symbolic. Technical vocabulary is made up of terms that are specific to mathematics, such as *fractal* or *quadratic*. Subtechnical vocabulary consists of words that have multiple meanings, such as *range*, *integral,* and *plane*, but which have specific mathematical meanings. General vocabulary in mathematics is more common in nature, such as *less than*, *gallon,* and *of*. The symbolic category of mathematical vocabulary distinguishes it from other disciplines' vocabulary, with the inclusion of symbols such as $\neq$, $\surd$, and π. The combinations of these types of terms exacerbate the vocabulary challenges for all students.

While the need for understanding mathematical vocabulary has been recognized for more than 100 years, it has become increasingly more important in recent years for students to perform well on high-stakes, state-mandated assessments. The growing emphasis on science, technology, engineering, and math programs (STEM) demands that students have a command of mathematical vocabulary so that they are able to negotiate mathematics text. Furthermore, it is paramount for educators and researchers to select the most appropriate instructional strategies that will help students surpass the barriers that mathematical vocabulary often presents.

The testing movement in the United States began with legislation in 1965 as part of Title 1 of the Elementary and Secondary Education Act (Congress of the U. S., 1965). The *Nation at Risk* report (National Commission on Excellence in Education, 1983), however, led to the high stakes testing movement along with increased accountability at school districts and state levels. Since their creation, high-stakes, state-mandated tests have become as much about reading as mathematics (Abedi & Lord, 2001). As the assessments have become more rigorous and conceptually based, the reading demands have greatly increased. Understanding the vocabulary embedded within the questions has become a confounding issue in determining student achievement in mathematics. Students have to negotiate the meaning of mathematical, academic, and general vocabulary to grasp the problems they are attempting to solve. Mathematical word problems have grown in length and complexity on high-stakes assessments (Daro, Stancavage, Ortega, DeStafano, & Linn, 2007). Answers to these tests have transitioned from numerical solutions to explanations and justifications of responses. This, too, has contributed to the vocabulary demand for mathematics students (Kovarik, 2010).

The major indicator of mathematical success in education today is performance on high-stakes assessments. There is a heightened demand for preparing students to be successful in mathematics through the growing emphasis on STEM (Greene, DeStefano, Burgon, & Hall, 2006; Kuenxi, 2008). This is being spearheaded by both private industry and legislation. There is a growing concern that the United States is not adequately preparing students in STEM areas (Kuenxi, 2008). Hackwood (2009) addressed the state of STEM in California:

> [D]espite projected growth in science and technology occupations, the supply of graduates in these fields is shrinking. California and the United States need a competent, creative science and technology workforce to stay competitive and advance innovation, but both the state and the nation remain far from being able to produce this type of skilled workforce." (p. 7)

The need to produce students who are well prepared in mathematics has added to the reliance on high-stakes assessments to determine the capabilities of those students. Due to the vocabulary demands of the test items in both the questions and the answer choices, it is difficult to determine students' true mathematical capabilities.

Educators who seek knowledge and advice on how to remove or lessen the barrier that mathematical vocabulary often presents are faced with an array of scholarly works that offer instructional strategies. There are many resources dedicated to general vocabulary instructional strategies across all content areas (Blachowicz & Fisher, 2014; Cunningham, 2013; Fisher & Frey, 2014; Graves, 2006; Johnson, 2001; Marzano & Carleton, 2010; Marzano & Pickering, 2010; Vacca & Vacca, 2008). However, there are fewer works written specifically for mathematical vocabulary instruction (Kenney, 2005; Murray, 2004; Molina, 2012; Pearce & Reynolds, 2005). An examination of these two groups of works raises questions, since many of the same strategies are suggested for all disciplines, with few specific to mathematics.

In order to better understand current mathematical vocabulary trends, one must first be mindful of previous research and historical movements. Nearly every concept that educators consider to be modern is linked to research in the past (Fitzgerald, 1990). Examination of past works provides the researcher a foundation for grounding future research. Stahl and Hartman (2011) argued that historical research is an interpretive and reconstructive process, as topics found within the historical body of works cannot be considered in isolation but must instead be viewed within the context in which they were written. They further stated, “History provides us with a sense of honor as a profession” (p. 218). The researcher embarked on a journey to pursue just such an understanding of the field of mathematical vocabulary.

Rationale

The goal of this study was to determine the major trends and topics in mathematical vocabulary research from the mid-1800s to the present. An examination of these trends and topics sought to identify a core group of foundational works that may influence current research conducted on mathematical vocabulary, as well as impact classroom instruction. An exploration of the reasons and factors that mathematics vocabulary experts use to identify significant mathematical vocabulary works may guide and influence future research studies and provide direction for those studying mathematical vocabulary.

Although much writing exists in the field of mathematical vocabulary, there is a lack of research that addresses the historical trends and major topics in the discipline. In an effort to determine the trends and topics of mathematical vocabulary research, a historical analysis of the literature and research was conducted. The initial search was confined to an exhaustive investigation at the local university library. This, in turn, led to a thorough analysis of scholarly works that were identified in the initial search. The references cited provided more sources to explore, including both scholars and works. The researcher employed the use of databases and Google Scholar. A systematic method was developed over time to increase the confidence that all possible information was located. When the researcher believed that all pertinent works had been identified, those works were read, analyzed, and summarized. The summary from each of the identified works was used to generate a three-by-five index card that represented the individual work. These cards were both bibliographic and content driven in nature, as supported by Stahl and Hartman (2011). The cards were sorted and resorted until stable categories were formed.

This immersion into previous thinking about mathematical vocabulary revealed the importance of connecting the past to the present line of scholarly inquiry. Therefore, this research design included a case study that consisted of a group of four mathematics education professors who were considered to be experts in mathematics vocabulary. They were interviewed in order to identify the significant works that they believed address the understanding of mathematics vocabulary. They were also asked to provide insight into the trends and topics found in the historical analysis of the body of literature in which they situated their scholarly works. Additionally, the experts explained why they identified particular studies as significant. The purpose of the case study was to make the findings from the historical analysis relevant and connect them to current lines of inquiry, thus bridging the past to the present and possibly even to the future.

Research Purpose

The purpose of this study was to fill a gap in the body of research concerning the understanding of mathematical vocabulary. This was accomplished by conducting a historical analysis of scholarly works published in the United States from the mid-1800s to the present. To further connect the past to the present, a case study was performed to determine the views of current mathematics education vocabulary experts and how they interpreted and made meaning from the history of mathematics vocabulary research and writings. This study addressed the following research questions:

1. What content information and trends arise from an examination of the literature and research on mathematical vocabulary?

2. What commonalities arise from interviewing experts about mathematical vocabulary?

3. What is the relationship between the historical and current themes in literature?

Operational Definitions

1. Experts – mathematics education professors who have published articles concerning mathematics vocabulary
2. Mathematics vocabulary – words, terms, and symbols necessary for understanding mathematics and navigating mathematics assessments

Methodological Framework

Two methodologies were employed for this study. The first was a historical analysis which was used to develop an understanding of the historical trends and content information of mathematical vocabulary literature and research. As Stahl and Hartman (2011) claimed, "Everything has a history. Everything that exists in the present comes out of the past" (p. 213). The past needs to be understood in order to create meaning in the present. Examining the past provides the opportunity for the identification and evaluation of evolving trends in mathematical vocabulary. The second methodology used for this study was a case study. The purpose of the qualitative inquiry was to develop an understanding of the way in which experts in the field positioned themselves within the body of research and how they viewed current trends in vocabulary classroom instruction. Glesne (2011) stated that qualitative researchers "seek to interpret people's constructions of reality and identify uniqueness and patterns in their perspectives and behaviors" (p. 19). Upon review of this study, readers will gain an understanding of the historical trends and concepts that have emerged in mathematical vocabulary. In addition, readers will recognize how current experts in the field negotiate the past understandings with current realities in mathematics vocabulary scholarship. It is assumed that through the combination of the historical analysis with the case study of current experts in the field, the readers of this research will see how mathematics vocabulary works are grounded within a solid foundation of literature and merit attention.

The case study consisted of participant interviews and an analysis of the works the participants have contributed to the field of mathematics vocabulary. The experts participated in detailed interviews to explore their perspectives concerning their research and how they connected themselves to the existing body of literature.

Limitations and Delimitations

The following are limitations of this study: limited field of potential participants, geographical constraints, and ability to locate primary sources of scholarly works pertaining to this study. The field of active mathematical vocabulary experts is narrow, and relationships exist within this circle. This may or may not have affected how the participants communicated with the researcher. The geographical distance between the researcher and the participants limited the opportunities for face-to-face meetings, which may have altered the depth of understanding that could have resulted from the interview processes. Initial interviews were conducted in person, with one exception due to significant weather conditions. This interview was thus conducted via Skype. Follow-up discussions occurred by phone and email, or used a digital environment, specifically Skype. A further limitation of this study was the inability to locate a full representation of scholarly works that are specific to mathematical vocabulary. To address this potential limitation, the researcher kept logs of search criteria used and repeated the searches by time periods in Google Scholar. Identified works were thoroughly reviewed for related or supporting works that were potential candidates for inclusion in this study. If they were pertinent to the study, they were included. All works that were identified for the study were printed, saved electronically, or purchased. This allowed for multiple examinations of each work during the historical analysis. On occasion, the researcher was unable to locate references or citations noted by scholars, and as this study only included primary sources, those works were excluded.

The following are delimitations of this study: selection of the participants and selection of scholarly works to be included or not included in this study. Because the field of active mathematical educators who have produced scholarly works involving mathematical vocabulary is narrow, the researcher made specific decisions in the solicitation of participants. To begin with, two participants from the same institution could not be interviewed. Also, potential participants' works were examined to avoid close partnerships with other potential participants, such as co-authoring or co-presenting. The researcher also determined a point at which to end the search for potential works to be included in this study and commence the analysis and crafting of this study. It is possible that the decision to conclude the search may have inadvertently led to the omission of works which may have been pertinent for this study. Furthermore, this study utilized only works published in the United States as opposed to those published in the English-speaking world.

Chapter Summary

This chapter established the importance of vocabulary, along with the uniqueness of mathematical vocabulary. It also identified the need for studying mathematical vocabulary, as it is requisite for unlocking the meaning within high-stakes, state-mandated assessments and the ability to communicate mathematical understandings. Chapter 1 further explained the methodology employed in this study, a historical analysis and case study, as well as defined the theoretical framework as symbolic interpretivism. Lastly, the chapter offered limitations and delimitations of the study.

CHAPTER II: LITERATURE REVIEW

This chapter presents a review of the scholarly works that are the focus of the historical analysis of mathematical vocabulary. The following headings and subheadings forecast the organizational makeup of this chapter:

- Introduction
- Historical Examination of Vocabulary by Decades
    - Prior to 1900s
    - 1900-1919
    - 1920-1939
    - 1940-1949
    - 1950-1969
    - 1970-1979
    - 1980-1989
    - 1990-1999
    - 2000-2009
    - 2010-Present
- Chapter Summary

Introduction

Interest in mathematics vocabulary needs has a long history in the United States, spanning from the 1850s to the present. The voices from the past define the journey that research has made over time. Reviewing the work of dedicated scholars and searching for an understanding of mathematical vocabulary over the years is humbling. Many of the works could have been written by their present-day contemporaries. It was essential to develop a means of organizing the literature so that it made sense. Initially, this was accomplished by delineating time periods. This made the vastness of scholarly work easier to examine in smaller pieces. This chapter consists of a thorough literature review of scholarly works pertaining to mathematical vocabulary in the United States. In this literature review, the researcher organized the information to help bring clarity to the field. The works are organized and described in Table 1 by time period. Each study includes the sample size and the findings. Articles, chapters in books, and books are briefly summarized, highlighting the main points. The researcher used three subheadings to divide the time periods into similar size units of time in order to assist the reader's interpretation of the nuances of the works that are identified. The following research questions defined this study:

1. What content information and trends arise from an examination of the literature and research on mathematical vocabulary?
2. What commonalities arise from interviewing experts about mathematical vocabulary?
3. What is the relationship between the historical development and current themes and perceptions of experts?

Table 1 is provided to give clarity and an overall sense of the works through a chronological lens. The table presents the time periods and the quantity of mathematical vocabulary works that were produced, and it also serves as a reference for the relationship of articles to studies.

Table 1

Scholarly Works by Time Period

Time period	Date	Author(s)	Type of publication
Prior to 1900	1855	Davies & Peck	Book
	1881	Peirce	Article
	1898	Bailey	Book
	1899	Speer	Book
1900-1919	1902	Smith	Book
	1907	Aley	Article
	1913	Betz	Article
	1914	Yocum	Article
	1918	Merrill	Article
1920-1939	1920	Young	Book
	1921	Minnick	Article
	1921	The National Committee on Mathematical Requirements	Article
	1922	Thorndike	Article
	1922	Wilson	Article
	1924	Monmouth, Isbell, Jenkins, & Pieters	Study
	1924	Pressey	Article
	1925	Lessenger	Study
	1926	Brooks	Study
	1928	Remmers & Grant	Study
	1929	Georges	Study
	1931	Buswell & John	Study
	1931	Monroe & Engelhart	Study
	1932	Pressey & Moore	Study
	1932	Pressey, Pressey, & Zook	Study
	1932	Pressey, Pressey, & Narragon	Study
	1933	Monroe & Engelhart	Study
	1934	Dresher	Study
	1934	Cowley	Study
	1937	Buckingham	Study
1940-1949	1940	Drake	Study
	1941	Bond & Bond	Book
	1941	O’Rourke & Mead	Study
	1941	Hastings	Study
	1943	Tiews	Article
	1944	Johnson	Study
	1944	Treacy	Study
	1944	Hansen	Study
	1948	Eagle	Study
	1949	Johnson	Study
1950-1969	1953	Brune	Article
	1956	Peeler	Article
	1956	Totten	Article
	1957	Johnson	Study

	1957	Strang & Bracken	Book
	1960	Repp	Study
	1961	Austin	Article
	1961	Bamman, Hogan, & Greene	Chapter
	1961	Faulk & Landry	Study
	1964	Vanderlinde	Study
	1966	Stauffer	Study
	1967	Lerch	Article
	1967	Strang, McCullough, & Traxler	Chapter
	1967	Wiegand	Study
1970-1979	1970	Earp	Article
	1970	Kane	Study
	1970	Harvin & Gilchrist	Study
	1970	Hater & Kane	Study
	1971	Byrne & Kane	Study
	1971	Olander & Elmer	Study
	1971	Wilmon	Study
	1972	Aiken	Article
	1973	Shepherd	Book
	1975	Robinson	Book
	1977	Dolgin & Schneider	Chapter
	1977	Hollander	Article
	1979	Austin & Howson	Article
1980-1989	1980	Krulik	Article
	1981	Kossack & Vigilante	Article
	1982	Thomas & Robinson	Chapter
	1983	Jackson & Phillips	Study
	1983	Milligan & Milligan	Article
	1984	Cox & Wiebe	Article
	1984	Cuevas	Article
	1985	Crandall, Dale, Rhodes, & Spanos	Article
	1985	Garbe	Article
	1987	Pimm	Book
	1988	Davison & Schindler	Study
	1989	Rothman & Cohen	Article
1990-1999	1990	Small	Article
	1993	Capps & Pickreign	Article
	1993	Miller	Article
	1996	Monroe & Panchyshyn	Article
	1996	Olivares	Book
	1996	Usiskin	Chapter
	1996	Monroe	Article
	1997	Monroe	Article
	1998	Brenner	Article
	1998	Krussel	Article
	1999	Garrison & Kerpa Mora	Chapter

	1999	Moschkovich	Article
	1999	Schwarz	Study
	1999	Steele	Article
2000-2009	2000	Abedi	Study
	2000	Kiplinger, Haug, & Abedi	Article
	2000	Monroe & Pendergrass	Article
	2000	Rubenstein	Article
	2000	Rubenstein & Schwartz	Article
	2000	Thompson & Rubenstein	Article
	2000	Zazkis	Article
	2001	Abedi & Lord	Study
	2002	Gay & White	Article
	2002	Monroe & Orme	Article
	2002	Raiker	Article
	2002	Rubenstein & Thompson	Article
	2003	Moschkovich	Article
	2003	O'Halloran	Chapter
	2004	Murray	Book
	2005	August, Carlo, & Dressler	Article
	2005	Barwell	Article
	2005	Hancewicz	Chapter
	2005	Kenny	Chapter
	2005	Leung	Article
	2005	Pearce & Reynolds	Chapter
	2006	Capraro & Joffrion	Study
	2006	Spencer & Guillaume	Article
	2007	Moschkovich	Article
	2007	Rubenstein	Article
	2008	Gay	Article
	2008	Riccomini, Sanders, & Jones	Article
	2008	Thompson, Kersaint, Richards, Hunsader, & Rubenstein	Book
	2009	Bay-Williams & Livers	Article
2010-Present	2010	Cirillo, Bruna, Herbel-Eisenmann	Article
	2011	Salinas & Ortlieb	Article
	2012	Ortlieb, Perkins, & Verlaan	Study
	2012	Smith & Stein	Book
	2013	Dunston & Tyminski	Article
	2013	Pearce, Bruun, Skinner, & Lopez-Mohler	Study
	2013	Roberts & Truxaw	Article
	2013	Vessel & Robillard	Study
	2014	Livers & Bay-Williams	Article
	2015	Bicer, Boedker, Capraro, & Capraro	Article
	2015	Pace & Ortiz	Article
	2015	Riccomini, Smith, Hughes, & Fries	Article

Historical Examination of Vocabulary by Decades

The primary sources located through exhaustive searches were organized by time period in order to provide a sense of the trends that emerged throughout the literature by time. Such organization also revealed the times when new lines of scholarly inquiry began, merged, disappeared, and sometimes reemerged throughout history. The following captures the essence of the works.

Prior to 1900s

The earliest publications that address vocabulary in mathematics in the United States date back to the mid-1800s. There were four works prior to 1900 that specifically addressed mathematical vocabulary. These works examined the following concerns: mathematical vocabulary was impeding student success, mathematical vocabulary needed to become standardized in public education, and vocabulary instruction needed to be situated within mathematical contexts and student experiences. It is important to note that while these works considered mathematical vocabulary, such vocabulary was not the primary focus of the works. Davies and Peck (1855) published a mathematics dictionary, addressing the concern that "Most of the difficulty experienced in the study of mathematics, has risen, we apprehend, from the use of terms in a vague or ambiguous sense" (p. 3). The authors argued that many of the difficulties experienced in the learning of mathematics were due to a misunderstanding of the technical terms. Benjamin Peirce (1881) gave a speech before the National Academy of Sciences in 1870 wherein he made the case that algebra was formal mathematics written in a symbolic language with a unique vocabulary. He stated, "Symbols must be adopted which may serve for the embodiment of forms of argument, without being trammeled by the conditions of external representation or special interpretation. The words of common language are usually unfit for this purpose" (p. 98). Bailey (1889) contended that teachers must "[D]evelop inductively, to present step by step a mathematical vocabulary, and to form a habit of clear mathematical thinking" (p. 4). Speer (1899) agreed with the need for students to learn mathematical vocabulary but cautioned that learning the words without conceptual development grounded in experiences had little value. He wrote, "A religious vocabulary without religious experiences is of little value; a mathematical vocabulary without mathematical experiences is of no more value" (p. iv).

1900-1919

During the early 1900s, there was continued discussion of the importance for students to learn the unique language and vocabulary of mathematics in order to properly prepare students for college. Smith (1902) argued against learning definitions and instead promoted robust learning in context. He stated, "The old argument that learning definitions strengthens the memory and gives a good vocabulary has too few advocates now to make it worth consideration" (p. 30). Betz (1913) purported, "Mathematics furnishes a language without which, it is not too much to say, our present civilization cannot be understood and cannot exist" (p. 226). Yocum (1914) discussed the unique vocabulary found in mathematics. He wrote, "Its vocabulary is not only limited, but largely limited to words that have only mathematical use" (p. 148). He addressed the concern that without a working knowledge of the vocabulary found in mathematics, one would be unable to follow the logic of the arguments made and miss the opportunity to develop mathematical understanding or application. Merrill (1918) pointed to the language demands within mathematics as one of the primary reasons for student failure. Young (1920) took a different approach from his contemporaries and suggested that the teacher should be expected to "practice accuracy of expression always and cultivate it in his pupils" (p. 131).

He advised that students need to be provided opportunities to use the newly acquired mathematical vocabulary and work toward the goal of developing full accuracy of expression.

The works generated in the time period of 1900-1919 were limited to scholarly work drawing attention to the issue of vocabulary, but they did not delve into research and may be classified as wisdom of practice. The mathematical vocabulary scholars appeared to draw from experience and observation when making their assertions in their publications. Educational research in the United States as a whole was in its infancy.

1920-1939

The 1920s provided a turning point in scholarly work. This time period marked the emergence of studies in the growing scholarly conversation dedicated to mathematical vocabulary. This decade added studies to the discussion that were conducted primarily to examine why students struggled with arithmetic. It may be useful to consider the definition of arithmetic held during this time period in history. "The purpose of arithmetic is to enable one to solve the problems of practical life" (Monmouth, Isbell, Jenkins, & Pieters, 1924, p. 279); therefore, arithmetic involved problem solving that did not include the formal disciplines of algebra or geometry. The studies during this time frame can be grouped by the relationship of reading to performance in arithmetic, readability of mathematics texts, and the creation of vocabulary lists. Minnick (1921) made the case that if education's main goal was to prepare students to be able to render effective service to society, students must be fluent in the language of mathematics. He specifically mentioned the need for students to learn the appropriate usage of mathematical vocabulary to "enable the child to express his thoughts more clearly and to understand written and spoken language more readily" (p. 300).

A major research focus during this time period was on the relationship between reading and performance in arithmetic. Wilson (1922) conducted a limited study of 17 sixth-grade boys and 17 sixth-grade girls. Students in the experimental group were provided with a variety of reading strategies for arithmetic problems, including vocabulary instruction, dramatization, and problem enhancement to add interest to the problems. She found that providing specific reading strategies improved students' mathematics scores on standardized tests. Wilson substantiated the concern that student failure in mathematics was due to poor reading and inability to understand the problems. She noted that in her observations, teachers had not required students to master the story of the arithmetic problems, which in turn, led to students focusing on numbers and computing answers. Monmouth et al. (1924) performed a study that sought to determine what vocabulary difficulties existed in solving arithmetic problems and if these difficulties were real or imagined. A sample of 32 elementary and junior high teachers was initially asked to create lists of words that troubled their students in mathematics. The next stage of the study involved approximately 800 students. Students were given a partial list of the terms that the teachers had generated and were asked to provide the meaning of the terms when used within arithmetic problems. The researchers found that technical words and phrases specific to mathematics gave students the greatest difficulty and were a hindrance to students' ability to solve arithmetic problems. They suggested that if teachers provided assistance in understanding the vocabulary, the students would then perform better in their solutions to arithmetic problems. This research was supported by the findings of Lessenger (1925). His large study, which included all third- through eighth-grade students in Radcliff, Iowa, examined special instruction and motivation in reading. These students were provided with an emphasis on instruction in reading with no changes to instruction in other content areas. Students were given the *Stanford Achievement Test*

(Kelley, Ruch, &Terman, 1924) in September and again at the end of the school year. An unexpected result emerged when the data was examined: arithmetic errors attributed to vocabulary and reading significantly decreased. Buckingham (1937) applied this line of research to 139 first-year algebra students in high school. Buckingham created a new taxonomy of mathematical vocabulary: non-technical words, such as *silo*; technical mathematical terms, such as *cube;* and technical algebraic terms, such as *monomial.* He then created a list of 49 of these terms as they were found in the textbook being used at the location of the study. He created algebra problems that incorporated the identified terms and then assessed the students on their ability to solve these problems. He compared this with the results of the *Cooperative Algebra Test Form 1934* (Long, Siceloff, & Lundholm, 1934) and found a significant relationship between vocabulary understanding and the ability to solve algebra problems.

A second avenue for research during this decade concerned the readability of textbooks. Brooks (1926) examined a variety of elementary through Grade 8 mathematics textbooks for usage of technical and semi-technical vocabulary of arithmetic. He determined that there was low repeat usage of the technical vocabulary, which did not support the learning of these critical words. Remmers and Grant (1928) followed up with a study that considered the vocabulary demands of secondary mathematics textbooks and found the same issue. In a related study, Georges (1929) investigated the nature of difficulties encountered when reading mathematics text. The participants were students in their first year in junior high school mathematics in the University High School of the University of Chicago. He found that 23.4% of all difficulties that students experienced were caused by a lack of understanding of mathematical vocabulary.

One group of researchers pursued a separate line of inquiry in mathematics vocabulary research during this time period; they worked to create lists of words and symbols that needed to be included as mathematical vocabulary terms. The National Committee on Mathematical Requirements (1921) focused on creating vocabulary lists for geometry, algebra, and arithmetic. Pressey (1924) worked with teachers to create word lists and determined by consensus the essential technical mathematical vocabulary that students needed to understand in order to be successful in mathematics. Her work led to the determination of which terms, at that time, should be considered the technical vocabulary of mathematics. She continued this work in the 1930s. Pressey's body of scholarly work was highly cited by other scholars for several decades.

Thorndike (1922) argued that students needed to be able to read formulas, including correctly interpreting the symbols therein, in order to appropriately use them in a specified context. He provided the following example to illustrate misunderstandings that may occur when this is not achieved, "$A = p + prt \ and \ I = \frac{E}{R} \ to \ conclude \ that \ AI = \frac{pE+prtE}{R}$" (p. 10). He pointed out that although algebraically correct, conceptually it did not make sense to combine the ideas of computing money invested over time with the relationship of electricity, current, and resistance. Thus, he proposed that students needed to be able to read and understand formula and the symbols within them.

Minnick (1921) made the case that if education's main goal was to prepare students to be able to render effective service to society, students must be fluent in the language of mathematics. He specifically mentioned the need for students to learn the appropriate usage of mathematical vocabulary to "enable the child to express his thoughts more clearly and to understand written and spoken language more readily" (p. 300).

The 1930s contributed to the conversation about vocabulary issues confounding the learning of mathematics. It also marked the emergence of research to identify mathematical vocabulary that was essential for success in a variety of mathematics courses taught in public school. Pressey, Pressey, and Zook (1932) examined the essential vocabulary necessary for understanding plane geometry, and Pressey, Pressey, and Narragon (1932) did the same for algebra. Both studies identified the technical words found within a group of textbooks and then had groups of teachers judge the importance of each word for instruction. Similarly, Cowley (1934) conducted a multiple-pronged study to determine the vocabulary necessary for plane and solid geometry. Her study utilized textbooks and involved teacher groups but also obtained student input from more than 3,000 students in public high schools. In a related study, Pressey and Moore (1932) identified what mathematical technical words students knew in a variety of grade levels. The study was conducted in five cities of varying size that the researchers regarded as typical. Students from Grade 3 through high school participated, with a minimum of 406 students at any grade level. They found that children's mathematical vocabularies did not mature over time and suggested that inadequate mastery of fundamental terminology was one of the most critical reasons students' encountered difficulty when dealing with anything mathematical in nature.

The 1920s and 1930s also included research which studied the role of vocabulary instruction in relation to mathematical achievement. Monroe and Engelhart (1931) performed a critical summary of research related to the teaching of arithmetic. They identified the work of Buswell and John (1926) and of Terry (1922) to have provided the "most significant evidence relative to the importance of instructing pupils to read arithmetic" (p. 77). They reported that a technical vocabulary was essential for children engaged in arithmetical learning activities. Monroe and Engelhart (1933) conducted a large study of 587 fifth-grade students to decide if systematic and specific instruction of reading verbal arithmetic problems, including vocabulary instruction, affected scores on the *Stanford Reading and Arithmetic* tests (Kelley, Ruch,

&Terman, 1924). They found that there were no meaningful differences between the experimental and control groups with the exception of students whose intelligence quotients were measured as less than 100. Dresher (1934) developed a study using a sample of 500 seventh-grade students. The measure of achievement was the results of three vocabulary tests over arithmetic, geometry, and algebra terms, all created first by Pressey and later by her fellow researchers. The fourth test was a concrete-problem test created by Dresher. All four tests were given as pre- and post-test. Dresher determined that extensive and specific instruction of mathematical vocabulary improved the students' ability to solve concrete problems. He stated, "The failure to know a word is evidence of failure to comprehend the idea represented by the word" (p. 203).

1940-1949

The 1940s highlighted a series of experimental studies that examined the effects of teaching vocabulary on mathematics scores. The studies had conflicting results; while some studies determined that there was a positive effect on mathematics scores associated with vocabulary instruction, others did not. Drake (1940) considered the effect of teaching algebra vocabulary. His study involved students in Grade 9 at seven schools in Minneapolis, Minnesota. The author of this highly-cited article noted that students who received vocabulary instruction had significantly higher algebra achievement. Specifically, there was a favorable effect on the understanding of simultaneous equations, special products and factoring, power, roots, and radicals. The concepts of positive and negative numbers were impacted the least. In a related study of 162 ninth-grade algebra students, Eagle (1948) found that a greater understanding of mathematical vocabulary provided greater success in mathematics achievement and course grades. His study relied on researcher-developed vocabulary tests, the *New Stanford Achievement Test* (Kelley, Ruch, &Terman, 1943) and students' semester grades in algebra.

In contrast, another frequently cited study of this time period was Johnson's (1944) work that involved 898 students in seventh grade. This study resulted in markedly different findings from Drake. Johnson found that vocabulary instruction improved performance on vocabulary assessments but did not transfer to other situations or result in improved performance in arithmetic problem solving. These findings were in agreement with the findings of Davis's (1944) work, specifically with regard to vocabulary instruction that improved performance on vocabulary assessments but did not transfer to reading comprehension. In addition, Hansen (1944) conducted a study of 681 sixth-grade students and determined that general reading ability and knowledge of general vocabulary were not essential for achievement in verbal problem solving in arithmetic.

Hastings (1941) conducted a study of 331 ninth graders in Indiana. His study explored the need to use multiple questions or types of questions to determine knowledge of mathematical concepts and the associated vocabulary. The results of his study indicated that multiple opportunities needed to be provided in order to determine if a student truly learned and mastered the meaning of mathematical vocabulary. Johnson (1949) studied all eighth-grade students in Chicago schools. He examined data collected from a series of standardized assessments and found that vocabulary was related to measures of intelligence and problem-solving in arithmetic.

The 1940s featured scholarly works that also addressed mathematical vocabulary within the larger framework of reading in content areas. Treacy (1944) performed a study of 244 seventh graders in Wisconsin. Rather than examining the effects of instruction on vocabulary, he examined the relationship between reading skills and the ability to solve arithmetic problems.

He found that the reading skills associated with vocabulary differed significantly between students who were successful problem solvers and students who were poor problem solvers. He also found that good achievers in problem solving tended to have better reading skills in general than poor achievers in problem solving. Bond and Bond's (1941) book focused on the difficulties of vocabulary in mathematics within a subsection of reading in mathematics. They cautioned that context clues were of little value to develop meaning of unknown terms in mathematics. The authors noted, "It is the task of the student with the aid of his teacher gradually to build the mathematical vocabulary needed and the ability to see the relationship in the statements read" (pp.186-187).

1950-1969

The 1950s was a time period where there was little scholarly activity concerning mathematics vocabulary. One of the exceptions was a study that considered the readability of mathematics books. Johnson (1957) determined that the Flesch formula (Flesch, 1948) was suitable for measuring the readability of mathematics textbooks. He also purported that mathematics textbooks tended to be of a higher readability than the grade level for which the book was intended. During this time, Peeler (1956) and Totten (1956) discussed what role vocabulary instruction should have in mathematics instruction. Strang and Bracken (1957) identified difficulties inherent in learning mathematical vocabulary. They provided strategies in their article that were focused on content area reading.

The 1960s continued to be a time period where vocabulary instruction did not receive much attention in mathematics education scholarly work. The impact of Sputnik may have influenced the direction that research took during this period (De Boer, 2000). Mathematical vocabulary instructional suggestions were embedded within scholarly works dedicated to reading improvement and were typically limited to a page or two. Instructional strategies were suggested and general in nature (Bamman, Hogan, & Greene, 1961; Lerch, 1967; Strang, McCullough, & Traxler, 1967). Primarily, they focused on using the terms correctly, explaining their meaning, and teaching the words in conjunction with teaching the associated mathematical concepts. Austin (1961) suggested much the same but also proposed the notion of two categories of mathematical vocabulary:

> Special attention should be directed also to two types of words: (1) those whose mathematical meaning is different from their general meaning (root, improper, rational), and (2) those whose mathematical meaning is more precise than the general meaning (opposite, direction, similar). (p. 392)

She also suggested that teachers create lists of mathematical vocabulary words on which to focus their instruction.

While few, some studies were conducted during the 1960s. The research topics during this decade focused on examining vocabulary issues related to problem solving and readability of mathematical texts. Weaver and Gibb (1964) included one such study in their review of educational research in mathematics. In this study, Faulk and Landry (1961) examined the effects of a four-pronged approach to teaching problem solving on 148 sixth graders. They included vocabulary study which consisted of teaching word meanings, syllabication of words, synonyms, and completing crossword puzzles. The problem solving approach also incorporated discussion of the problem situation, diagram drawing, and computation. The authors administered the *California Arithmetic Reasoning Achievement Test* (Tiegs & Clark, 1957) as a pre- and post-test. The experimental group performed slightly better than the control group. Vanderlinde (1964) studied 394 fifth-grade students in Michigan. He utilized the *Iowa Test of*

Basic Skills (Lindquist & Hieronymous, 1964) as both a pre- and post-test and determined that direct study of quantitative vocabulary was linked to higher achievement on arithmetical problem solving. The four subtests were vocabulary, reading comprehension, arithmetic concepts, and arithmetic problem solving. These studies reconfirmed that direct instruction did not translate to reading comprehension or improvement in general vocabulary understandings.

Over the years, readability of mathematics textbooks reemerged as a topic of interest (Repp, 1960; Stauffer, 1966; Wiegand, 1967). Repp (1960) examined five third-grade arithmetic textbooks for new vocabulary introduced. She determined that 3.98 to 6.78 new mathematical vocabulary words were introduced per page. Stauffer (1966) reviewed three arithmetic textbook series for Grades 1, 2, and 3. He determined that there was very little overlap of vocabulary within the series. This revealed that from 1951-1953, three major textbook series did not consistently use mathematical vocabulary within their series from grade level to grade level. Wiegand (1967) considered a study conducted by the Communication Skills Division of the Pittsburgh Public Schools (year) to address the concern that students were coming to high school unprepared to read mathematics. The reading levels of all children in mathematics classes from Grade 8 to 12 were determined using the *Metropolitan Achievement Tests* (Durost, Bixler, Hildreth, Lund, & Wrightstone, 1964), and the *Iowa Silent Reading Tests* (Greene, Jorgensen, & Kelley, 1939) To ascertain textbook readability, 160 samples were taken from nine textbooks and analyzed with the *Dale-Chall Formula for Predicting Readability* (Dale & Chall, 1948). The results showed that students were asked to read mathematics textbooks that were consistently above their reading levels, in some cases as much as six years beyond.

1970-1979

The 1970s witnessed an increase in the quantity and diversity of research and articles concerning mathematical vocabulary. These included the following: the reoccurring topics of readability of mathematics textbooks, instructional strategies for teaching mathematical vocabulary, a comparison of students' vocabulary understanding in 1968 to that of students in the 1930s, and language and reading factors in mathematics learning. Although there was greater interest in mathematical vocabulary, a preponderance of this scholarly work revisited earlier research.

During the time period of 1970-1979, several researchers revisited the readability of textbooks. Kane (1970) argued that previous studies which examined the readability of mathematics textbooks were flawed. He reasoned that applying the readability formulas, such as *Dale-Chall* (Dale & Chall, 1948), *Flesch* (Flesch, 1948), and *Spache* (Spache, 1953), were inappropriate due to the nature of the language of mathematics. However, another study by Hater and Kane (1970) reported that the Cloze tests were found to be highly reliable predictors of comprehensibility of mathematical English passages for Grades 7-12. Cloze tests were positively correlated with comprehension test scores. In a related study, Wilmon (1971) reviewed primary grades textbooks and found that children were introduced to approximately 500 new technical mathematical vocabulary words by the end of the third grade.

Earp (1970) and Hollander (1977) studied instructional strategies for reading mathematical text. Earp suggested instructional strategies for vocabulary, problem-solving, and interpreting the mathematical text found in textbooks. Hollander expanded on Earp's work and cautioned that unless teachers instruct students on how to read textbooks, students will be forced to rely solely on oral instruction. Robinson (1975) argued that "In mathematics, more than in any other subject area, the meanings of language units are carefully and precisely dealt with *in context*" (p. 161), even though "some writers have concluded that context clues are, therefore, useless in math" (p. 161). His argument centered on the idea that mathematical terms had specific and exact meanings that were unique to the context within which they were used. He was not referring the vocabulary strategy of using context clues. Moreover, he stated that mathematics concepts and terms needed to be situated in an appropriate context to build foundational understanding in mathematics. Shepherd (1973) developed instructional strategies to promote the understanding of vocabulary words in four categories of mathematics. He classified vocabulary as technical words, which are "peculiar to some area of the mathematics" (p. 259), general words that have specific mathematical meanings, words which signal mathematical processes, and general words which can determine students' comprehension, such as *before*, *of*, or *compare*.

Olander and Ehmer (1971) conducted a study involving 1,200 students from the fourth, fifth, and sixth grades. They investigated how current students' mathematical vocabulary understandings compared to those of students in the 1930s. The researchers utilized the *Buswell-John Vocabulary of Arithmetic Test* (Buswell & John, 1931) that was used in the 1930s research, as well as a *Contemporary Mathematical Vocabulary Test* (Olander & Ehmer, 1971), which was developed by the researchers for this research. The students outperformed their counterparts from the 1930s in Grades 4 and 5 but not in Grade 6. On a side note, the students from the 1930s outperformed the students from 1968 in arithmetic computational skills.

Harvin and Gilchrist (1970) explored the reading and language factors associated with achievement in mathematics. They examined standardized test results from 121 third graders in western New York and 93 third graders in rural Indiana. They found a positive relationship between problem solving in arithmetic and reading for both groups. However, the relationship was not considered sufficient enough to be a predictor. Specific reading skills were not investigated to determine which skills were necessary in a mathematics setting. Aiken (1972) reviewed mathematical educational research and focused on language factors associated with learning mathematics. His highly-cited work addressed vocabulary, the unique language of mathematics as a mix of symbolic and written words, and the readability of text.

1980-1989

During the 1980s mathematical vocabulary works addressed the issue of English language learners (ELLs) and students with disabilities learning vocabulary, as well as instructional strategies necessary for their success. This time period was dominated by scholarly articles. In contrast only one study was conducted. Cummins (1979), who was concerned with linguistic considerations for ELLs, in conjunction with other scholars' works published in the 1970s, may have been the catalyst that inspired the research activity in the realm of mathematical vocabulary. Published articles addressed the mathematics register, teaching of cognates, focus on polysemic words, and the specific mathematical meanings of words such as *a, an, and, the,* and *or* (Crandall, Dale, Rhodes & Spanos, 1985; Cuevas, 1984; Garbe, 1985; Milligan & Milligan, 1983; Pimm, 1987). There was also a focus on targeted vocabulary instruction for students (Rothman & Cohen, 1989). Content area reading books and articles provided suggestions for mathematical vocabulary instruction, including writing definitions, making lists, examining word parts, and rewriting the terms in students' own words (Krulik, 1980; Thomas & Robinson, 1982). Milligan and Milligan (1983) suggested using a linguistic approach as an instructional strategy for teaching mathematical vocabulary. They recommended structural analysis of words which focused on teaching students prefixes, suffixes, and root words as means of building an understanding of vocabulary. Jackson and Phillips (1983) studied 213 seventh graders to examine the relationship between vocabulary instruction and achievement tests. They found that the students who received vocabulary instruction achieved higher verbal and computational scores. Cox and Wiebe (1984) created the Wiebe/Cox Mathematical Vocabulary Reading Inventory for first through third grades.

1990-1999

Vocabulary research and publications of the 1990s continued the focus on issues pertaining to the language of mathematics with the specific goal of meeting the needs of ELLs as evidenced by the works of Brenner (1998), Moschkovich (1999), Garrison and Kerpa Mora (1999), and Steele (1999). Brenner examined discourse patterns in two algebra classes with ELLs. She found that effectively utilizing small groups and computers resulted in greater student discourse than standard classroom structures. Moschkovich, Steele, and Garrison and Kerpa Mora discussed strategies for creating learning environments that fostered greater opportunities for all students, especially ELLs, to actively participate in mathematical discourse to learn and appropriately use mathematical vocabulary. There was also a call for research to determine which strategies would prove most beneficial for this specific student population.

This decade also featured the use of graphic organizers to learn vocabulary. Monroe and Pendergrass (1997) studied the integrated use of Concept of Definition (Schwartz & Raphael, 1985) and the Frayer model (Frayer, Frederick, & Klausmeier, 1969) and found that this combination of graphic organizers was effective in increasing the use of mathematical vocabulary in fourth-grade writing. They also found that these tools could improve the affective learning of mathematical vocabulary. This was expanded upon in a subsequent work by Monroe

(1997) where she looked at how available research could inform mathematics vocabulary instruction.

Throughout this decade, scholars expressed a need to reimagine the teaching and learning of vocabulary (Capps & Pickreign, 1993; Krussel, 1998; Monroe, 1996; Small, 1990; Monroe & Panchyshyn, 1995). Instead of looking at vocabulary instruction as a separate issue in mathematics, researchers suggested that it should be integrated into the structure of the classroom, the design of lessons, and the culture of the classroom. “[R]egardless of students’ abilities, the content of mathematics is not taught without language” (Capps & Pickreign, 1993, p. 12). Vocabulary is learned intentionally and through frequent usage in student discourse over time. Small (1990) stated, “Students rarely speak more than a few words in mathematics class” (p. 29). She suggested that teachers design instruction with a heavy emphasis on student talk in primary mathematics classrooms. Little pencil time should occur, with the exception of student writing, for computation. Krussel (1998) suggested that students use dictionaries when they encounter unknown mathematical vocabulary. She also recommended that students practice using the newly learned words through speaking and writing sentences.

2000-2009

The turn of the century sustained the trends of ELL issues, vocabulary instructional strategies, and discourse. Vocabulary was reaffirmed as critically important, and it was noted that ELLs lagged behind their English-speaking peers. Kenny (2005) explored the difficulty of learning mathematical vocabulary, especially for ELLs. The concern was the lack of research conducted to investigate the acquisition of vocabulary in students learning English as a second language, as well as the need to determine the effectiveness of vocabulary instructional strategies for this group of learners (August, Carlo, Dressler, & Snow, 2005).

Abedi (2000) and Abedi and Lord (2001) published studies that examined differences between ELLs and native English speakers. Abedi's (2000) study compared the performance of ELLs and native English speakers on mathematics problems from the *National Assessment of Educational Progress* (NAEP) (U.S. Department of Health, Education & Welfare, Office of Education, 1970) from 1990 and 1992 and found that ELLs performed significantly lower on mathematics word problems. Abedi and Lord (2001) evaluated 1,174 eighth-grade students to determine how they performed on math problems with modified linguistic complexity as compared to the released items from NAEP. They discovered that the linguistic structures of the word problems affected students' performance specifically that of ELLs and students identified as having low socioeconomic status.

The topic of vocabulary instructional strategies was an active line of scholarly work during this time period. Thompson and Rubenstein (2000) presented instructional strategies that were organized around learning styles. Vocabulary strategies were linked with the development of mathematical concepts by several scholars. They viewed vocabulary development as an incremental activity that develops and expands as meaning is created (Leung, 2005; Monroe & Orme, 2002; Murray, 2004; Thompson, Kersaint, Richards, Hundsader, & Rubenstein, 2008). Several scholars explored the concept of mathematical discourse (Barwell, 2005; Moschkovich, 2003, 2007; Raiker, 2002) and how such discourse could promote learning mathematical vocabulary (Hancewicz, 2005). Generally the works cautioned that a narrow definition of discourse could distract the focus from the usage of technical mathematical vocabulary and impede student mathematical competence. They suggested the creation of learning environments that would support the building of mathematical meanings through speaking and listening. Researchers also promoted the use of intentional student groupings and practices, such as thinkpair-share, roundtable, and silent teacher, in order to provide students the opportunity to actively use the vocabulary in meaningful ways (Thompson et al., 2008).

Three works focused on teachers' use of vocabulary. O'Halloran (2003) observed that teachers of working class students and female students tended to use more informal and non-technical vocabulary in the classroom discourse, which limited access to knowledge in those circumstances. Zazkis (2000) reflected on four years of working with preservice teachers. She suggested teaching students to use code-switching to learn mathematical language and then requiring students to use the formal mathematics register in order to internalize it. Gay (2008) focused on intentionally including vocabulary instructional strategies when developing lesson plans with preservice teachers.

A group of scholarly work offered general instructional strategies for teaching mathematical vocabulary that included ideas such as concept maps, word walls, word origins, analogies, and games (Pearce & Reynolds, 2005; Rubenstein & Thompson, 2002; Salinas & Ortlieb, 2011; Spencer & Guillaume, 2009). Riccomini, Sanders, and Jones (2008) recommended the use of mnemonic instructional strategies with an emphasis on the keyword strategy for learning mathematical vocabulary. Schwarz (1999) conducted the only study within this group of scholarly work. In his study of 36 fifth graders, Schwarz found that students who had the opportunity to use vocabulary and mathematics journals and vocabulary word walls improved their understanding of mathematical vocabulary as evidenced by a vocabulary word pre- and post-test.

2010-Present

Instructional strategies have been the major topic of the current time period. Instructional practices and activities reverse vocabulary deficits and allow the connection to be made between words and concepts (Cirillo, Bruna, & Herbel-Eisenmann, 2010). Smith and Stein (2012) argued that mathematically productive discussions can be orchestrated where students are "authors of their own ideas" (p. 2). Riccomini, Smith, Hughes, and Fries (2015) recommended explicit instruction of mathematical vocabulary, mnemonic strategies, and the need for multiple exposures to the new words. Dunston and Tyminski (2013) suggested the use of graphic organizers as an instructional strategy. Pace and Ortiz (2015) identified vocabulary chart activities as an instructional strategy for kindergarten students. Roberts and Truxaw (2013) advocated that teachers create vocabulary lists, pre-teach the vocabulary, and use word walls graphic organizers to support ELLs in learning mathematical vocabulary.

This time period has included four studies. Pearce, Bruun, Skinner, and Lopez-Mohler (2012) conducted a study of 70 elementary teachers to determine their perceptions as to why students have difficulty with word problems. They found that the teacher-reported student difficulties were attributed to the following: 45% to reading and understanding the problem, 35% reported making a plan, and 13% reported vocabulary, 3% background knowledge, 2% determining reasonableness, 1% computation, and 1% higher level thinking. They also sought to identify teacher perceptions about the causes of those problems, their classroom practices, and strategies they used to teach problem solving. The results showed that 21% of the teachers taught students to identify key words, as well as 10 additional strategies. Ortlieb, Perkins, and Verlaan (2012) studied 99 high school students in Algebra 2 to investigate the effectiveness of two instructional strategies: the Modified Cloze Procedure (Greene, 1965) and the Concept of a Definition word map (Schwartz & Raphael, 1985). They found that students who were taught these strategies increased their comprehension of mathematical vocabulary as measured by a pre- and post-test. In addition, their understanding of the mathematical concepts improved as measured by a unit test. Bicer, Boedeker, Capraro, and Capraro (2015) conducted developed a study of 53 eighth-grade students who were employing STEM and Project Based Learning methods during a summer camp and found that mathematical and science vocabulary understanding increased according to a pre- and post-test of vocabulary words. Vessel and Robillard (2013) utilized eight teachers and 31 students with hearing impairments in fourth through eighth grades to investigate mathematics instruction and learning with and without an interactive signing mathematics dictionary. This case study revealed an increase in access to mathematical vocabulary, learner independence, teacher differentiation , students' motivation to learn mathematics, and access to standardized signs.

Chapter Summary

The body of scholarly works concerning mathematical vocabulary spanned over 150 years. The works were categorized by time period to show what areas received more attention by scholars during different time periods and also to reveal areas that are open to additional research. The classification of the works as studies or articles/books may be helpful when considering the findings.

CHAPTER III: METHODOLOGY

This chapter provides the methodologies of the study. The following headings and subheadings organize the chapter and assist the reader.

- Introduction
- Subjectivity
- Research Design
    - Historical Analysis
    - Case Study
- Participant and Site Selection
- Ethical Considerations and Reciprocity
- Data Collection Methods
    - Historical Analysis
    - Case Study
- Data Collection Procedures
    - Historical Analysis
    - Case Study
- Documents
    - Historical Analysis
    - Case Study
    - Journals and Memos
- Data Management and Analysis
    - Historical Analysis
    - Case Study

- Data Analysis
    - Historical Analysis
    - Case Study
- Trustworthiness and Rigor
- Chapter Summary

Introduction

The purpose of this study was to examine the historical perspective of mathematics vocabulary research, as well as its current and future directions. The researcher analyzed the history of mathematics research and organized the findings by major categories and themes. The researcher also interviewed a small group of experts in mathematics vocabulary research to gain insight into current and future directions the research will take. The following research questions guided this study:

1. What content information and trends arise from the examination of literature and research on mathematical vocabulary?
2. What commonalities arise from interviewing experts about mathematical vocabulary?
3. Is there a relationship between the historical and current themes in literature?

The initial plan was to use a Modified Delphi Method to find consensus among the leaders in the field regarding the most significant works in vocabulary research and then conduct a content analysis on the identified works to find common themes and understandings. The results of a pilot study conducted prior to this research determined that there were too few current mathematics education professors working in the field of mathematics vocabulary to conduct a Delphi study. This was meaningful arguments that lead to the decision to conduct a historical analysis and the case study of four participants.

Subjectivity

Cheater (1987) stated, "We cannot rid ourselves of this subjectivity, nor should we wish to; but we ought, perhaps, to pay it very much more attention" (p. 172). Subjectivity is present through the entire process of research, and researchers need to be acutely aware of it (Peshkin, 1982). Therefore, I must acknowledge my subjectivities regarding this topic and how I managed them throughout the research process. I admit that I entered this research study with preconceived ideas that developed over my 28 years as a certified public school teacher. Much of that time I spent focused on improving mathematics instruction, especially for students who struggled with mathematics. I strived to create meaningful learning experiences that would benefit this vulnerable segment of the student population. As a curriculum specialist, I developed curriculum and assessments for students in Grades 6 through 12 and provided professional development to teachers and administrators specifically designed to improve mathematics instruction. As part of my position as a curriculum specialist, I also had the opportunity to visit and observe the secondary mathematics classrooms in a large urban school district in South Texas. As a result, I witnessed students struggle when attempting to discern meaning from mathematical vocabulary within texts and word problems. I admit that it was impossible to enter this research without preconceived ideas, but throughout the research process, I moved forward with an open mind and willing heart in order to learn from the participants in this study and embrace the meaning that the results created. I kept a self-reflective journal throughout the study in order to document my belief system and keep myself in check. I positioned myself as a learner and as a participant-observer (Spradley, 1980) as I approached this study and allowed the data and experiences to teach me the meaning I was intended to gain.

Research Design

The design for this research was a historical analysis coupled with a case study. The historical analysis provided a means to examine the entire body of works concerning mathematical vocabulary in the United States from the mid-1800s to present. The case study revealed how current mathematics education professors view mathematical vocabulary instruction. Combining the historical analysis with the case study gave voice to the past, the present, and the future directions of the field of mathematical vocabulary. A discussion of both methodologies follows. Additionally, it is important to note that throughout the remainder of this chapter, both the historical analysis and the case study portions of the study are discussed in terms of their procedures, documents, data management, and analyses.

Historical Analysis

The first aspect of this study was a historical analysis, which constitutes Chapter II. Johnson and Christensen (2004) stated, “[C]urrent issues in education are frequently the stimulus for the research” (p. 395). The emphasis on the need for students to be able to read and communicate mathematical understandings on high-stakes, state-mandated assessments is the current issue that provided the stimulus for this study. Borg and Gall (1963) explained, “Historical research gives us an insight to some education problems that could not be gained by any other technique” (p. 260). Historical research strives to find patterns and trends within a field; an analysis of these patterns and trends has the potential to reveal new information and direction for future research. “Everything has a history. Everything that exists in the present comes out of the past" (Stahl & Hartman, 2011, p. 213). Much like an archeologist, the historical researcher digs into documents to uncover the historical roots on which our current beliefs are based. Stahl and Hartman (2011) proposed a working definition of history as the "interpretative reconstruction of the known past" (p. 216). They broke this definition down into three elements:

1. History is an interpretation of the past.
2. History is reconstruction of the past.
3. History is about the known past.

History as an interpretation of the past involves a careful analysis of evidence which reveals patterns that produce meaning. History as a reconstruction of the past is the "making again" (p. 217) of an event, thus re-presenting that which happened in the past. It is not the same as the original event but can add meaning and understanding to that which has occurred. History as the known past simply conveys that many events have occurred over time with nothing being saved or recorded; hence, such events have not survived the march of time. It is the known past that can be interpreted and reconstructed to find meaning.

As Smith (1974), Atkinson and Coffey (1997), and Perakyla (2008) suggested, much of social life is recorded and preserved by written documents. These provide a wealth of raw material for qualitative researchers. Perakyla (2008) suggested three main categories of text analysis: discourse analysis, including critical discourse analysis; historical discourse analysis; and finally membership categorization analysis. For the purpose of this study, historical discourse analysis was employed. Armstrong's (2002) research on hygienic rules supported this decision. His work focused on propositional content situated in time and informed by theory and the belief that texts and practices are interwoven.

There exists a commonly held belief that historical research is simply a register of past events. In addition, there are those who believe that history is an impartial and unprejudiced discipline. However, as Berkhofer (1969), Gaddis (2004), and Rury (2006) pointed out, historical research is far more; it includes interpretation (Bloch, 1964). In fact, it may have more room for interpretation than most other fields of research. Gottschalk (1969) went as far as to say that history can be described as a creative reconstruction of bygone eras. Furthermore, it should be noted that historical research is set apart in its methodology. Arthur (2011) stated, "Comparative-historical methodology transcends the qualitative-quantitative division. It is neither qualitative nor quantitative in its approach to data collection and analysis but instead has its own approach" (p. 173).

It is important to state that the historical analysis is situated within the boundaries of the literature review found within Chapter II. This purposeful decision resulted in a literature review crafted in an interrogative format. Most often, literature reviews demonstrate the researcher understands of the foundation upon which the study is based. The literature review for this study, in contrast, extends beyond what is typical in order to capture the depth and richness of the history of mathematical vocabulary scholarly works from the United States.

Case Study

The second aspect of this study was the case study. Creswell (2007) stated, "case study research involves the study of an issue explored through one or more cases within a bounded system" (p. 73). The bounded system in this study was limited to current mathematical professors who had produced scholarly work which addressed mathematical vocabulary. This selection was made to obtain the perspective on mathematical vocabulary from experts within the field of mathematics. Much of what has been written about mathematical vocabulary comes from scholars of other disciplines. According to Yin (2006), the case study provides a means to "illuminate a particular situation, to get a close (i.e., in-depth and firsthand) understanding of it" (p. 112). A goal of this study was to learn how the participants situated themselves in the field of research by understanding what they considered to be the foundation of their scholarly works. The participants further identified the current trends in mathematical vocabulary instruction they deemed worthy of celebration and of concern, as well as their viewpoints of future directions for research concerning mathematical vocabulary. Because the purpose of a case study is to portray the realities of the case, not the world, the results of this study provide insight into this individual case and may not speak as to how all current mathematical professors view mathematical vocabulary research and current practices.

The researcher chose this study design to promote a deep understanding of who and what mathematics education professors identified as foundational, as well as the reasons behind their choices, and to highlight the current trends and future directions of research in the field of mathematical vocabulary instruction. The case study design served as the means to develop a rich understanding, to "come to know it well, not primarily as to how it is different from others but what it is, what it does" (Stake, 1995, p. 8). Building this deep awareness could potentially lead to an understanding of the scholars and scholarly work that are foundational to the field of mathematics vocabulary research. It is not enough to know what is happening within this small group of experts; rather, it is crucial to recognize how meaning is made and why.

Yin (2006) stated, "[G]ood case studies benefit from having multiple sources of evidence" (p.115). This study included participant interviews, discussion of the participants' scholarly works, and the scholars and scholarly works identified by the participants as foundational to their own works. Using multiple sources of evidence provided an avenue for triangulation that lead to robust findings.

The potential roles of the researcher in case studies are teacher, advocate, evaluator, biographer, and interpreter (Stake, 1995). In this study, the researcher's role was that of an interpreter who interviewed the participants to find meaning in their experiences, and together craft a clear image from their words, actions, and artifacts.

Participant and Site Selection

Creswell (2007) maintained that purposeful sampling is used in qualitative research. It is important to select individuals and sites for research because "they purposefully inform an understanding of the research problem and central phenomenon in the study" (p. 125). It is imperative to select information-rich cases that could provide insight into the phenomenon being studied (Patton, 1990). Therefore, the researcher used purposeful sampling strategies for this study.

The mathematics education professors selected for this study were current faculty members at a variety of universities. They were actively participating in research and creating scholarly works which addressed mathematics vocabulary. In addition, they were identified as experts by colleagues. These criteria narrowed the potential participants to those who could provide the greatest insight into the phenomenon that was under investigation. Three of the participants had been identified in a pilot study previously conducted by the researcher. The inclusion of these participants helped to create a common space from which to conduct this research. Stake (1995) supported such action with his statement, "we need to pick cases which are easy to get to and hospitable to our inquiry, perhaps for which a prospective informant can be identified and with the actors (the people studies) willing to comment" (p. 4). In this study, “easy to get to” was figurative and not literal, as the participants were geographically spread out across the United States.

Ethical Considerations and Reciprocity

Ethical considerations must be addressed in a qualitative study. The researcher is charged with protecting the anonymity and confidentiality of the participants, obtaining consent, maintaining transparency about the purpose of the study, and not engaging in deception (Creswell, 2007). To address these issues, the participants were made aware of the purpose of this research through discussion and the use of informed consent. Self-selected pseudonyms protected the participants’ confidentiality, and the artifacts of practice contained no personal information to link the items to the participants. All data collection procedures and documentation used the participants’ pseudonyms to protect their confidentiality. The researcher’s willingness to listen to the participants and work to make their voices heard addressed the issue of reciprocity. The participants received small gifts of appreciation after the interviews, some in-person and others by mail. These gifts consisted of items from the researcher’s home state and gift certificates to book stores. The duration of the interviews was limited so as to honor the valuable time the participants made available for this research. The member checks occurred by phone and email due to geographical and travel time constraints.

Data Collection Methods

Historical Analysis

Historical analysis data collection involves multiple procedures as the researcher seeks to collect all pertinent works to be included in the study. Stall and Hartman (2011) described the process as:

> Looking for historical signs in these readers is like being an archeologist or geologist. But instead of digging into the earth to uncover the past, the historical researcher digs into the visible, everyday elements of reading materials to find the historical roots from which they sprang. (p. 213)

The researcher used all primary resources, as supported by Johnson and Christensen (2004), who indicated that "primary sources are generally viewed as more valuable sources of information" (p. 405). The works included in this study were primary sources which were purchased, printed, copied, or downloaded. "History is the interpretative reconstruction of the known past" (Stall & Hartman, 2011, p. 216) and these works constituted the known past. The analysis was the interpretative reconstruction.

Case Study

Case study data collection includes multiple procedures as the researcher works to build a rich picture of the case. Yin (2003) referred to six forms of data that can be a part of a case study. These include "documentation, archival records, interviews, direct observations, participant observation, and physical artifacts" (p. 85). The goal of data collection procedures is to capture an insider's perspective on the research participants' individual and shared experiences (Stake, 2006). For the purposes of this study the researcher used interviews, artifacts such as scholarly works published by the participants, and the scholarly works identified by the participants as foundational to their own work. Table 2 represents the timeline followed throughout the pilot study. Table 3 represents the data inventory collected during the study.

Table 2

Timeline for Mathematical Vocabulary Case Study

Date	Project item	Participants' roles

July 2014	Submitted IRB	
July 2014	Identified Participants and secured agreement to conduct the study	Agreed to study and signed consent forms
July 2014	Composed interview questions and maintained a researcher's journal.	
October 2014 – March 2015	Conducted interviews and maintained researcher's journal	Provided desirable dates and locations for interviews
October – April 2015	Transcribed and analyzed data Maintained a researcher's journal	Verified meanings and accuracy of narratives researcher captured
November, 2014 – May 2015	Conducted member checks and maintained a researcher's journal	Verified meanings and accuracy of narratives and findings
December 2015	Submitted initial draft of manuscript to chair for reflection and critique and maintained a researcher's journal	Nothing
December- March 2016	Made revisions and maintained a researcher's journal	Nothing
April 2016	Set date for defense	Nothing
April 2016	Submitted dissertation to committee	Nothing

Table 3

Data Inventory

Data source	Pages per data source	Actual total pages
1 interview per participant 1 hour each interview	7 hours 15-20 pages per one hour interview	95
2 peer debriefings with 2 different peers	2x2=4 peer debriefings 3-5 pages per debriefing	18
Journaling and memoing Researcher journaling prior	35 pages of journals	35

and post each interview and observation		
Researcher memoing as needed for reflection and insight	5 pages of memoing	5
Total		153

Data Collection Procedures

Historical Analysis

The essential steps in historical research include "defining the problem, gathering the data, and evaluating and synthesizing the data into an accurate account of the subject investigated" (Borg & Gall, 1963, p. 261). The researcher used Google Scholar to locate scholarly works which addressed mathematical vocabulary. Initial searches yielded an abundance of returns that needed to be examined; for example, using the search term *mathematical vocabulary* resulted in approximately 385,000 results in Google Scholar. Over time, the researcher developed a systematic method for searching and recording data. A log housed the search terms used. The following is a sample of such terms: *math vocabulary, vocabulary math, mathematical vocabulary, math literacy, mathematical literacy, vocabulary of mathematics,* and *mathematic vocabulary.* The researcher scrutinized valuable works to locate additional scholarly works for this study. The search by time period feature within Google Scholar narrowed the search parameter to ensure a thorough examination of potential works. The possibility exists that works were missed; however, the researcher's systematic search patterns, repeated throughout the study, minimized the risk of missing works.

Case Study

The data collection process began with the initial contact to arrange interview times and locations. As suggested by deMarrais (2004), the ultimate success of the qualitative interview rests on the rapport between the participant and the researcher. The initial contacts from the pilot study laid the groundwork for the interactions between the researcher and the other participants. These initial contacts established the relationship, provided an opportunity to explain the nature of the study, and allowed for the collection of signed consent forms. The subsequent interviews explored the perceptions the participants had in regard to mathematical vocabulary historical research, current trends in classroom mathematics vocabulary instruction, and the direction of future research. The interviews were scheduled to be under an hour to clearly demonstrate respect for the participants' valuable and limited time. However, in two of the cases, the participants expanded the time considerably. The researcher captured all interviews with a digital recorder and recorded field notes and reflections in a researcher's journal. Three of the interviews were face-to-face, while one interview occurred via Skype. This was a result of adverse weather conditions that confounded air travel. Two of the in-person interviews took place in the participants' own campus offices. One transpired at a conference that the participant and researcher attended. Digital member checks thereafter resulted in brief follow-up conversations by phone, Skype, and email. Again, these respected the participants' valuable time.

Documents

Historical Analysis

The researcher analyzed the works in order to determine possible categories for classification. A follow-up search using these categories revealed additional scholarly works that had not previously been found. The category word or phrase served as the new search term in Google Scholar. In many instances, it was difficult to obtain articles found during these searches. Upon finding a potential title that would fit into this research, the researcher conducted follow-up searches using the data bases in the university library system. This process resulted in the successful location of the most scholarly works that became a part of this historical analysis. The researcher also canvassed the National Council of Teachers of Mathematics (NCTM) publication site after obtaining the required membership that grants full access to all archived journal and research articles published by NCTM.

The researcher printed, purchased, or saved the scholarly works as eBooks and wrote summaries of each in order to capture their central ideas or, as in the case of a published study, to reveal the findings, methodologies, sample sizes, limitations, and references. Copies of the articles and studies were organized by decade in binders, and in some cases, articles could not be printed. The researcher retyped these into Word documents and then added them to the binders. The books received a similar treatment, whether physical or digital.

Case Study

The researcher categorized the participants' scholarly works and compared them to the overall body of scholarly work within the field of mathematical vocabulary in order to provide points of discussion for the interviews and to drive the conversation to a deeper, more meaningful place. This also ensured that the participants felt valued and respected as experts.

Reference to and understanding the participants' bodies of work allowed the researcher to demonstrate preparedness and propose questions that were situated within the body of research.

Journals and Memos

Charmaz (2006) pointed out that memos are used to record the details during the journey of collecting data, and journals are a window into the interpretations of the researcher in relation to the data. Memos provide a summary of what was seen and what was deemed important. Journals are more reflective and help record the thought processes of the researcher. In this study, both journals and memos helped the researcher to clarify thoughts and feelings and to reflect upon and record the data. The journal provided not only self-reflection, but also a sense of direction that guided the researcher's steps throughout the research process. It also supported the treatment and analysis of the data which clarified the meaning of the researcher's observations.

Data Management

Historical

The data management of the historical works evolved during the collection of the artifacts. Initially, the works largely consisted of downloaded articles. As the number of potential works being considered for inclusion into this study grew, the management system had to adapt. The researcher printed the works and organized them in binders according to general topic. This method, too, had to change as the study progressed. The final data management system was comprised of printed articles and title pages from books, which were organized by time periods and stored in binders.

Case Study

The data management of the case study also evolved from storage in folders to organization within a binder that was locked in a file cabinet in the researcher's home office. The binder was divided by participant and contained the transcriptions of interviews, emails, and copies of the participants' scholarly works. In the cases where the scholarly works were books, summaries took the place of the entire work.

Data Analysis

Historical

The researcher wrote a summary for each primary source, either an article, a chapter in a book, or a book. The summaries included the sample sizes, treatments or measures employed, and findings. Bibliographic cards contained typical reference information, such as title, author, date, publisher, and identified the work as an article, chapter, or book. Content cards provided a brief summary of each work. Cards for studies included a brief description of the type of study, sample size, treatment, and findings as appropriate for each study. Content cards for non-studies included the topic of the work, the author's main points, a brief summary, and sometimes quotes.

Leedy and Ormrod's (2009) model served as a reference for the analysis. Although Leedy and Ormrod advocated for creating three of each card for the purpose of organization by chronological order, by author, and by subject, the researcher elected to organize by chronological order and by subject only. There were a few instances of multiple works by the same author but not so many that there was a need for organizing cards.

According to Stahl and Hartman (2011), the process for constructing history consists of three parts:

> [T]he philosophical stance held by the writer, the evidence that was examined in the available primary and secondary sources, and the researchers' interpretation of the data. The facts that are gathered by the historian say little in and of themselves…The process of inductively interpreting the facts individually or in sets depends on having both mastery of the data and an understanding of the significance of that material." (p. 225)

The summaries of each work and the organization of the content cards represent the researcher's interpretation of the data. The researcher organized and reorganized the content cards until the resulting structures were stable categories and could be arranged into themes.

Case Study

The case study utilized multiple sources of data: interviews, the participants' scholarly works, and the scholars and scholarly works the participants identified as foundational to their own work. Using three sources of data provided the opportunity to triangulate and have more robust findings (Yin, 2006).

The researcher captured the interviews using a digital voice recorder. The interviews ranged from 48 minutes to two hours, the longest of which included several side conversations that were not contained within the final transcription. The interviews occurred over a span of five months. Three interviews took place in the participants' offices, and one happened at a research conference. The participants, all mathematics education professors, resided in four different states situated in the northeast, south, and the western regions of the United States. The geographical challenges, coupled with scheduling and travel conditions, resulted in one interview conducted using Skype. The in-person interviews were significantly longer than the Skype interview and gave rise to more personal connections that were difficult to achieve through a digital experience. A digital voice recorder captured all of the interviews so they could be saved as media files on the hard drive of the researcher's computer. To protect the confidentiality of the participants, the researcher erased the digital voice recorder and deleted the media files upon completion of the transcriptions.

The researcher transcribed each of the interviews over a span of weeks following each of the conversations. Although there are various ways to transcribe interviews, the researcher decided to transcribe them verbatim (Carlson, 2010). Using a verbatim method of transcribing the false starts, pauses, and repetitive statements present in the interviews preserved the natural flow of the dialog between the participant and researcher. The transcription process gave life to the experiences and assisted with the analysis process. The researcher also recorded reflections and insights in a researcher's journal throughout the experience. Upon completion of the transcriptions, the researcher transferred the data to a flash drive and stored it in a locked file cabinet and deleted the computer files that contained the transcriptions. The researcher did not identify the participants in the transcriptions; hence further safe guarding the participants' confidentiality.

The researcher emailed the transcriptions to the participants for member checking. The participants reviewed the transcriptions, added additional comments, edited, and/or deleted comments as necessary to fully capture their unique thoughts and insights. In order to protect their confidentiality, the researcher deleted all emails after receiving responses from the participants.

The researcher used in vivo coding to analyze the interview transcriptions. This coding style lifts words and phrases from the actual language used in the transcription of the interviews (Saldana, 2009). In vivo coding honored the participants' voices. The process of coding began with an examination of each interview question separately and involved reading line-by-line and highlighting words and phrases in each interview to establish the codes. The researcher coded and analyzed the transcripts for each participant individually and recorded the codes on sticky notes in order to arrange and rearrange them to identify emerging categories. Once stable categories surfaced, the researcher photographed the organization and recorded the information in a table. Then the researcher grouped the sticky notes containing the codes from each participant by interview question and reanalyzed. Again, the researcher photographed the organization and recorded the information in a table to reveal any emerging themes.

The researcher utilized Wordle to analyze each participant's response to the interview questions. McNaught and Lam (2010) identified Wordle as a useful research tool. In this study it supplemented the more traditional analysis typically found within a case study.

The participants' scholarly works functioned as a second source of data in the case study. The researcher read, summarized, and recorded all works on index cards, which were both bibliographic and content-focused. The researcher organized and reorganized the content cards until the resulting structures formed stable categories. Many of the participants' works had already been analyzed as part of the historical analysis but not all, due to the fact that the participants also authored works that did not focus on mathematical vocabulary.

The scholars and scholarly works identified by the participants as foundational to their own work supplied the final source of data in the case study. The researcher identified and summarized each of these works in much the same way as the historical analysis. The summaries were briefer and were labeled according to their type such as *reading, mathematical content,* and *vocabulary.* This offered perspective on the types of works that these scholars produced. The researcher then categorized the resulting information by author and also by type.

The three sources of data, the interviews, participants' scholarly works, and works identified as foundational to the participants provided triangulation for this study. The researcher studied the individual analysis of each data source to determine if there were any overarching themes which could describe the entire case study.

Trustworthiness and Rigor

Qualitative researchers are concerned with consistency between the collected data and the results. In qualitative research, trustworthiness needs to be established. Lincoln and Guba (1985) used terms such as credibility, authenticity, transferability, dependability, and confirmability to describe trustworthiness of qualitative research. Angen (2000) suggested that validation is "a judgment of the trustworthiness or goodness of a piece of research" (p. 387). The researcher addressed trustworthiness throughout the study.

To begin with, the researcher spent adequate time in the field collecting data and scholarly works. The course of this study spanned two years, from the initial contact to the conclusion of engaging with the topic and participants. This has been reported to be an essential aspect of research to build trust with participants and to learn the culture of the research site(s) (Glesne & Peshkin, 1992; Lincoln & Guba, 1985; Merriam, 1988). The researcher also utilized peer review and debriefing to provide an external check on the study (Erlandson, Harris, Skipper, & Allen, 1993; Glesne & Peshkin, 1992; Lincoln & Guba, 1985; Merriam, 1988). The researcher periodically checked in with other doctoral students who were writing dissertations in the field of education to discuss the on-going research and kept written records of these peer debrief sessions.

The use of multiple sources of data provided corroborating evidence (Ely, Anzul, Friedman, Garner, & Steinmetz, 1991; Erlandson et al., 1993; Glesne & Peshkin, 1992; Lincoln & Guba, 1985; Merriam, 1988; Miles & Huberman, 1994; Patton, 1990). Three data sources, which were the interviews, participants' scholarly works, and the works the participants identified as foundational to their own, triangulated the data (Merriam, 1998). The researcher analyzed each piece of data as it was collected, thus allowing for constant comparison of the data. In addition the code-recode strategy enhanced the dependability of the case study (Chilisa & Preece, 2005). The researcher coded the same data twice with a week or two in between each experience and then compared the results of the two coding sessions. This allowed for consistency in the coding practice and a deeper understanding of the patterns that developed.

Member checking allowed for insight into the participants' viewpoints and determined the credibility of the researcher's interpretations and findings. Lincoln and Guba (1985) considered this to be the "the most critical technique for establishing credibility" (p. 314). The participants had the opportunity to examine the researcher's rough drafts and provide feedback regarding what was written and what was left out. This was also critical to establishing authenticity with the participants. Through member checking, the participants established whether or not their viewpoints and experiences had been treated faithfully and fairly.

Chapter Summary

This chapter communicated the research purpose and questions, as well as firmly situated this study in historical analysis and framed the qualitative analysis in the theoretical framework of symbolic interactionism. The method revealed the processes of participant selection and data collection and analysis. This chapter also provided the procedures for considering ethical implications and reciprocity.

CHAPTER IV: RESEARCH FINDINGS

Overview of Results

This chapter provides the results of the study. The following headings and subheadings organize the chapter and assist the reader.

- Introduction
- Results of the Historical Analysis
- Findings from Research Question One
- Categories that Emerged
- Discussion
    - Annotated Bibliography
    - Discourse
    - ELLs' Issues
    - Instructional Strategies for Mathematical Vocabulary
    - Mathematics as a Language
    - Readability of Mathematics Textbooks
    - Reading Inventory
    - Taxonomy of Mathematical Vocabulary
    - Vocabulary Knowledge and Mathematical Success
    - Vocabulary Lists
    - Vocabulary Relationship to Problem Solving
- Development of Themes
- Results of the Case Study
- Data Management and Analysis

- Interviews with Participants

- Findings from Research Question Two
    - From Codes to Themes
    - Participants’ Scholarly Works
    - Development of Themes
    - Participants’ Foundational Works
- Chapter Summary

Introduction

The purpose of this study was to examine the historical perspective of and the current and future directions for mathematics vocabulary research through the use of historical analysis and a case study. A historical analysis of literature and research pertaining to mathematics vocabulary in the United States revealed major themes that have emerged over the years. Stahl and Hartman (2011) asserted that “Everything has a history. Everything that exists in the present comes out of the past” (p. 213). The case study determined the views of current mathematics education vocabulary experts and how they situated themselves within the body of research, as well as what they considered to be current trends in mathematical vocabulary instruction and future directions of research in the field of mathematical vocabulary. Glesne (2011) stated that qualitative researchers “seek to interpret people’s constructions of reality and identify uniqueness and patterns in their perspectives and behaviors” (p. 19). Four mathematics education professors participated in the case study. Data was collected from interviews, participants’ observations and scholarly works, and the scholarly works identified by the participants. The use of multiple data collection methods triangulated the data and enhanced the validity of this study. Pseudonyms preserved the confidentiality of the participants.

Results of the Historical Analysis

Research Question One

Research Question One – *What themes arise from a historical analysis of mathematical vocabulary research?* The historical analysis examined scholarly works pertaining to mathematical vocabulary in the United States. Two primary themes emerged: Instructional Strategies and ELLs' Issues. Instructional Strategies included the works that addressed how to teach mathematical vocabulary to students. ELLs' Issues consisted of the works that addressed mathematical vocabulary in terms of English Language Learners. This category contained works specifically dedicated to ELLs and the unique issues pertaining to the teaching and learning of mathematical vocabulary for this particular population. Three categories did not fit into these themes: Annotated Bibliography, Vocabulary Lists, and Reading Inventory.

Categories that Emerged

An array of scholarly work has been written about mathematics and vocabulary in the United States since the mid-1800s. The researcher used an interpretive and reconstructive process to ensure that the historical themes were not considered in isolation, but were situated within the context of the times in which they were written (Stahl & Hartman, 2011). Table 4 contains the categories of works that resulted from the historical analysis. The categories are as follows: Annotated Bibliography, Discourse, ELLs' Issues, Instructional Strategies for Mathematical Vocabulary, Mathematics as a Language, Readability of Mathematics Textbooks, Reading Inventory, Taxonomy of Mathematics Vocabulary, Vocabulary Knowledge and Mathematical Success, Vocabulary Lists, and Vocabulary Relationship to Problem Solving. The category of Mathematical Success includes the scholarly work that referred to good performance on a variety of assessments and passing scores in mathematics courses.

The table is organized alphabetically by category and also by publication date. Each work is identified as an article, chapter in a book, a book, or a study. In addition, a brief summary is provided for each work. It is important to note that some scholarly works are listed in more than one category or theme. The ultimate identification was based on the best fit(s) for the individual work. The groupings were reexamined throughout the study in an attempt to capture deeper understandings that developed. The groupings evolved as additional works were added and new categories emerged.

Table 2

Categories of Scholarly Works

Category	Author(s)	Type of publication	Date	Summary
Annotated Bibliography	Austin & Howson	Article	1979	Writings on the interaction of language and mathematical education
Discourse	Young	Book	1920	Worked towards accuracy of expression and used the newly acquired mathematical vocabulary
	Pimm	Book	1987	Viewed mathematics and its teaching in linguistic terms. Examined and illustrated the components of language to develop conceptual understanding and vocabulary
	Small	Article	1990	Suggested heavy reliance on student talk in primary mathematics classrooms; little pencil time with the exception of writing – not computing
	Brenner	Article	1998	Discourse in problem solving groups for ELLs
	Garrison & Kerper Mora	Chapter	1999	Use of discourse to develop mathematical vocabulary with ELL students
	Moschkovich	Article	1999	Discourse as means to enhance development of mathematical vocabulary with ELLs
	Steele	Article	1999	Discourse as a means to learn vocabulary
	Zazkis	Study	2000	Suggested moving between mathematics register and everyday

				English register
	Gay & White	Article	2002	Teaching vocabulary to enhance ability to communicate mathematically
	Raiker	Article	2002	Use of correct terminology did not equate to conceptual understanding on the novice to expert continuum
	Moschkovich	Article	2003	Explored what constituted mathematical discourse and what to listen for in student talk
	O'Halloran	Chapter	2003	Mathematical discourse limitations were observed by teachers of working class students and of female students
	Barwell	Article	2005	Argued the necessity of both non-formal and formal mathematical language used in discourse
	Hancewicz	Chapter	2005	Using discourse to develop mathematical meaning and vocabulary
	Moschkovich	Article	2007	Understanding features in discourse; distinguishing between every day and mathematical discourse
	Thompson, Kersaint, Richards, Hunsader, & Rubenstein	Book	2008	Provided students with intentional grouping groupings and practices that promoted the active usage of the mathematical language through reading, listening, speaking, and writing
	Smith & Stein	Book	2012	Orchestrated mathematically productive discussions
ELLs' Issues				
	Cuevas	Article	1984	Need for mathematics curriculum that attends to second-language skills
	Garbe	Article	1985	Instruction for the culturally diverse students needed to develop mathematical language
	Davison & Schindler	Study	1988	Examined mathematical vocabulary acquisition in Crow reservation schools
	Brenner	Article	1998	Discourse in problem solving groups for ELLs
	Garrison & Kerper Mora	Article	1999	Use of discourse to develop mathematical vocabulary with ELL students
	Moschkovich	Article	1999	Discourse as means to enhance development of mathematical

				vocabulary with ELLs
	Abedi	Study	2000	Students classified as ESL had considerably lower mathematics performance than other students on NAEP
	Kiplinger, Haug, & Abedi	Study	2000	Student performance on assessments with high proportions of word problems was directly related to their proficiency in reading in English
	Zazkis	Article	2000	Shifting between formal mathematic register and the register of everyday English as a tool for learning mathematical language
	Abedi & Lord	Study	2001	ELLs performed lower on word problem assessments – vocabulary issues
	August, Carlo, Dressler, & Snow	Article	2005	Leverage understanding of first language, ensure ELLs know the meaning of basic words, and review and reinforce vocabulary
	Kenny	Chapter	2005	Explored the difficulties involved with learning math vocabulary; particularly for ELLs
	Bay-Williams & Livers	Article	2009	Providing support as math vocabulary is learned and used by ELLs
	Roberts & Truxaw	Article	2013	Pre-teach, work walls, graphic organizers, and vocabulary lists were suggested
	Livers & Bay-Williams	Article	2014	Providing support to construct meaning and being cautious to not obstruct meaning
Instructional Strategies for Mathematical Vocabulary				
	Bailey	Book	1898	Suggested a step-by-step development of vocabulary
	Speer	Book	1899	Learning math vocabulary without experiential and conceptual development had little value
	Smith	Book	1902	Argued again learning definitions; but advocated contextual learning of mathematics and associating the terms to the concept
	Bond & Bond	Book	1941	The teacher needs to help develop vocabulary and understanding of the

			concepts
Hastings	Study	1941	Examined how vocabulary grew during year based on instructional practices
Brune	Article	1953	Emphasized correct teacher usage of mathematical terms
Peeler	Article	1956	Role of vocabulary instruction by mathematics teachers
Totten	Article	1956	Role of vocabulary instruction within mathematics instruction
Strang & Bracken	Book	1957	Identified difficulties inherent to mathematical vocabulary and addressed strategies through reading instruction
Hogan, Green, & Bamman	Chapter	1961	Write definitions, use context clues, use word parts
Lerch	Article	1967	Suggested developing speaking and reading vocabularies of math terms both in words and symbols. Focused on oral development of the language of mathematics.
Strang, McCullough, & Traxler	Chapter	1967	Created a pretest and posttest of terms, should say terms slowly and have students repeat them, use terms repeatedly
Earp	Article	1970	Suggested instructional strategies for vocabulary, reading, and problem solving in mathematics
Robinson	Book	1975	Suggested that in mathematics, concepts and terms must be taught in context
Dolgin	Chapter	1977	Suggested strategies for vocabulary development and reading comprehension for students to be able to read mathematics textbooks
Hollander	Article	1977	Expanded on Earp's work. Cautioned that without reading strategies for mathematics textbooks students must rely on oral instruction alone
Krulik	Article	1980	Discussed terms, constructing a math dictionary, use of acrostic puzzles, and matching definitions to terms
Thomas & Robinson	Chapter	1982	Suggested that students look up definitions, look at word parts, rewrite definitions in student words, make a list of math terms

Milligan & Milligan	Article	1983	Suggested a linguistic approach to help students learn vocabulary
Rothman & Cohen	Article	1989	Suggested strategies for vocabulary development and use of task analysis to alleviate math difficulties related to vocabulary and problem solving
Capps & Pickreign	Article	1993	Recommended that teachers discuss words, provide matching activities, use manipulatives to learn concepts and to focus discussion
Monroe	Article	1997	Graphic organizers to teach vocabulary
Krussel	Article	1998	Suggested the use of dictionaries in mathematics classes, practice using words by speaking and writing sentences
Schwarz	Study	1999	Analyzed the use of word walls and journaling on vocabulary understanding
Monroe & Pendergrass	Article	2000	Graphic organizers to teach vocabulary (Frayer)
Rubenstein	Article	2000	Learning word origins to enhance the learning of math vocabulary
Rubenstein & Schwartz	Article	2000	Integrating history into vocabulary instruction to build understanding
Thompson & Rubenstein	Article	2000	Provided a list of potential pitfalls and proposes a spectrum of strategies to address them
Monroe & Orme	Article	2002	Suggested using mathematical contexts, explicit instruction of words, and a combination of both
Leung	Article	2005	Strategies connected with concepts expand as meaning is created
Pearce & Reynolds	Chapter	2005	Concept maps, possible sentences (writing activity), word wall, analogies, games
August, Carlo, & Dressler	Article	2005	Concerned over little research on effectiveness of vocabulary instructional strategies
Capraro & Joffrion	Study	2006	Examined students' ability to translate between words in problem solving situations and algebraic representations using symbols. The relative success was attributed more too conceptual understanding than the procedural approach strategies that

				were taught.
	Rubenstein	Article	2007	Strategies to address challenges of learning math vocabulary and using it fluently
	Gay	Article	2008	Worked with preservice teachers to address instructional strategies in lesson plans
	Riccomini, Sanders, & Jones	Article	2008	Mnemonic instructional strategies with the specific attention on keyword strategy
	Thompson, Kersaint, Richards, Hunsader, & Rubenstein	Article	2008	Suggested to introduce vocabulary after concept development, creation of personal dictionary, use word origins and word parts to develop word meaning, graphic organizers, use invented language and transition to formal
	Murray	Book	2004	Teach vocabulary in context through intentional learning design with a focus on immersion
	Spencer & Guillame	Article	2009	Provided 35 strategies for developing content area vocabulary – general strategies
	Cirillo, Bruna, Hebel-Esienmann	Article	2010	Strategies making connections between words and concepts
	Salinas & Ortlieb	Article	2011	Graphic organizers and word walls were suggested strategies
	Ortlieb, Perkins, & Verlaan	Study	2012	Investigated effectiveness of strategies: Modified Cloze Procedure and Concept of a Definition word map.
	Dunston & Tyminski	Article	2013	Graphic organizers were suggested
	Roberts & Truxaw	Article	2013	Pre-teach, word walls, graphic organizers, and vocabulary lists were suggested
	Vessel & Robillard	Study	2013	Examined mathematics instruction with and without use of an interactive signing mathematics dictionary with students who have a hearing impairment
	Bicer, Boedeker, Capraro, & Capraro	Study	2015	STEM Project Based Learning methods beneficial for math and science vocabulary learning

	Pace & Ortiz	Article	2015	Vocabulary chart activity for kindergartners
	Riccomini, Smith, Hughes, & Fries	Article	2015	Explicit instruction, mnemonic strategies, multiple exposures
Mathematics as a Language				
	Brune	Article	1953	Emphasized using language of mathematics effectively to help students think critically and communicate understanding
	Capps & Pickreign	Article	1993	Content of math cannot be taught without language
	Olivares	Book	1996	Identified three characteristics that make communication in mathematics as a foreign language.
	Usiskin	Chapter	1996	Instructional practices were affected when considering mathematics as a language
	Cirillo, Runa, Herbel-Eisenmann	Article	2010	Recommended instructional practices to develop the language of mathematics
Readability of Mathematics Textbooks				
	Brooks	Study	1926	Low repeat use of vocabulary which did not support learning new terminology; Grades 3-8
	Remmers & Grant	Study	1928	Found same as Brooks; occurs in secondary mathematics textbooks
	O'Rourke & Mead	Study	1941	Vocabulary difficulties in Grade 3 textbooks
	Johnson	Study	1957	Readability of mathematics textbooks using Flesch
	Repp	Study	1960	Examined new mathematical vocabulary introduced in five textbooks
	Stauffer	Study	1966	Examined the vocabulary demands in mathematics textbooks
	Wiegand	Study	1967	Readability of mathematics textbooks using Dale-Chall
	Hater & Kane	Study	1970	Readability of mathematics textbooks using Cloze method
	Kane	Study	1970	Readability of mathematics textbooks flawed. Flesch, Dale-Chall, others

				not appropriate tool
	Byrne & Kane	Study	1971	Measured vocabulary and symbol familiarity for readability measures of text
	Wilmon	Study	1971	Vocabulary demands over time Grades 1-3
Reading Inventory				
	Cox & Wiebe	Article	1984	Authors created the Wiebe/Cox mathematical vocabulary reading inventory for first through third grade
Taxonomy of Mathematical Vocabulary				
	Buckingham	Study	1937	Three categories: 1) non-technical words (silo), 2) technical mathematical terms (cube), 3) technical Algebraic terms (monomial)
	Austin	Article	1961	Two categories: 1) terms with a different mathematical meaning than their general meaning, 2) terms with a mathematical meaning that is more precise than general meaning
	Shepherd	Book	1973	Four categories: 1) technical, 2) general word having a mathematical meaning, 3) signals a mathematical process, 4) general word which can determine comprehension
	Kossack & Vigilante	Article	1981	Proposed a taxonomy consisting of standard, transitional, technical, changeable, and phrases
	Monroe & Panchyshyn	Article	1996	Four major categories: technical, subtechnical, general, and symbolic
Vocabulary Knowledge and Mathematical Success				
	Davies & Peck	Book	1855	Proposed that the difficulty of the study of mathematics is due to misunderstanding of mathematical vocabulary
	Peirce	Article	1881	Specific academic language issues of vocabulary interferes with algebra success
	Aley	Article	1907	Emphasized the importance to teach vocabulary in mathematics for

				students to be successful
	Betz	Article	1913	Importance of learning vocabulary of mathematics for success in mathematics
	Yocum	Article	1914	Importance of vocabulary understanding for success in mathematics
	Merrill	Article	1918	Purported that vocabulary issues in mathematics are primary reason students fail
	Minnick	Article	1921	Argued that the language of math is necessary to be able to learn and communicate understanding of concepts
	Georges	Study	1929	23.4% of all reading difficulties in mathematics text were due to vocabulary
	Monroe & Engelhart	Article	1933	Systematic instruction focused on vocabulary found to be not effective with bright students and relatively effective with slower students
	Pressey & Moore	Study	1932	Not mastering terminology most critical reason for lack of success in mathematics
	Buckingham	Study	1937	Significant correlation found between vocabulary and ability to perform Algebra
	Aiken	Article	1972	Review of mathematics education research focused on language factors and learning mathematics
	Jackson & Phillips	Study	1983	Students who received vocabulary instruction achieved higher verbal and computational scores on achievement assessments
	Capraro & Joffrion	Study	2006	Students without deep understanding of vocabulary relied on procedures to solve problems that may or not be appropriate to the problem
Vocabulary Lists				
	The National Committee on Mathematical Requirements	Article	1921	Created a list of terms and symbols for elementary mathematics (math taught prior to college)
	Monmouth, Isbell, Jenkins, &	Study	1924	Examined lists of words teachers submitted as troublesome for students in solving arithmetic problems.

	Pieters			
	Buswell & John	Study	1931	Collection of arithmetical terms
	Pressey, Pressey & Zook	Study	1932	Essential vocabulary list for plane geometry
	Pressey, Pressey, & Narragon	Study	1932	Essential vocabulary list for algebra
	Cowley	Study	1934	Vocabulary list for plane and solid geometry
	Austin	Article	1961	Suggested teachers make a list of the mathematical terms that needed to be taught in their course(s)
Vocabulary Relationship to Problem Solving				
	Thorndike	Article	1922	Connection between being able to read the formula and the symbols and the appropriate context for usage
	Wilson	Article	1922	Reasons for failure in mathematics: poor ability to read word problems
	Monmouth, Isbell, Jenkins, & Pieters	Study	1924	Examined lists of words teachers submitted as troublesome for students in solving arithmetic problems.
	Lessenger	Study	1925	Arithmetic errors attributed to vocabulary and reading decreased significantly with focus on instruction in reading
	Monroe & Engelhart	Study	1933	Extensive and specific instruction of reading and math vocabulary only improved arithmetic problem solving with students whose IQ was less than 100
	Dresher	Study	1934	Extensive and specific instruction of math vocabulary improved the ability to solve arithmetic problems
	Drake	Study	1940	Students receiving instruction of vocabulary improved algebra achievement
	Tiews	Article	1943	English teacher's perspective of vocabulary hindering students' ability to problem solve
	Johnson	Study	1944	Vocabulary instruction improved performance on vocabulary

			assessments and did not transfer to other situations or improve problem solving
Treacy	Study	1944	Vocabulary was the reading skill that separated good from poor problem solvers
Hansen	Study	1944	General reading and vocabulary instruction not essential in verbal problem solving
Eagle	Study	1948	Students with greater understanding of mathematical vocabulary had greater achievement on standardized mathematics assessments and course grades
Johnson	Study	1949	There was a relationship between vocabulary knowledge and intelligence; and with arithmetic problem solving
Faulk & Landry	Study	1961	Four-pronged approach to problem solving resulted in slight improvement on standardized assessment
Vanderlinde	Study	1964	Knowledge of quantitative vocabulary linked to greater achievement in arithmetic problem solving.
Harvin & Gilchrist	Study	1970	Relationship between problem solving and reading
Olander & Ehmer	Article	1971	Relationship between vocabulary and arithmetic problem solving
Capraro & Joffrion	Study	2006	Students without a deep understanding of vocabulary relied on procedures that may or not have been appropriate to the Algebra problem they were attempting to solve
Pearce, Bruun, Skinner, & Lopez-Mohler	Study	2013	Perceptions of teachers concerning student difficulty with word problems

Annotated Bibliography and Reading Inventory

The smallest categories were Annotated Bibliography, which contained one work by Austin and Howson (1979), and Reading Inventory, which also had one work by Cox and Wiebe (1984). These two categories did not fit well with the other categories but were important enough that they need to be included. The reading inventory article reported on the creation and testing of the Wiebe/Cox Mathematical Vocabulary Reading Inventory for first through third grades.

Discourse

Both the categories of Discourse and ELLs' Issues contained the most recent works. Several of the works overlapped the two categories. Young (1920) advocated that students needed opportunities to verbally use newly acquired mathematical vocabulary to work toward accuracy of expression. Silence in this category lasted for 67 years and reemerged with Pimm (1987) who supported the notion that mathematics and its teaching should be conducted with the linguistic lens. Subsequent works examined discourse as a means of enhancing mathematical vocabulary understanding, aspects of the discourse to listen for within a mathematics classroom, and suggestions on how to orchestrate productive mathematical discussions.

ELLs' Issues

The ELLs' Issues category began in 1984 with Cuevas' work that identified the need to attend to second-language skills in mathematics curriculum. Every work in this category specifically addressed issues or strategies recommended for ELLs. Some works included discourse concerns, the relationship between vocabulary and performance on assessments, and problem solving. This category was created to house works that may exist in other categories, but focused on specific needs or concerns that students learning English encountered in mathematics.

Instructional Strategies

The category Instructional Strategies for Mathematical Vocabulary had the greatest longevity. It emerged as a topic in 1898 and continued in all but one decade through the 2000s. It also housed the most works, with a total 50, only ten of which studies. This category intertwined with the categories of ELLs' Issues and Discourse. It is important to note that nearly every one of the works in this category suggested strategies which were not necessarily based on the findings of research. This category consisted primarily of articles that proposed suggested practices. The recommended instructional strategies included: writing the vocabulary words on the board, student math journals, student-created dictionaries, word walls, discussion, group work, using meaningful mathematics tasks, and more.

There were six studies in this category. Hastings (1941) studied 331 ninth graders to determine if singular testing or multiple opportunities were needed to ascertain students' mastery of mathematical vocabulary meanings. He concluded that mathematical vocabulary needed to be assessed in multiple ways to determine if a student has solidly connected the concept with the word. Although this study concentrated on the assessment of mathematical vocabulary, it fit best into the Instructional Strategies category as compared to others. This decision was justified by the relationship between instruction and assessment. Capraro and Joffrion (2006) performed a study of 668 algebra students in middle school to determine students' ability to translate mathematical words to symbolic representations. They found that 9% of the students were able to answer all three assessment questions correctly; two were multiple-choice and one question was a short answer response. They concluded that the relative success that students displayed might have been attributed to conceptual understanding rather than procedural. They recommended that teachers expose students to multiple problems with decreasing teacher support over time. Additionally, they suggested that teachers equip students with a method for analyzing equations that students generate from problem situations. Vessel and Robillard (2013) examined 31 students who were hearing impaired and eight teachers in fourth through eighth grade. The study considered the effectiveness of using an interactive signing mathematics dictionary for students with hearing impairments. They found that students who had access to the interactive signing mathematics dictionary used it to learn mathematical vocabulary. They also noted that having this technology promoted individualized instruction and allowed for student self-learning. Ortlieb, Perkins, and Verlaan (2012) organized a study of 99 high school students to investigate the effectiveness of using a Modified Cloze Procedure and a Concept of a Definition word map with Algebra 2 students. They reported that students' comprehension of mathematical vocabulary improved as demonstrated by pre- and post-tests. They also observed that students improved their understanding of content when using the instructional strategies that were taught in this six-week study. Bicer, Bodecker, Capraro, and Capraro (2015) conducted a two-week study of 53 eighth-grade students during a STEM summer camp. Students took pre- and post-tests of 24 academic vocabulary words, 12 of which were mathematical and 12 that contained scientific terms. They found that students' understanding of the words grew due to the combination of contextual and direct instruction.

Mathematics as a Language

The category of Mathematics as a Language contained work which argued that mathematics needed to be thought of as a language due to its unique vocabulary (Olivares, 1996). Brune (1953) and Capps and Pickreign (1993) emphasized that language needed to be employed effectively to foster students' ability to think critically and effectively communicate their understanding of mathematics. Cirillo, Runa, and Herbel-Eisenmann (2010) recommended specific instructional practices that would help to develop the language of mathematics for students. Usiskin (1996) cautioned that instructional practices are affected when mathematics is considered as a language. All of the works in this category were articles, with the exception of one that was a book. This category of works overlapped with works in the Discourse category.

Readability of Mathematics Textbooks

The category of Readability of Mathematics Textbooks was represented in five decades of scholarly works ranging from 1926 to 1971. These 11 publications measured readability in a variety of ways using tools such as Flesch, Dale-Chall, Cloze method, vocabulary and symbol familiarity, and repeated use of mathematics vocabulary.

Reading Inventory

One of the two smallest categories, Reading Inventory, contained a single work by Cox and Wiebe (1984). This category did not fit well with the other categories; however it needed to be included and was important enough to stand on its own. The reading inventory article reported the creation and testing of the Wiebe/Cox Mathematical Vocabulary Reading Inventory for first through third grades.

Taxonomy of Mathematical Vocabulary

Taxonomy of Mathematical Vocabulary was also a small category with five scholarly works. This category could have been eliminated or identified as *other*; however, the decision to include it was based on the need to draw attention to this category and ensure that it was not overlooked. Buckingham (1937) identified three categories of mathematical vocabulary. Austin (1961) named two categories, while Shepherd (1973) and Monroe and Panchyshyn (1996) proposed four categories. Kossack and Vigilante (1981) acknowledged five categories. With the exception of Austin, the other scholars identified *technical* as a specific category of mathematical vocabulary within their taxonomies.

Vocabulary Knowledge and Mathematical Success

The category of Knowledge and Mathematical Success launched in 1855 and continued with multiple scholarly works through the 1930s. It reemerged in the 1970s through the 1980s and included one work in the 2000s. This category joined that of Bilingual Issues and intertwined with Vocabulary Relationship to Problem Solving. Mathematical Success was a general title created to capture scholarly works that linked vocabulary to assessment results and the ability to pass mathematics course work. The works prior to the 1920s were not studies and were instead based on experiences and observations that could be considered wisdom of practice. The works from the 1920s and later were all studies, and generally they found that mathematical vocabulary knowledge was related to mathematical performance on assessments. One such work was from Capraro and Joffrion (2006). It examined how well students were able to translate between words and symbols in algebraic situations. They cautioned that if students did not internalize the meanings of the vocabulary encountered in word problems, they would resort to procedural work that may or may not be appropriate in the given situation, thus limiting their opportunity to be successful in mathematics.

Vocabulary Lists

The category of Vocabulary Lists was primarily limited to the 1920s and 1930s, with one article in the 1960s. Overall, the works within this category sought to determine the lists of mathematical vocabulary needed for students to be successful in mathematics. The studies Pressey and colleagues (Pressey, 1924; Pressey & Moore, 1932; Pressey, Pressey, & Narragon, 1932; Pressey, Pressey, & Zook, 1932) conducted were unique within this category because they sought to find consensus amongst classroom teachers to establish the lists. The final work in this group by Austin (1961) suggested that teachers create a list of terms needed for the course(s) they teach.

Vocabulary Relationship to Problem Solving

The category of Vocabulary Relationship to Problem Solving emerged in 1922 and was a frequent focus of scholarly work through the 1940s. The final work included in this analysis was published in 2013. This category of scholarly works intertwined with the category of Vocabulary Knowledge and Mathematical Success and later overlapped with the category of Bilingual Issues. It is important to state that the term *arithmetic* refers to problem solving in the grades prior to high school and the more formal mathematics categories, such as algebra. This category was distinct in the fact that all but three of the 18 works were studies. Overall, the collection of works found that students with larger vocabularies performed better when solving problems, just as students who were identified as good readers. According to Faulk and Landry (1961), vocabulary instruction did not transfer to students' ability to solve mathematics problems. These findings paralleled those within the field of literacy, which explored the role of vocabulary with comprehension.

A strong relationship between vocabulary and reading comprehension has been demonstrated through both correlational and experimental studies (Carroll, 1993; Davis, 1942; Freebody & Anderson, 1983; Marks, Doctorow, & Wittrock, 1974; Spearitt, 1972). While some studies have shown a relationship between vocabulary instruction and comprehension (Beck, Perfetti, & McKeown, 1982; Kameenui, Carnine, & Freschi, 1982; Stahl, 1983, Stahl & Fairbanks, 1986), many others have not found that vocabulary instruction impacts comprehension (Elleman, Lindo, Morphy, & Compton, 2009; National Reading Panel, 2000; Pany & Jenkins, 1978; Tuinman & Brady, 1974; Wixson, 1986). In a meta-analysis, Elleman et al. (2009) revealed that the studies which showed a relationship between vocabulary instruction and comprehension were largely based on researcher-created measures, whereas standardized assessments showed no such relationship. One study examined teachers' perceptions about students' difficulties with word problems (Pearce, Bruun, Skinner, & Lopez-Mohler, 2013). The study of 70 elementary teachers reported that 45% believed reading and understanding the problem was the primary difficulty, and 13% named vocabulary as the primary obstacle.

A commonality across all of the categories was that there were few research studies within each group of scholarly works. Thus, while many voices have spoken about mathematical vocabulary, relatively few have been grounded in or verified through research. Table 1 provides insight into this finding.

Conclusions from the research determined that ten categories of mathematical vocabulary scholarly works have been generated in the United States. Some of the categories intertwined and merged. For example, Vocabulary Relationship to Problem Solving merged with both Vocabulary Knowledge and Mathematical Success and later with ELLs' Issues. Vocabulary Lists was a category that was, for the most part, short lived. It surfaced in the 1920s and with the exception of one outlier, concluded in the 1930s. The category of Annotated Bibliography described a single work that did not readily fit into any other category, but needed to be included. Readability of Mathematics Textbooks lasted for five decades, from the 1920s to the early 1970s. The category of Instructional Strategies for Mathematical Vocabulary emerged early and continues today. This category ranked higher in importance over all the others because of its value to practitioners in the classroom as they seek ways to improve the vocabulary knowledge and application for their students. Nearly every work in this category offered strategies, whereas only a few, five of the 48 works, were based on the findings of research in mathematics vocabulary instruction. In contrast, 14 out of 17 of the Vocabulary Relationship to Problem Solving works were studies, and all of the Readability of Mathematics Textbooks works were studies.

Development of Themes

Analysis of the categories sought to determine the presence of an overarching theme or themes that could define the works. The category Instructional Strategies connected to most categories, with the exception of Annotated Bibliography, Vocabulary Lists, and Taxonomy. The other categories related due to their works identifying or suggesting instructional strategies for improving mathematical problem solving, discussing the ability to read the mathematical texts, addressing needs for ELLs, and improving discourse opportunities. ELLs' Issues was also connected to these same categories with the same exceptions. The works grouped as ELLs' Issues included instructional strategies, problem solving, mathematical success, discourse, the language of mathematics, and improving the ELLs' ability to read mathematical text. Therefore, these two categories rose to the level of overall themes. The categories of Annotated Bibliography and Vocabulary Lists were outliers and each stood alone. Figure 1 portrays the relationships between the categories and the themes which emerged. The body of scholarly works concerning mathematical vocabulary spanned over 150 years. These works were characterized to help discern the areas that received more attention by scholars, as well as topics that are open to additional research. The classification of the works as studies or articles/books is helpful when considering the findings. The category of Instructional Strategies for

Mathematical Vocabulary was the most extensive and is most likely the category practitioners would turn to for enhancing teaching practices, yet this category contained only five studies.

Figure 1 shows the relationships between the categories and the themes, Instructional Strategies and ELLs' Issues. The three categories that did not fit into the themes were Annotated Bibliography, Reading Inventory, and Vocabulary Lists. By far, the largest category of scholarly work was Instructional Strategies for Mathematical Vocabulary. The most recent scholarly works resided predominantly in the category of ELLs' Issues. Figure 1 utilizes shape to help discern the themes and categories. The ovals represent themes, and the rounded rectangles signify categories. The overlapped rounded rectangles denote the categories that shared some works. The lines connect the themes with the categories, as well as connect the two themes. The three rounded rectangles that are not connected to any shape correspond to the categories that did not fit well under the two themes and did not connect well with other categories.

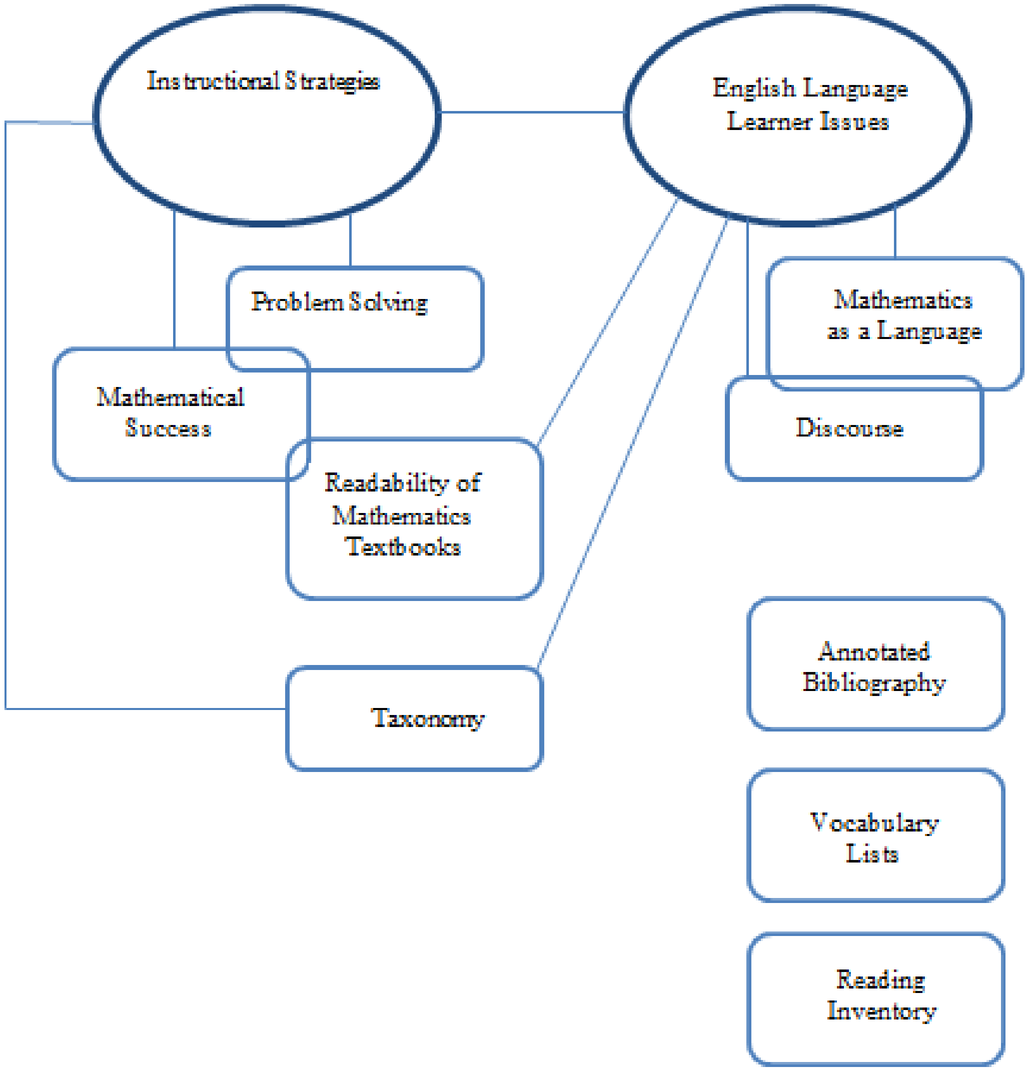

Figure 1. Themes from Historical Analysis

Research Question One – *What themes arise from a historical analysis of mathematical vocabulary research?* Instructional Strategies and Bilingual Issues emerged as the dominant themes. While three categories did not fit into these two themes, they represented a small body of works which fit nicely into the categories of Annotated Bibliography, Vocabulary Lists, and Reading Inventory. The theme of Instructional Strategies was associated with a majority of the categories. It was also the single largest category, representing 46 works. Additionally, this category was the most likely to be used by practitioners for enhancing teaching practices, yet it contained few studies.

Analysis of Studies in Historical Analysis

Analysis of the studies identified in the historical analysis portion of this research study strived to bring clarity to what has been studied and what the findings were. The following table displays the studies organized chronologically. The table includes the publication date, the author(s) of the study, the sample size, type of study, purpose of the study, and the findings.

Table 5

Studies by Time Period

Date	Author(s)	Sample size	Type	Purpose	Findings
1924	Monmouth, Isbell, Jenkins, & Pieters	32 teachers	Survey	Determine what are the problematic words in mathematics	Technical words and phrases were sources of difficulty and hindered the solution of arithmetic problems.
1924	Pressey	100 teachers	Survey	Determine the technical vocabularies of school subjects	Created a vocabulary list for school subjects.
1925	Lessenger	All 3^{rd}-8^{th} graders in Radcliff, Iowa	Treatment of emphasis on reading instruction; Measured by pre-posttest of Stanford Achievement Test Form A	Determine the role of reading as a factor of success in arithmetic (included word knowledge)	The gains in arithmetic scores were statistically significant. "Transfer of reading skills to reading in arithmetic was extensive" (p. 289).

1926	Brooks	5 textbooks for 3rd grade math	Counts of words	Examined 5 textbooks vocabulary content	Found low repeat usage of terms not allowing for the learning of the terms.
1928	Remmers & Grant	12 math textbooks	Counts of words and indexing the commonness using Thorndike's Word Book	Measure the vocabulary load in 12 geometry and algebra books	No significant difference between the algebra books. The geometry books had larger technical vocabulary loads than algebra books. Great differences among the geometry books.
1929	Georges	40 students	Observational study	Determine the nature of reading difficulties in a junior level math class	23.4% of difficulties attributed to mathematical vocabulary. Difficulties were: vocabulary, mathematical understanding, lack of intensity in reading, inability to analyze, carelessness, statement of the arithmetic problem.
1931	Buswell & John	1500 students in grades 4-6	Administered the Buswell-John vocabulary test	Determine what types of words were problematic to students	Determined top 10 least and most known mathematics vocabulary.
1932	Pressey & Moore	All students in 5 cities in grades 3-highschool	Vocabulary tests were administered	Determine the growth of math vocabulary from 3rd grade through high school	Inadequate mastery of fundamental terms. 89 words were never mastered by more than 50% tested. Only 36 words

					mastered by 95% of those tested.
1932	Pressey, Pressey, & Zook	Teachers in summer college classes	Consensus from rounds of discussion	Determine a vocabulary test for geometry	Created a test of 77 terms selected by frequency of usage in texts, importance for teaching, technical value, social value, and usefulness in other subjects.
1932	Pressey, Pressey, & Narragon	Teachers and college professors in summer college classes	Consensus from rounds of discussion	Determine a vocabulary test for algebra	Created a test of 52 terms selected by frequency and importance.
1933	Monroe & Engelhart	587 students in grade 5	Standardized assessments given mid-year and at the end of the year	Determine the effectiveness of a systematic program of reading instruction on arithmetic problems	Not effective with bright students and relatively effective with low-performing students.
1934	Dresher	500 students	Pre-Posttests. Pressey & Elam for arithmetic. Pressey, Pressey, & Zook for geometry. Pressey, Pressey, & Narragon for algebra	Determine the effects of extensive and specific vocabulary instruction	Essentially no difference between the experimental and control groups.
1934	Cowley	1200 students in plane geometry,	Survey	Determine students' geometry word knowledge	Found words successfully understood and those that were

		1500 students in plane geometry , 350 students in solid geometry			misunderstood. Greatest difficulty was with the words: *apothem, locus, median, proportional,* and *ratio.*
1937	Buckingham	139 students in Algebra I	Vocabulary test and standardized algebra test	Determine the relationship between vocabulary and ability in algebra	Significant relationship between vocabulary knowledge and ability to solve algebra problems.
1940	Drake	716 students in Algebra 1	Vocabulary tests and standardized algebra test	Determine what effect the teaching algebra vocabulary will have on achievement in algebra	The experimental group achieved higher on the achievement tests than the control groups. The unit with the least effect was focused on positive and negative numbers.
1941	Hastings	331 students in grade 9	A variety of four vocabulary tests over two weeks	Determine the need for assessing vocabulary understandings in a variety of ways	A variety of opportunities were needed to determine if students learned the meaning of mathematical words.
1941	O'Rourke & Mead	5 third-grade books	Frequency of vocabulary	Determine the vocabulary difficulties of beginning arithmetic textbooks	Repetition of vocabulary words was not sufficient for students to learn the words.
1944	Hansen	681 students in grade 6	5 Standardized tests	Determine if there are certain abilities associated with	Lack of relationship between certain reading abilities and problem

				achievement in problem solving	solving, including knowledge of vocabulary.
1944	Johnson, H. C.	898 students in grade 7	Vocabulary and problem solving pre- and posttests	Determine the effect of mathematical vocabulary instruction on problem solving	Vocabulary instruction improved knowledge of words; does not translate to improved problem solving.
1944	Treacy	244 students in grade 7	3 types of standardized tests: problem solving, mental ability, and reading ability	Determine the relationship of certain reading skills and ability to solve word problems	Good achievers out performed low achievers in arithmetic vocabulary and vocabulary in general.
1948	Eagle	157 students in grade 9	2 types of standardized test results	Determine the relationship of reading abilities to achievement in mathematics	Where general vocabulary was found to have less relationship to success in math than any other factor, mathematical vocabulary was associated with math success.
1949	Johnson, J. T.	6 schools in Chicago (age or grades and number of students involved unknown)	3 types of standardized test results	Determine the factors associated with problem solving ability	Vocabulary was associated with problem solving ability and general intelligence.
1957	Johnson, D. A.	Textbooks for: seventh grade, eighth grade algebra,	Variety of readability measures were used	Determine what is the appropriate tool for readability measures of	Determined that the Flesch formula was appropriate to determine readability of

		plane geometry		mathematics texts	mathematics texts.
1960	Repp	5 third grade textbooks	Word counts	Determine a common basis for comparison of the vocabularies of several arithmetic textbooks	Widely ranging. Inconsistent results between books.
1961	Faulk & Landry	148 students in grade 6	Pre-posttest of standardized test	Determine the effect of a particular problem-solving method on achievement. (focus on vocabulary study, talking through the problem, drawing a diagram, solving the problem)	Experimental group performed slightly better than control but was statistically insignificant.
1964	Vanderlinde	394 students in grade 5	Pre-posttest standardized tests	Determine the effect of direct study of technical vocabulary on other abilities	Direct study of quantitative vocabulary was associated with higher achievement in problem solving.
1966	Stauffer	3 textbook series in mathematics	Examined the introduction of new vocabulary and conducted word counts	Determine word counts in textbook series so that vocabularies can be compared	Little overlap of vocabulary between series within content areas.
1967	Wiegand	Textbooks for high school mathematics courses in a district	Standardized test and Dall-Chall Formula	Determine the relationship between readability of textbooks and reading levels of children in math classes.	The readability levels of the districts' mathematics textbooks were generally higher than the reading ability of students.

1970	Harvin & Gilchrist	214 students in grade 3	Standardized tests	Determine the relationship between reading and arithmetic performance. (vocabulary being one of the key reading skills)	Positive relationship was found between reading and arithmetic performance but not significant enough to be a predictor of performance from one to the other.
1970	Hater & Kane	1717 students in grades 7-10	Cloze test and comprehension test by subject	Determine the validity of the cloze procures as a measure of comprehensibility and difficulty of mathematical English	Cloze tests were found to be highly reliable measures and valid predictors of reading comprehensibility.
1971	Byrne & Kane	350 students in grades 7-8	Tests for familiarity with math terms and symbols	Determine measures of familiarity of mathematical terms and symbols	Set measures of familiarity.
1971	Olander & Elmer	1200 students in grades 4-6	Buswell-John Vocabulary Test and Contemporary Mathematical Vocabulary Test	Determine how students in 1968 compare to those in 1930 concerning mathematical vocabulary	The 1968 students outscored the 1930 students in Grades 4 and 5. The 1930 students outscored Grade 6 students from 1968.
1971	Wilmon	8 textbook series for 1st-3rd grade	Word identification and frequency counts	Determine the technical vocabulary of arithmetic textbooks in the primary grades	500 technical vocabulary mathematics words and phrases introduced in the textbook series. Most did not have the frequency of usage to adequately support learning the

					words.
1983	Jackson & Phillips	213 students in grade 7	Standardized test	Determine if vocabulary instruction specific to ratio and proportion would affect mathematics achievement.	Students who received vocabulary instruction achieved higher verbal and computational scores than control group.
1988	Davison & Schindler	30 students in grades 4-6 bilingual Crow speaking students	Interviews	Determine the needs of teachers to instruct bilingual Crow-speaking students in mathematics classes.	The Crow language was not being leveraged to assist students in learning math concepts. Students had incomplete mastery of mathematics concepts in both Crow and English. Textbooks were unconnected to Native Americans experiences.
1999	Schwarz	36 students in grade 5	Pre-postest researcher created, parent survey, problem solving logs, and daily arithmetic problems	Determine if the usage of word walls and journaling impact the student acquisition of math vocabulary	Increased performance of vocabulary understandings and problem solving
2000	Abedi	36 students in grade 8 graders in phase 1 of the study, 1,174 students in grade 8in phase 2 of the study	NAEP math items, student perception study (phase 1), and modified parallel math items to NAEP	Determine if modifying the linguistic structures in test items affected performance on math NAEP.	ELLs had considerably lower math performance than other students. Some students performed better with the modified linguistic parallel items.
2001	Abedi &	1,174	NAEP	Determine the	Student language

	Lord	students in grade 8		importance of language on test performance with math word problems.	background affected mathematics performance. Limited English associated with lower performance on standardized math tests.
2006	Capraro & Joffrion	668 algebra students in middle school	Pre-posttest algebra tests	Determine the extent students were able to translate English into mathematical symbols and vice versa.	Those taught with a procedural approach performed lower than those taught with a conceptual approach.
2012	Ortlieb, Perkins, & Verlaan	99 students in Algebra 2 students	Teacher created pre- and posttests for vocabulary and conceptual understanding	Compare the effectiveness of two vocabulary instructional strategies in Algebra 2.	No difference in vocabulary scores between the two strategies. Both groups improved their vocabulary knowledge and conceptual knowledge of Algebra 2.
2013	Pearce, Bruun, Skinner, & Lopez-Mohler	70 teachers of grades 2-5	Interview guide	Investigate teachers' perspectives of difficulties students have solving mathematical word problems and the causes of those difficulties.	Students' ability to read and understand word problems was the most frequently cited difficulty. The causes were: standardized testing, text difficulties, previous teachers, and student factors.
2013	Vessel & Robillard	8 teachers of grades 4-8	Case study	Examine the use of a signing math dictionary for students who use sign language to	Students used the signing dictionary to access and use vocabulary in math class.

		communicate access vocabulary required to learn math being studied

Four of the studies were qualitative in nature (Davison & Schindler, 1988; Pearce, et al., 2013; Vessel & Robillard, 2013). Two studies examined the difficulties students experienced due to vocabulary (Davison & Schindler, 1988; Georges, 1929). One study investigated teachers' perceptions regarding students' challenges with problem solving. One of the most frequently cited issues focused on vocabulary and the ability to read and understand word problems (Pearce et al., 2013). The fourth study was a case study which considered how the usage of a digital signing math dictionary impacted the teaching and learning of mathematics for students with profound hearing loss (Vessel & Robillard, 2013).

Six studies investigated vocabulary lists, five of which could be grouped as survey or consensus building. Four of these studies developed vocabulary lists (Monmouth, Isbell, Jenkins, & Pieters, 1924; Pressey, 1924; Pressey, Pressey, & Narragon, 1932; Pressey, Pressey, & Zook, 1932). One study in this group surveyed students to determine the mathematical vocabulary they considered to be troublesome. There was also one study in this group focused on vocabulary lists that did not use survey or consensus building; rather, it used vocabulary tests to determine lists of words that were problematic for students to learn (Buswell & John, 1931).

Eight studies addressed the vocabulary demands in textbooks. Two of these involved high school textbooks (Remmers & Grant, 1928; Wiegand, 1967), and five examined elementary textbooks (Brooks, 1926; O'Rourke & Mead, 1941; Repp, 1960; Stauffer, 1966; Wilmon, 1971). One study was very similar in nature to those examining the readability of textbooks; it was designed to determine the appropriate tool to measure the readability of mathematics textbooks from Grade 7 through geometry (Johnson, 1957).

There were ten studies that highlighted mathematical vocabulary instruction and the relationship between vocabulary and problem solving. Among these, four found that vocabulary instruction had little to no effect on students' ability to solve word problems (Dresher, 1934; Faulk & Landry, 1961; Monroe & Engelhart, 1933; Johnson, 1944). Six studies revealed that vocabulary instruction had an effect on students' performance on standardized tests (Drake, 1940; Jackson & Phillips, 1983; Lessenger, 1925; Ortlieb, Perkins, & Verlaan, 2012; Schwarz, 1999; Vanderlinde, 1964).

Another line of research looked at standardized test results to determine if there were relationships between vocabulary and achievement. Six studies focused on achievement in problem solving (Buckingham, 1937; Eagle, 1948; Hansen, 1944; Harvin & Gilchrist, 1970; Johnson, 1949; Treacy, 1944).

Several studies did not fit into groups of related studies. Pressey and Moore (1932) performed a study to define the growth of student vocabularies from the third grade through high school. One study showed that students needed to be tested in a variety of ways to determine the extent to which they learned new vocabulary (Hastings, 1941). Two studies examined standardized test results with a focus on ELLs (Abedi, 2000; Abedi & Lord, 2001). One study compared students' vocabulary test and computational test results from 1930 and 1968 (Olander & Elmer, 1971). Students' word knowledge established the validity of cloze tests for comprehension (Hater & Kane, 1970) and defined measures of familiarity of terms which impacted the readability measures used for textbooks (Byrne & Kane, 1971).

The studies' sample sizes ranged from eight participants in a qualitative study (Vessel & Robillard, 2013) to 3,050 participants in quantitative studies (Cowley, 1934). One study's sample included all third- through eighth-grade students from Radcliff, Iowa (Lessenger, 1925).

Results of the Case Study

During the time frame of this research study, the four participants involved in the case study were mathematics education professors serving as active faculty members at four different universities in the United States. Each had produced scholarly work that addressed the teaching and/or learning of mathematical vocabulary. The participants resided in four distinct geographical locations within the United States which required delicate scheduling to meet with them at their convenience. Two of the interviews transpired in the participants' offices at their respective universities. One took place at a conference that both the participant and researcher attended. The fourth interview occurred via Skype as a result of severe weather conditions and scheduling concerns.

Research Question Two

Research Question Two – *What commonalities or themes arise from interviewing experts about mathematical vocabulary?* The resulting themes were Discourse and Mathematical Vocabulary Instruction.

The researcher used Wordle (Feinberg, 2009) to perform the initial analysis of each individual interview question. McNaught and Lam (2010) suggested that Wordle is a useful research tool that "can allow researchers to quickly visualize some general patterns in text" (p. 641). Wordle was the most appropriate supplement to other forms of analysis. Through this preliminary analysis, the researcher developed a sense of the words each participant used most frequently to answer the questions. The researcher entered the participants' responses to each interview question into the online Wordle system. The web-based interface treated each of the words within the transcript as individual units and represented them in varying font sizes to indicate their frequency of use within the text. Figures 1 through 16 display the Wordles. It is important to note the limitations of using Wordles as a research tool. Wordles, also known as word clouds, treat each word as an individual unit; therefore phrases are not captured. For example, the phrase *open ended questions* has a different meaning than the individual words *open*, *ended*, and *questions*.

The Figures are grouped together by interview question. Figures 2 through 5 are the word clouds that represent the participants' responses to the first interview question. Because Wordle is used for quick analysis, the only touchups made to the word clouds kept the formatting uniform (McNaught, 2010). Interview question one was, "Tell me about the research or researchers, in the field of mathematical vocabulary, that form the foundation of your scholarly work."

*Figure 2***.** Word Cloud of Participant One Responding to Interview Question One

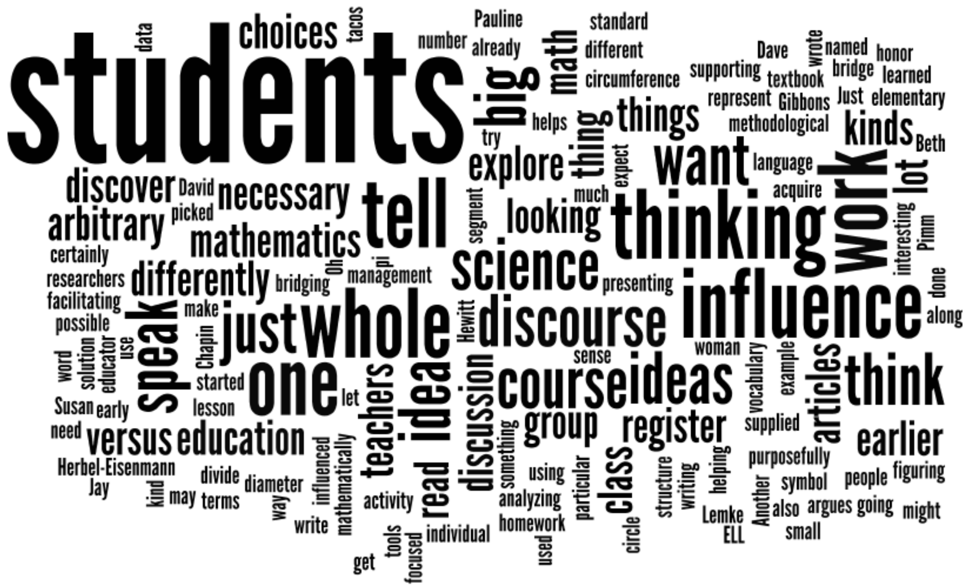

Figure 3. Word Cloud of Participant Two Responding to Interview Question One

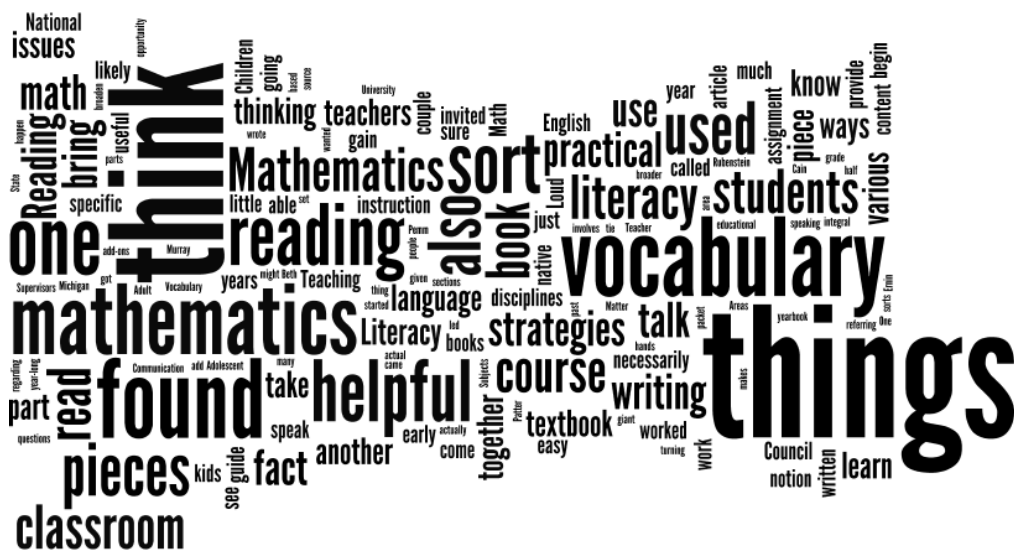

Figure 4. Word Cloud of Participant Three Responding to Interview Question One

Figure 5. Word Cloud of Participant Four Responding to Interview Question One

Even a perfunctory look at the four figures reveals that the participants had commonalities and differences in the high frequency words they used to describe what they pointed to as forming the foundation of their scholarly work. Participants 1 and 3 shared a focus on literacy and vocabulary. Participants 2 and 4 similarly focused on language, discourse, and students. Participants 1, 2, and 3 identified a great number of researchers in their responses. All four participants were prolific with their responses to the initial interview question. Participant 1 used the words *literacy* and *vocabulary* at a markedly greater frequency than all other words. Participant 2 used the word *students* with the greatest frequency.

Figures 6 through 9 display participants' responses to the second interview question: "What celebrations or concerns about the current trends in mathematical vocabulary instruction do you see?"

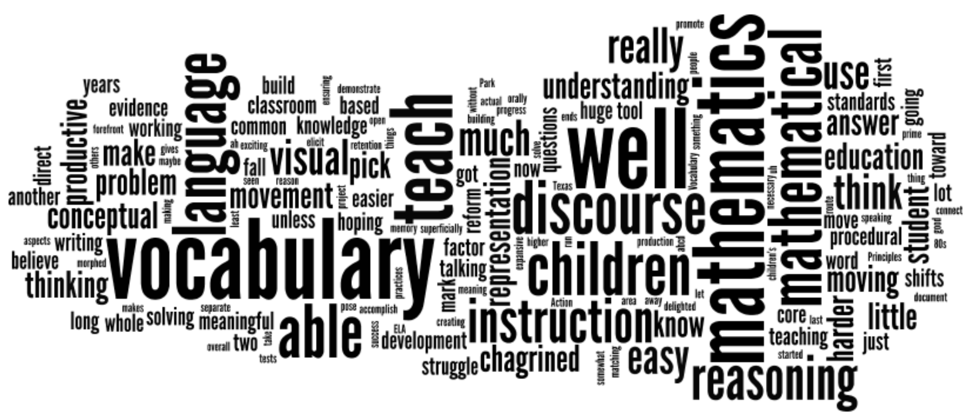

Figure 6. Word Cloud of Participant One Responding to Interview Question Two

Figure 7. Word Cloud of Participant Two Responding to Interview Question Two

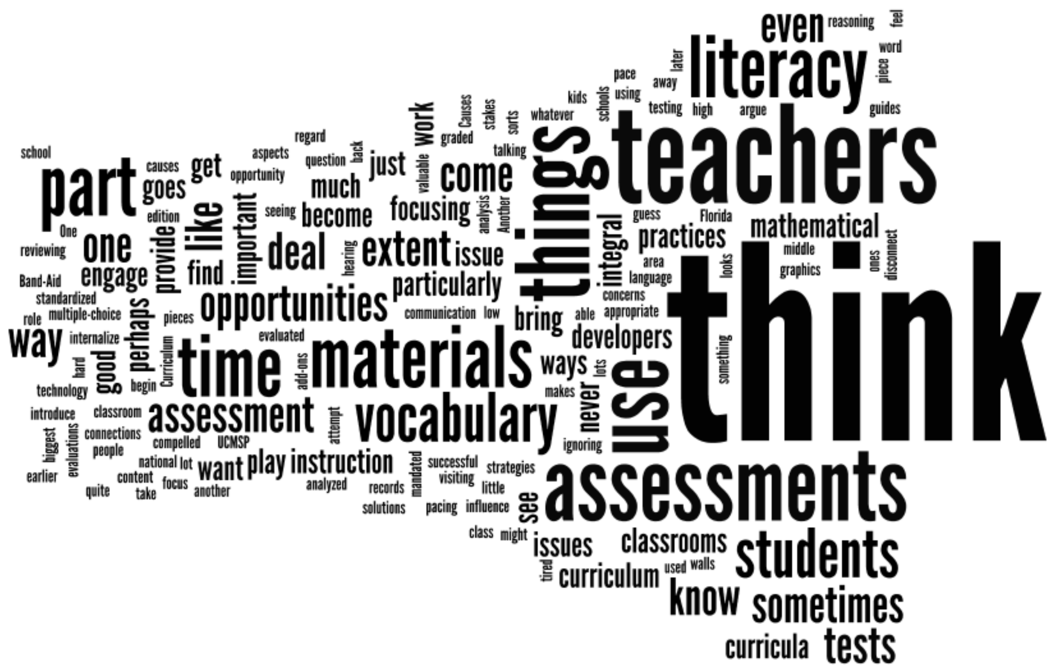

Figure 8. Word Cloud of Participant Three Responding to Interview Question Two

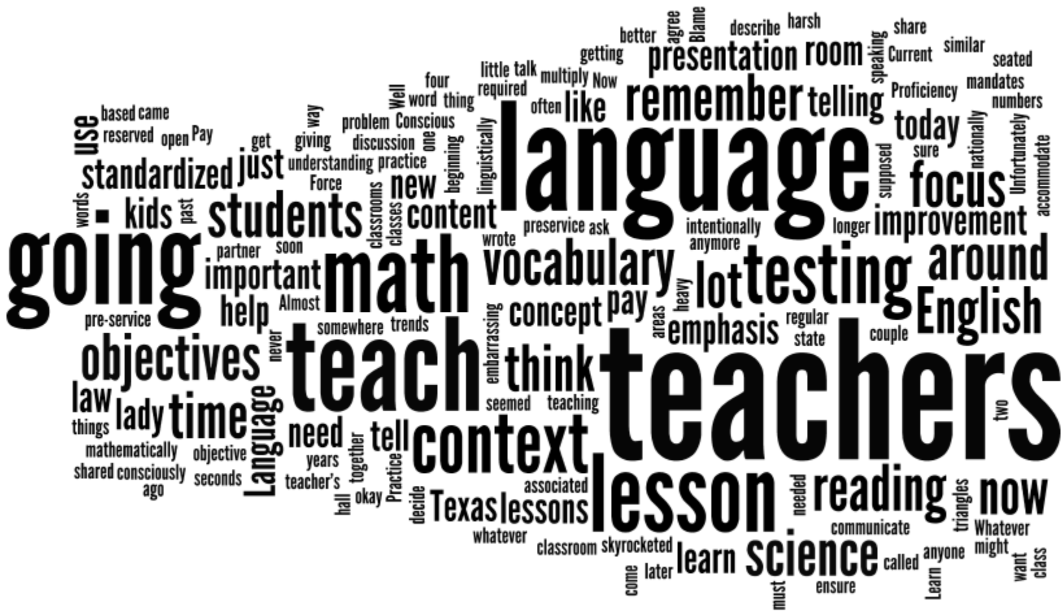

Figure 9. Word Cloud of Participant Four Responding to Interview Question Two

The Wordles that represent the participants' responses from the second interview question revealed a heavy use of the words *teachers*, *vocabulary*, *discourse,* and *language*.

Participant 2 used the word *talk* and *talking* frequently which was tied to this participant's discussion of discourse practices. Participant 3 often included the word *think* when responding to the interview question. It was primarily used in the context "I think…," thus capturing the dialogue patterns of the participant's speech. Participant 3 also referred to *assessment* repeatedly when answering this question, just as Participant 4 used the word *testing*. All of the participants' responses to this question were profuse. Figures 10 to 13 are the word clouds formulated from the participants' responses to the third interview question, "What do you believe is the most important message that classroom teachers need to hear to inform their instruction of mathematical vocabulary?"

Figure 10. Word Cloud of Participant One Responding to Interview Question Three

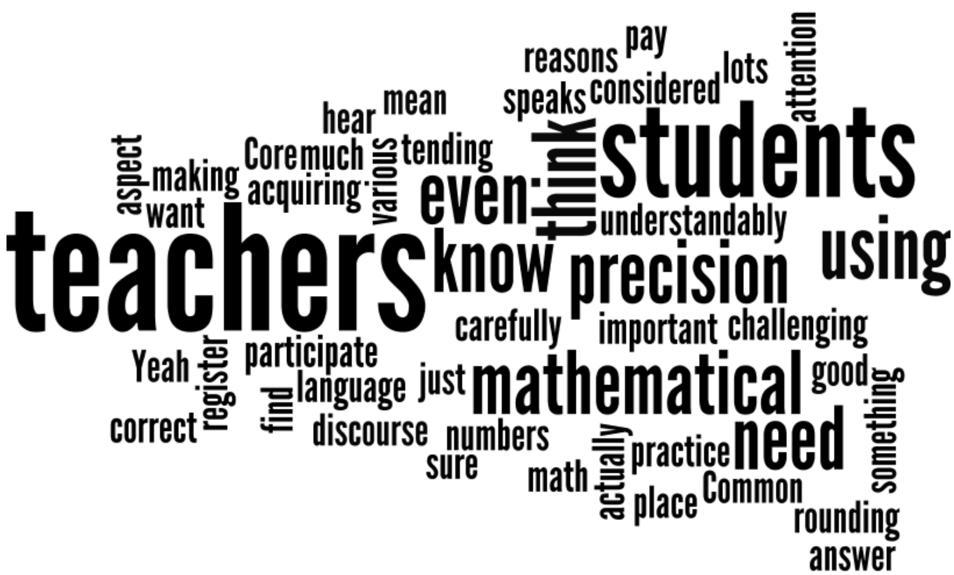

Figure 11. Word Cloud of Participant Two Responding to Interview Question Three

Figure 12. Word Cloud of Participant Three Responding to Interview Question Three

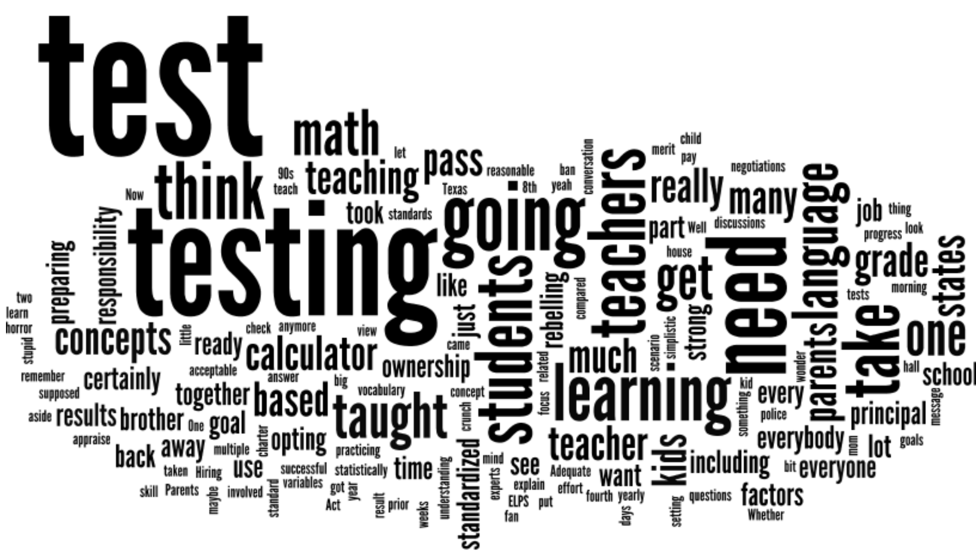

Figure 13. Word Cloud of Participant Four Responding to Interview Question Three

The analysis of the Wordles generated from participants' transcripts of interview question three exposed some similarities as well as striking differences. Three of the participants had a lot to say in response to this interview question while one participant was more concise. In addition, three of the participants frequently used the words *students* and *teachers*. Both Participants 1 and 2 repeatedly included the word *precision* in their responses. One striking difference appeared in the Wordle from Participant 4 who mentioned *test* and *testing* with great regularity, whereas the other three participants did not. Figures 14 to 17 show the word clouds formulated from the participants' responses to the fourth interview question, "What do you anticipate the direction of mathematical vocabulary research will be in the next five to ten years?"

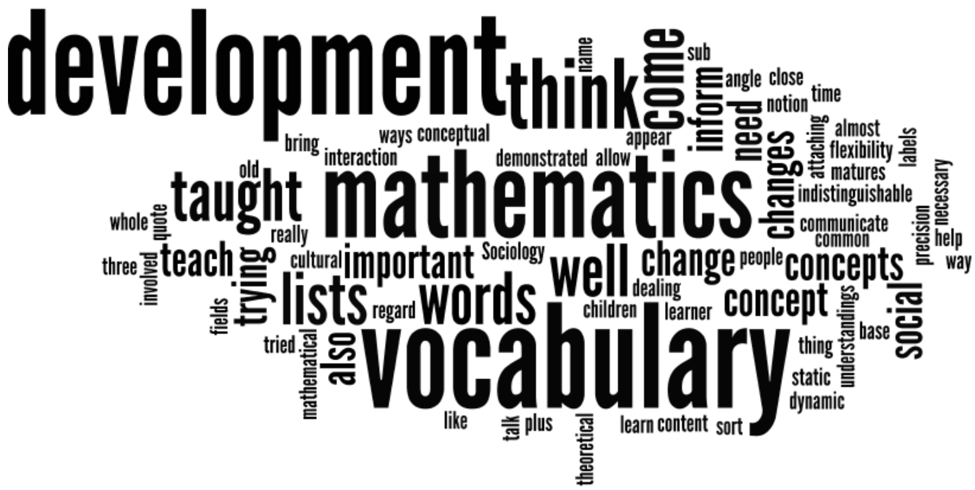

Figure 14. Word Cloud of Participant One Responding to Interview Question Four

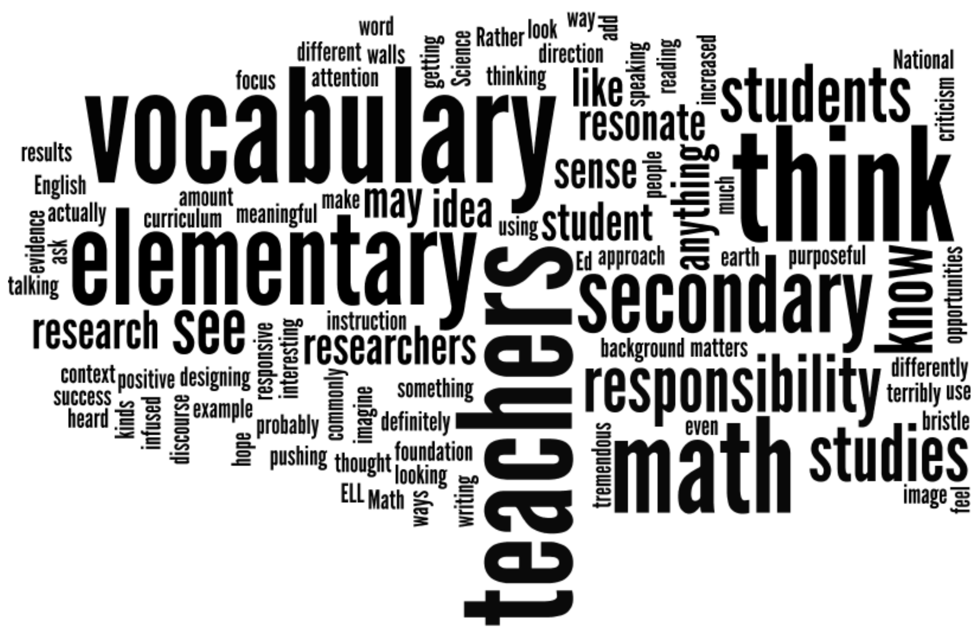

Figure 15. Word Cloud of Participant Two Responding to Interview Question Four

Figure 16. Word Cloud of Participant Three Responding to Interview Question Four

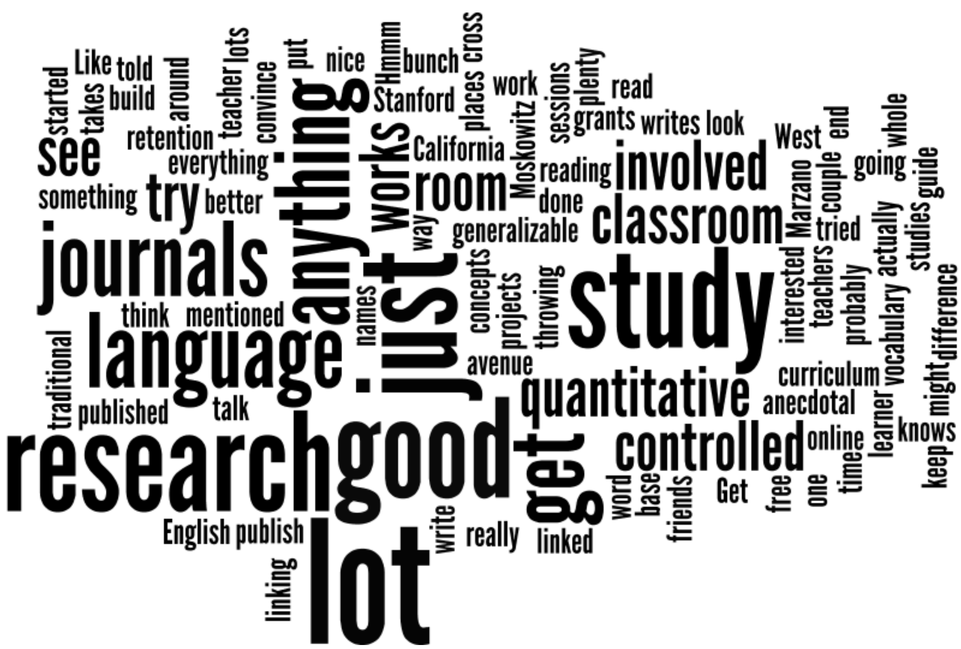

Figure 17. Word Cloud of Participant Four Responding to Interview Question Four

Obvious similarities and differences surfaced in the analysis of the Wordles generated from the transcripts of the fourth interview question. They predominantly included the words *vocabulary* or *language*. Participants 2 and 4 shared the words *research* and *study*; Participants

2 and 3 used the word *students*. Participants 1and 2 focused on fewer words, and the length of their text was quite different. The first participant's response was much longer than that of the second participant and consistently used the word *development*.

As an analysis tool, Wordle established the high frequency words used by the participants to answer each of the interview questions. This prompted a search for the commonalities and differences among the participants. As suggested by McNaught and Lam (2010), it further provided the opportunity to quickly visualize general patterns and diversities in the text. Wordle also helped to focus the analysis on participants' responses and kept the researcher's subjectivities in check.

From Codes to Themes

The researcher coded the data according to the In vivo method. Saldana (2009) recommended In vivo Coding as appropriate for many forms of qualitative studies and "studies that prioritize and honor the participant's voice" (p. 74). The researcher analyzed each of the interview questions separately in an attempt to capture the major ideas that the group of participants expressed. During the coding process, the researcher recorded codes on sticky notes and then grouped the sticky notes into categories by participant for each interview question. Photos of the results allowed the data to be organized in a table. Then the researcher gathered all of the sticky notes from each participant for one of the interview questions and reflected upon the process in a journal. After a week or two, the researcher repeated the process in order to determine if new insights occurred or if the coding was similar using the code-recode strategy (Chilisa & Preece, 2005). The researcher once again arranged the sticky notes into categories, posted them on chart paper, and took photos of the resulting organization. This process occurred for each interview question. Tables 6 through 9 show the categories that emerged during the second cycle coding process for the interview questions.

The first interview question established the works that the participants considered to be foundational to their participation in the field of mathematical vocabulary. The category *Researchers* represented the scholars that the participants identified as significant, just as the category *Scholarly works* characterized the specific works that the scholars named.

Table 6

Interview Question One and Categories

Interview question one	Categories
Tell me about the research or researchers, in the field of mathematical vocabulary, that form the foundation of your scholarly work?	Informal ways of knowing
	Researchers
	Scholarly works
	Language/discourse
	Literacy
	Mathematical conceptual development

The second interview question addressed the current trends in mathematical vocabulary instruction. The categories summarized the trends about which the participants were excited and hopeful, as well as those that caused concern. They were all pleased with the growing emphasis on discourse in mathematics classrooms that is being called for by national, state, and Common Core objectives. The participants were also enthusiastic about the increased expectation to include language objectives in lesson planning. They commented positively on the growing interest in using engaging tasks in mathematics instruction and viewed this as an avenue that would naturally lend itself to increased discourse and use of mathematical vocabulary by students. They were also hopeful regarding language-rich assessments. Such open-ended questions would necessitate the use of written communication about student understanding of mathematics. Participants' concerns about the predominate use of multiple-choice, high-stakes assessments ranged from the over emphasis on high-stakes testing to the limited information that is gained from the results of this style of testing. The participants shared the worry that although there is a growing expectation concerning the need for discourse in mathematics classrooms, they have observed that for the most part students remain silent. All but one of the participants indicated the need for direct instruction.

Table 3

Interview Question Two and Categories

Interview question two	Categories
Thinking about classroom instruction, what celebrations or concerns about the current trends in mathematical vocabulary instruction do you see?	Celebrations
	Emphasis on discourse
	Language objectives
	Engaging tasks
	Changes in testing (language rich)
	Concerns
	Multiple-choice high stakes assessments
	Students are mostly silent
	Direct instruction of vocabulary

The third interview question determined the message teachers needed to hear regarding mathematical vocabulary instruction. All of the participants addressed the need for students to be actively using language in the classroom to read, write, speak, and listen. The general consensus was that if students actively discuss mathematics, they will have greater opportunities to interact with the vocabulary of the discipline and wrestle with both the word meanings and mathematical concepts. The participants also mentioned instructional strategies to avoid, such as using word finds, copying definitions, writing the words multiple times, and writing sentences using the words. They cautioned that such activities do not result in the learning of mathematical vocabulary and instead take valuable time away from learning opportunities.

Table 4

Interview Question Three and Categories

Interview question three	Categories
What do you believe is the most important message that classroom teachers need to hear to inform their instruction of mathematical vocabulary or register?	Teach through discourse Journey towards precision of mathematical language Additional benefits of having a discourse rich learning environment in mathematics A list of things to avoid

The final interview question expanded the participants' views on the trends in mathematical vocabulary by asking them to predict future developments. Discourse surfaced as a predominant line of thought, as many of their responses connected either directly or indirectly to classroom dialogue. One participant expressed the hope that a theoretical framework could be developed for mathematical vocabulary instruction.

Table 5

Interview Question Four and Categories

Interview question four	Categories
What do you anticipate the direction of mathematical vocabulary research will be in	Development of a theoretical framework for math vocabulary instruction

the next five to ten years?	
	Discourse analysis
	Evidence based research
	Determine if discourse translates to student achievement
	How technology affects discourse and vocabulary development

Figure 19 shows the relationships between the categories, subthemes, and the themes of Discourse and Mathematical Vocabulary Instruction which resulted from the analysis of the interviews.

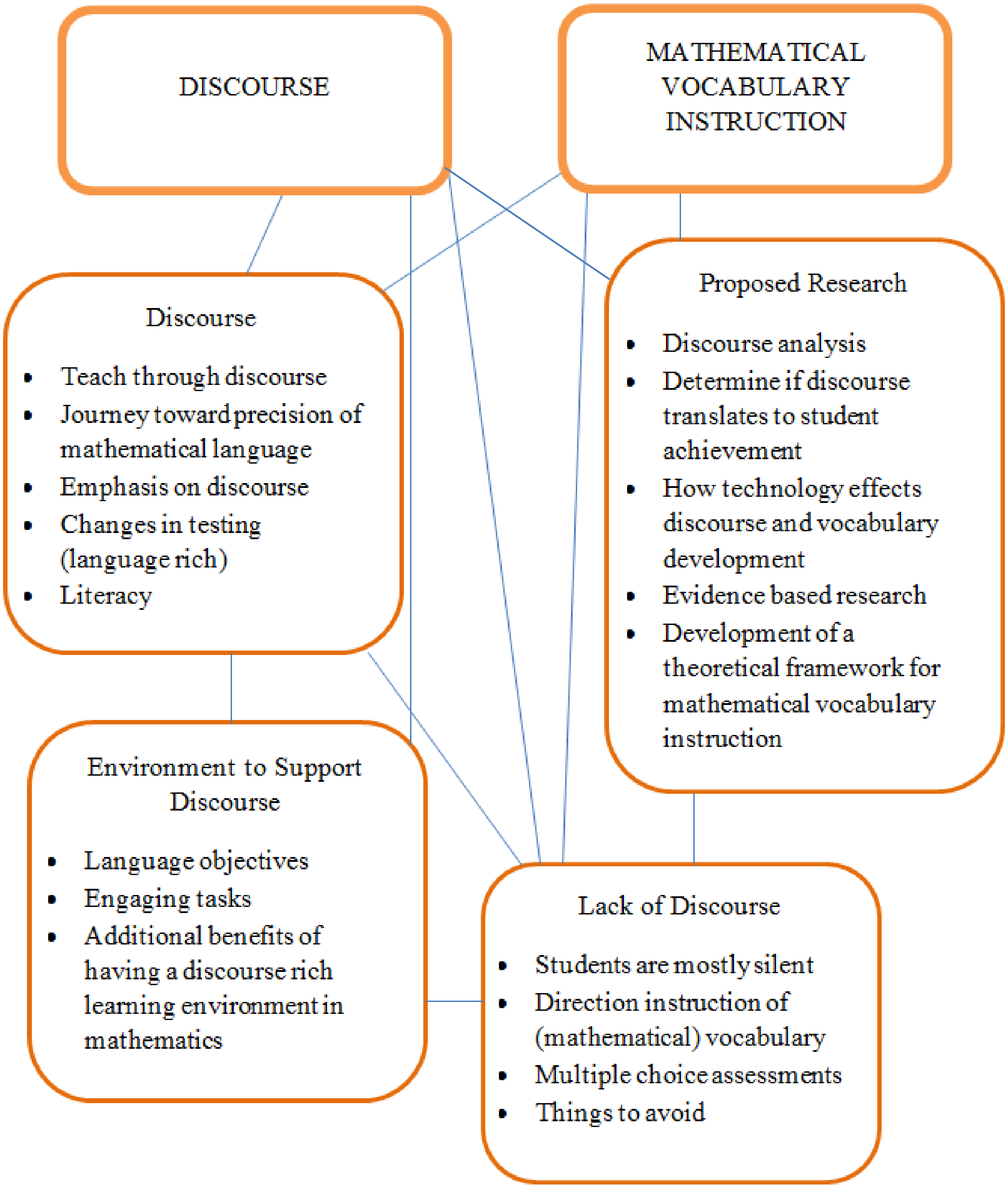

Figure 18. Relationships of the Categories into a Theme

The themes of Discourse and Mathematical Vocabulary Instruction captured the essence of the other subthemes of discourse, environment to support discourse, proposed research, and lack of discourse. All of the subthemes fit under the overall umbrella that the theme provided.

Participants' Scholarly Works

To provide additional insight into how each participant situated themselves in the field of mathematical vocabulary, the researcher recorded all of the scholarly works attributed to each. The researcher read, identified each type of work, and verified whether or not there was a connection to mathematical vocabulary. The researcher collected, charted, and examined the resulting information for commonalities and differences, which allowed the analysis to evolve naturally as a result of the process. The researcher photographed the analysis to preserve the findings and discovered that most of the works identified in this context were present within the works identified in the literature review. Reexamining them as a separate group added insight to this study. Additionally, this analysis determined whether or not the same categories and theme(s) emerged as those found within the interviews and the historical analysis. Table 10 displays the organization of works by the following categories: Assessment, Discourse, ELLs Considerations, Mathematical Literacy, Proof, Use of Literature, and Outliers.

Table 6

Analysis of Participants' Scholarly Works Associated with Vocabulary

Categories	Participant	Type of publication	Summary
Assessment			
	1	Article	Suggested 2levels of integration of writing in mathematics to enhance vocabulary and language of mathematics
	1	Article	Suggested an intentional focus on building vocabulary at the same time as mathematics content

	1	Article	Recommended the use of portfolios in mathematics classes; opportunity to write and speak about math authentically
	2	Article	Recommended changing questioning to encourage student ability to express understanding – verbally and in writing
	2	Article	Argued for questioning that assesses and guides student thinking in the context of visual representations
	2	Article	Asserted that rubrics need to be designed for high school mathematics courses to assess student thinking and writing
	2	Article	Called for changing assessment to have students explain reasoning in a calculator rich environment
	2	Article	Recommended for a shift in assessment to allow students opportunities to justify and explain reasoning
	2	Study	Examined changes in assessments to assess student mathematical thinking
	2	Article	Mathematical literacy – defined it and offered suggestions for instruction
	2	Book	Mathematical literacy – meaning making, communication, reading and language issues, instruction
	2	Article	Argued for the need to develop the language of mathematics in students (including vocabulary)
	2	Study	Explored words and concepts and determined where the breakdown in understanding occurs within the context of area and perimeter
	2	Article	Addressed the communication and representation of mathematics concepts
Discourse			
	1	Study	Reported on a teacher self-study concerning the usage of productive discourse in mathematics classroom
	1	Study	Analyzed discourse-based instruction and outcomes
	1	Study	Analyzed discourse patterns of student groups working on mathematics webquest
	2	Article	Explored opportunities for communication in mathematics lessons

	2	Article	Addressed NCTM Standards based instruction – focused on communication in mathematics classrooms
	3	Study	Examined classroom discourse in the context of going over homework
	3	Article	Used discussion groups to promote discourse in mathematics
	3	Article	Suggested the need to listen to student discourse to gain insight into their understanding
	3	Article	Argued for promoting discourse in mathematics teachers professional development
	3	Article	Discussed promoting purposeful discourse in secondary mathematics classrooms
	3	Article	Recommended ideas on how to begin using discourse in mathematics classrooms
	3	Article	Argued that active listening is the foundation for building up to discourse in mathematics classrooms
	3	Article	Recommended the need to develop mathematics teacher discourse moves
	3	Study	Examined revoicing – discourse strategy in mathematics
	3	Article	Suggested using purposeful discourse in mathematics classrooms
ELLs' Considerations	2	Book	Recommended strategies for teaching mathematics to ELLs
	3	Article	Shared activities to help the acquisition of mathematics language
	4	Study	Analyzed problem-solving skills of teachers before and after PD; intentional focus on ELLs and vocabulary
	4	Study	Examined the reflective practices of teachers providing instruction of mathematics language to ELLs
	4	Study	Examined ELL focused instruction
Mathematical Literacy			
	1	Article	Argued for developing mathematical literacy
	1	Article	Suggested two levels of integration of writing in mathematics to enhance vocabulary and language of mathematics

	1	Article	Recommended an intentional focus on building vocabulary at the same time as mathematics content
	1	Article	Discussed using portfolios in mathematics classes; opportunity to write and speak about math authentically
	2	Article	Mathematical literacy – defined it and offered suggestions for instruction
	2	Book	Mathematical literacy – discussed meaning making, communication, reading and language issues, instruction
	2	Article	Recommended developing the language of mathematics in students (including vocabulary)
	2	Study	Explored words and concepts and where the breakdown in understanding occurred within the context of area and perimeter
	2	Article	Explored mathematical communication and representation
Proof			
	1	Study	Examined mathematics tasks and questioning – students deep thinking and clear usage of mathematical language when explaining reasoning; informal proofs
	2	Study	Analyzed the use of reasoning and proofs in precalculus and discrete mathematics
	2	Study	Analyzed using proof to elicit students explanation and reasoning in mathematics
	2	Study	Proofs – measured its inclusion in textbooks
	2	Article	Using proofs to explore, make conjectures, and convince others of truth or falsity of conjectures
	2	Study	Analyzed task design and proof building
	2	Article	Discussed using mathematical language activities to support students when reading, writing, drawing, and explaining mathematical ideas – informal proofs
	3	Article	Argued for the posing of interesting problems and move to authentic proof practice in geometry
	3	Article	Explored student thinking through alternative geometry proof tasks
	3	Study	Examined the nature of reasoning and proving and curriculum analysis in mathematics education

	3	Article	Explored the inconsistencies in definition of argumentation, justification, and proof
	3	Article	Suggested tools to support student usage of formal proofs
	3	Article	Argued for the use of authentic proof practices in geometry building from authentic tasks
	3	Study	Recommended the need for supporting teachers while introducing formal proofs to students
	3	Article	Suggested the teaching authentic proofs
	3	Study	Examined teacher reflection of teaching proofs
Use of Literature			
	1	Article	Discussed the mathematics language development using children's literature (including vocabulary)
	1	Article	Evaluated potential children's literature through the lens of NCTM Principles and Standards – focusing on communication (including vocabulary)
	1	Article	Discussed the use of trade books for mathematics instruction – opportunity to develop vocabulary at the same time as skills and concepts
	2	Article	Addressed using children's literature to develop context for learning probability – both the concepts and the vocabulary
	2	Article	Argued for the use of children's literature to develop proportional reasoning – both the concepts and the vocabulary
	2	Article	Explored algebraic reasoning through literature – explaining and writing using the language of the discipline
Outliers			
	1	Book	Created a Mathematical Dictionary
	3	Article	Argued for developing the usage of mathematics register in secondary mathematics educators

Note. The table does not include the publication date of the scholarly works to help assure participants' confidentiality.

Similar to the historical analysis, the researcher recorded the following information about each scholarly work on index cards: the title of the work, the date of publication, type of publication, participant identification, and a summary. The researcher organized the cards into common groups, charted, and photographed them to capture the results and repeated this process on separate occasions to determine if the resulting groups were a consistent finding. Two fellow doctoral students examined the groupings to verify the conclusions, and discussed any differences with the researcher. They were not provided with the title or participant designation in order to ensure anonymity. Once the groupings were stable, the researcher labeled each with a category name. Two outliers did not fit neatly into any of the other groups. Table 10represents the results of this process.

Development of Themes

The categories of Assessment and Proofs intertwined, though they remained separate. While they both offered insight into student understanding, the Proofs category consisted of only a few works that pertained to assessment. This category largely addressed both formal and informal reasoning and justification of student understanding about mathematical ideas. The Assessment category focused on creating and using assessments that were deeper than correct and incorrect answers, more rigorous than a multiple choice or true-false assessment, but also required students to explain their answers.

The category of ELLs included works that specifically addressed the instructional and learning concerns of this particular group of students. These works suggested using language supports such as sentence stems, word banks, and visual representations of mathematical words. They also included ideas for student grouping that would provide opportunities for ELLs to listen, speak, and write with the support of their peers.

The category Use of Literature contained works that discussed using literature in mathematics instruction. Such works could be used to build mathematical concepts and language, as well as to provide an approachable context in which to learn new material. These works focused on mathematics learners from primary grades through upper middle school and addressed patterns, geometry, algebraic reasoning, and probability.

The categories of Discourse and Instructional Strategies overlapped. Discourse was considered an instructional strategy that allows students the opportunity to listen and speak the language of mathematics. However, some of the participants viewed discourse as much broader than an instructional strategy, more a culture for exploring and developing an understanding of mathematics. The Instructional Strategies category included discourse as one of the strategies that was suggested by the participants' works. Some of the other recommended strategies for developing understanding of mathematical vocabulary were the use of graphic organizers, discussion, word walls, and non-linguistic representations.

The category of Mathematical Literacy focused on meaning making and the ability to communicate such meaning. The scholarly works in this section described how to structure learning environments that would encourage students to share their ideas, ask questions, and develop their understandings. Some of the scholarly works also addressed building concepts and vocabulary in concert rather than as separate activities. The category Discourse linked with Literacy; discourse tended to address listening and speaking, and literacy included reading and writing with a focus on meaning making.

Figure 20 illustrates the organization of the categories as a group. Discourse and Literacy connected to all of the other categories and were thus elevated to themes. .

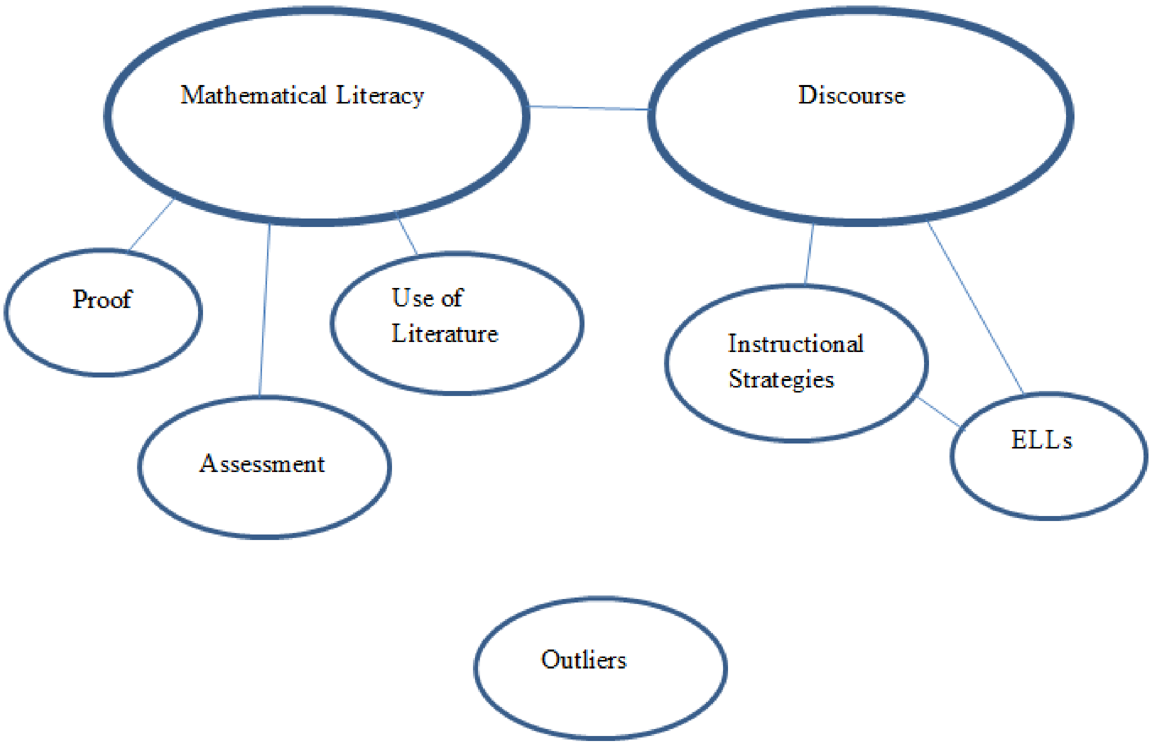

Figure 19. Participants' Scholarly Works Organized by Category

Participants' Foundational Works

The researcher analyzed the works identified by the participants in their responses to the first interview question, "Tell me about the research or researchers in the field of math vocabulary you believe form the foundation of your scholarly work." The participants' responses not only included researchers but also specific works they produced. Examining the scholarly works of the named researchers provided additional insight into the participants' stances concerning mathematical vocabulary. The researcher evaluated these using the same process as the historical analysis with the goal of determining how these works aligned to the categories and theme(s) that emerged in the participants' responses, as well as the aforementioned analysis. The tables that were generated characterized the results and were useful in delineating the commonalities and differences amongst the participants. The tables were not included in this study in order to maintain participants' anonymity. Several of the participants coauthored some of the scholarly works they attributed to the named researchers. The tables were also lengthy in nature and added up to 46 pages of information. Figures 21 and 22 were added to help the reader visualize the foundational works identified by the participants. Figure 21 specifies the categories of works the participants considered foundational. Figure 22 displays the scholars or specific scholarly works that the participants had in common. The ovals represent the categories, and the circles with thick borders denote the participants.

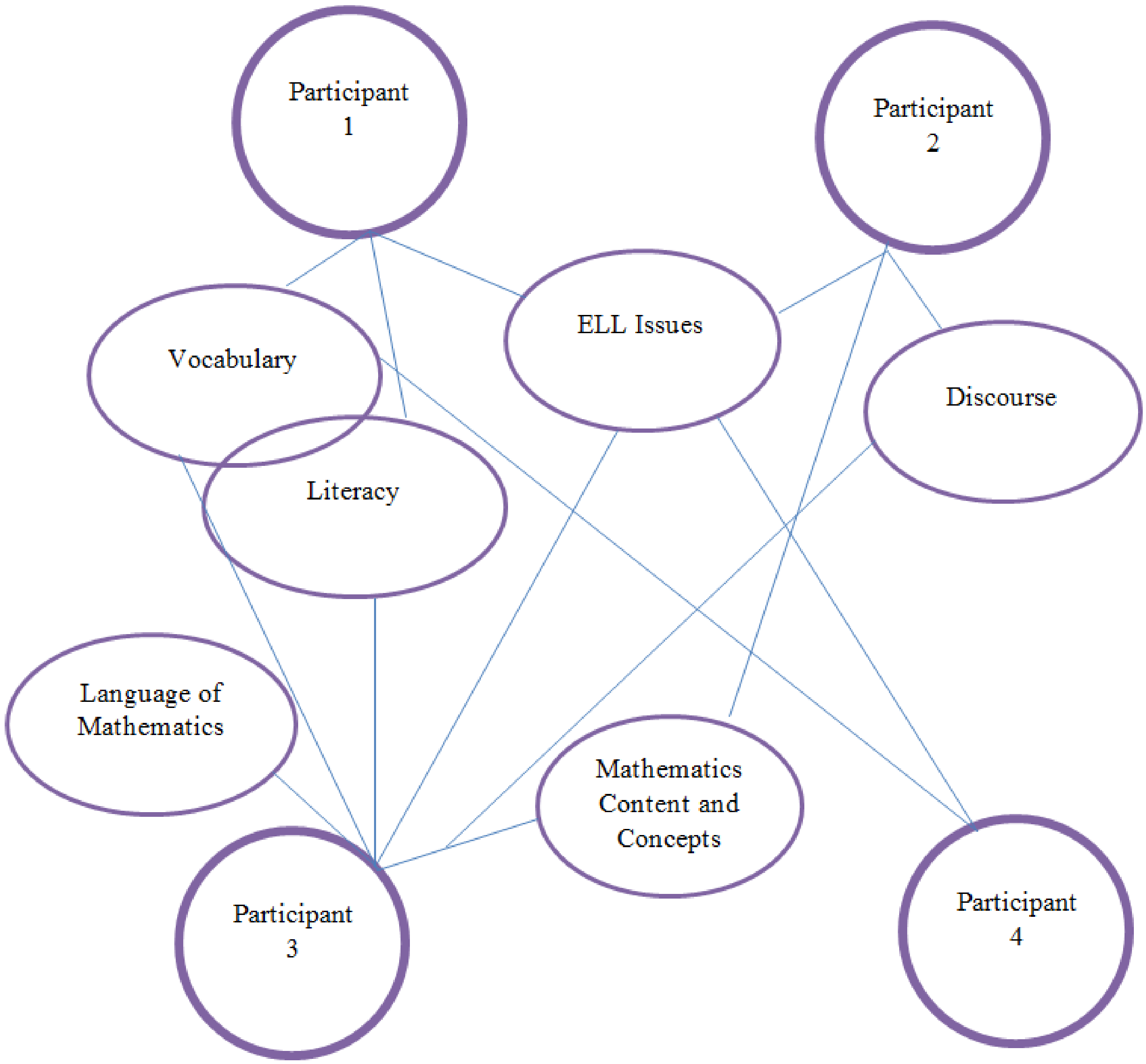

Figure 20. Categories of Foundational Works Identified by Participants

Participant 1 identified scholars whose major works could be categorized as vocabulary, literacy, and ELLs. These works were primarily associated with the field of literacy and included the authors Dale Johnson, P. David Pearson, Isabel Beck, William Nagy, Alden Moe, and Camille Blachowicz. While Dale Johnson's body of work is predominantly situated in vocabulary issues, P. David Pearson's work is positioned in the broader field of literacy. Isabel Beck, William Nagy, Alden Moe, and Camille Blachowicz have produced numerous works specifically addressing matters concerning vocabulary, in addition to other issues within the field of literacy. Note that very little of the work generated by these foundational scholars focused explicitly on mathematical vocabulary. The works that did attend to mathematical vocabulary incorporated vocabulary learning in other content areas. Some examples of these works include, but are not limited to, *Teaching Vocabulary in All Classrooms* (Blachowicz, 2014), *Vocabulary in Content Area Lessons* (Nagy, 1992), and *Vocabulary in the Elementary and Middle School* (Johnson, 2001). Participant 1 also named Janna Echevarria and Jennifer Bay Williams, whose scholarly works are associated with ELLs. Furthermore, Participant 1 cited three particular books as being fundamental: *Developing Mathematical Ideas* (Schifter, 1997), *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011), and *Principles and Standards for School Mathematics* (NCTM, 2000). These three works were mostly about mathematics and mathematics teaching; however, they did target the communication of mathematical ideas and the development of the precision of mathematical language.

Participant 2 referred to scholars who largely focused on discourse and use of the mathematics register. J. Lemke and Herbel-Eisenmann built their bodies of work on discourse; additionally, they addressed equity issues with flavors of critical pedagogy. Chapin's work concentrated on classroom discussions and the conducting of academic productive talk. Gibbon's work focused on the needs of ELLs. Pimm's work dealt with communication in mathematics, including the mathematics register. Register is a linguistic term that Halliday (1975) described as having a "set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p. 65). Unlike dialect, people can choose to use a particular register in certain situations and not in others.

Pimm argued that learning to speak like a mathematician or correctly using the mathematics register is one aspect of learning mathematics.

Participant 3 referenced foundational scholars whose work included mathematical vocabulary, discourse, ELLs' issues, and specific works. Rubenstein, Pimm, Murray, Gay, Herbel-Eisenmann, and Bay-Williams addressed vocabulary, discourse, and mathematical language. Adams's work dealt with reading mathematics. Rubenstein's work also targeted the learning of mathematical concepts. Of the four participants, only Participant 3 named foundational works that targeted mathematical vocabulary.

Participant 4 pointed to Janna Echevarria and Robert Marzano's works as fundamental. Participant 4 identified closely with Echevarria because of her work with sheltered instruction protocols for ELLs and with Marzano due to his research on instructional practices for vocabulary learning.

Figure 22 depicts the scholars or scholarly works that were identified by more than one participant. Each participant identified works they considered crucial as well; however, those were not included in the figure for the sake of clarity. The red rounded rectangles represent publications from the National Council of Teachers of Mathematics (NCTM). The orange rounded rectangles portray scholars that were commonly identified by the participants.

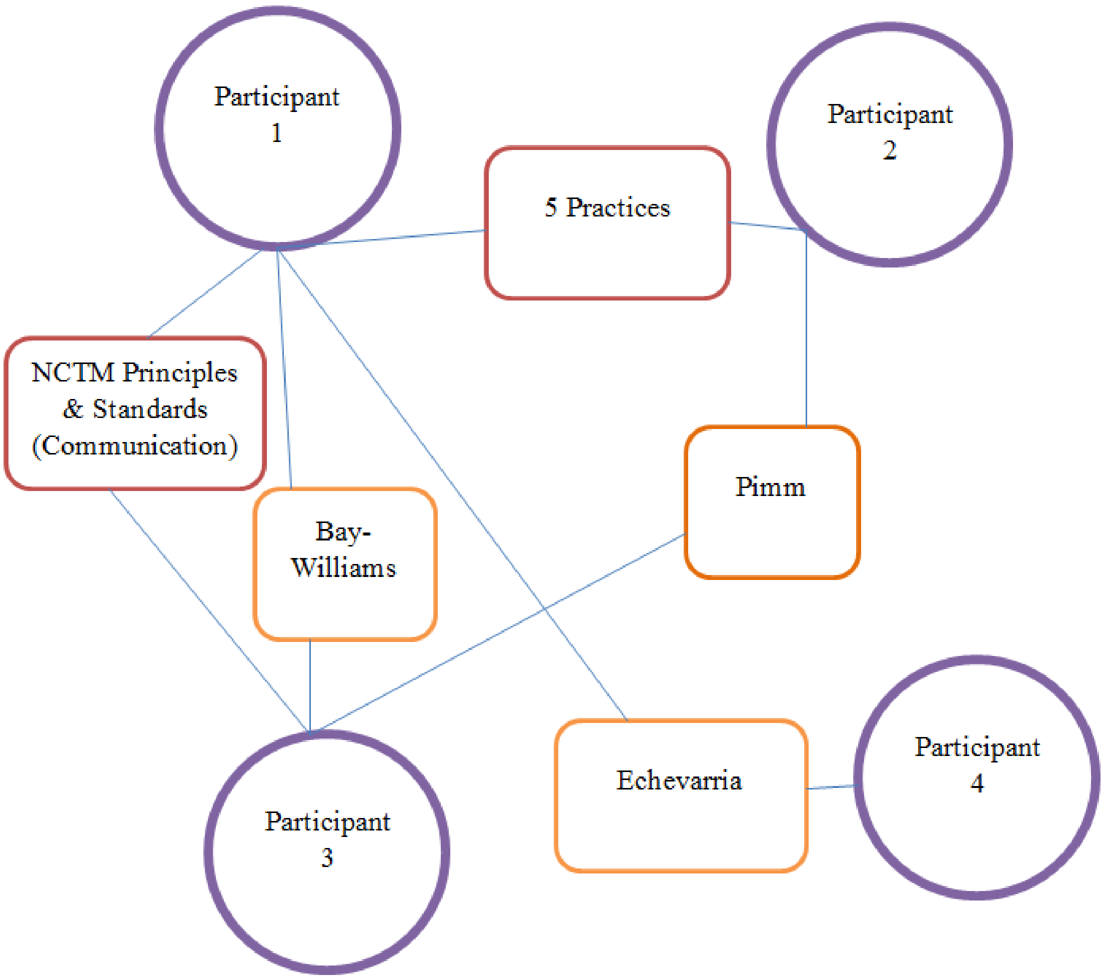

Figure 21. Relationship of Participants' Shared Foundational Works

The works and/or scholars identified by the participants helped the researcher further understand their responses to the interview questions. The works showed how their views of the field of mathematical vocabulary were grounded by the works they considered foundational. Participants 1 and 3 focused on mathematical vocabulary instruction. Participant 1 identified a scholar who produced a large body of work devoted specifically to literacy and vocabulary. Participant 3 mentioned scholars who authored scholarly work explicitly about mathematical vocabulary. During their interviews, Participants 2, 3, and 4 addressed issues pertaining to ELLs, as well as named scholars and scholarly works that addressed those students' needs. Participant 4 referred to the fewest scholars and scholarly works, but those identified fit well with the interview responses and the Participant's work with teachers in the field.

Overall, the participants pointed out the scholars and scholarly works that focused on vocabulary, literacy, and ELLs' issues in learning the language of mathematics, and mathematics content and concepts. The majority of the works were situated in the field of literacy, followed by vocabulary, and then ELL issues.

Chapter Summary

This chapter presented the results of the historical analysis and the case study. The historical analysis of the scholarly works in mathematical vocabulary in the United States uncovered the themes of Instructional Strategies and Bilingual Issues. The historical analysis included 126 works spanning from 1855 to 2015 and included only primary sources. Although the researcher worked diligently to capture all possible works pertaining to mathematical vocabulary, some may have been inadvertently left out. Several works identified through references and citations were not included in this study because they could not be located. The researcher conducted repeat searches using Google Scholar, as well as the data bases available through the Texas A&M University – Corpus Christi library, and kept a record of the search phrases used.

The case study consisted of interviews and follow up emails and phone calls to clarify original responses to the interview questions. The researcher examined the participants' scholarly works, as well as the scholars and scholarly works each participant identified as foundational to their own work. The researcher also analyzed the participants' interviews using

In-Vivo, Descriptive, and Emotion coding. Second cycle thematic coding resulted in the theme of Discourse. The researcher examined the participants' scholarly work in the same manner as the historical analysis, and the resulting themes were Mathematical Literacy and Mathematical Discourse. Similarly, the researcher studied the scholars and scholarly works identified as foundational to the participants and identified three major categories of works: literacy, vocabulary, and issues pertaining to ELLs.

CHAPTER V

This chapter focuses on the results of the historical analysis, the case study, and the combination of the two. The following headings and subheadings provide structure to the chapter and clarity to the reader.

- Introduction
- Findings
    - Historical Analysis
    - Case Study
    - Findings from the Historical Analysis and Case Study
- Implications
- Limitations
- Chapter Summary
- Recommendations

Introduction

The goal of this study was to determine the major trends and topics in mathematical vocabulary research in the United States from the mid-1800s to present. An examination of those trends and topics identified a core group of foundational works that may influence research conducted in mathematical vocabulary, as well as impact classroom instruction. An analysis of the reasons mathematics vocabulary experts identified significant mathematical vocabulary works may guide and influence future researches studies and provide direction for those studying mathematical vocabulary.

The purpose of the study was to fill a gap in the body of research concerning the understanding of mathematical vocabulary. The historical analysis identified trends, major topics and ideas, the emergence and development of those ideas, and gaps in the research that currently exist. In addition, the case study determined the views of current mathematics education vocabulary experts and how they interpreted and made meaning from the history of mathematics vocabulary research and writings. This study addressed the following research questions:

1. What content information and trends arise from an examination of literature and research on mathematical vocabulary?
2. What commonalities arise from interviewing experts about mathematical vocabulary?
3. What is the relationship between the historical and current themes in literature?

Findings

Historical Analysis

Rury (2006) stated, “A critical stage, of course, is actually constructing a historical narrative or an analysis of historical data with a distinctive interpretation” (p. 331). That critical stage in this historical analysis of scholarly works published in the United States from the mid-1800s to present addressed Research Question One: *What content information and trends arise form an examination of literature and research of mathematical vocabulary?* Two themes and 11 categories resulted. The themes were Instructional Strategies and ELLs’ Issues. The categories included Annotated Bibliography, Discourse, ELLs’ Issues, Instructional Strategies for Mathematical Vocabulary, Mathematics as a Language, Readability of Mathematics Textbooks, Reading Inventory, Taxonomy of Mathematical Vocabulary, Vocabulary Knowledge and Mathematical Success, Vocabulary Lists, and Vocabulary Relationship to Problem Solving. Figure 1 displays the relationships between the categories and between the categories and the themes.

The category of Instructional Strategies consisted of more scholarly works than any other. This category was one that classroom practitioners are likely to look to in order to find vision for improving the instructional practices employed in classrooms. While this body of work may provide insight and suggestions for instruction, it is worthy to note the limitation of this category; only five of the works were studies. The other scholarly works offered suggestions and examples of instructional strategies including word walls, games, graphic organizers, word parts, and several more. The studies in this category were conducted in 1941, 2006, 2012, 2013, and 2015.

The category of ELLs' Issues contained 15 works that specifically addressed the unique issues ELLs experience when learning mathematical vocabulary, as well as instructional strategies designed to support and enhance their learning. It is also important to point out that only four of these scholarly works were studies.

Examining the scholarly work that was identified in this study revealed a rich history of mathematical vocabulary works. A key point is that 40 of the 126 total works examined for this research were studies. The remaining works were not, some of which were from a time before research occurred in the field of mathematical vocabulary (Aley, 1907; Bailey, 1898; Betz, 1913; Davies & Peck, 1855; Merrill, 1918; Minnick, 1921; The National Council of Mathematical Requirements, 1921; Pierce, 1881; Smith, 1902; Speer, 1899; Thorndike, 1922; Wilson, 1922; Yocum, 1914; Young, 1920). There were also works grounded in studies that had been conducted in other content areas which applied the findings to the content area of mathematics (Dolgin, 1977; Hollander, 1977; Krulik, 1980; Monroe, 1997).

Situating the mathematical vocabulary works into the broader history of education and world events helps explain some of the trends that surfaced. As Rury (2006) stated, "The explanations historians offer, in that case, usually are not formal and are not intended to be testable or subject to strict replication. Rather, they often are situational, describing how events grow out of given contexts" (p. 325). For example, no studies in the field of mathematical vocabulary existed until Wilson's (1922) study that addressed the concern that poor reading resulted in students' inability to understand arithmetic problems. Educational research was in its infancy in the United States at that time. The American Educational Research Association had been established only a few years earlier in 1916. The 1950s had a small representation of scholarly works dedicated to mathematical vocabulary and contained four articles and one study. This lull in scholarly work may in part be attributed to the end of World War II in 1945 and the Korean War that lasted from 1950-1953. After the launch of Sputnik, in 1957, there was a call for enhancing mathematics and science education in the public schools. This might explain the surge of scholarly activity pertaining to mathematical vocabulary in the 1960s and 1970s. The work of Cummins (1979) and other scholars in the 1970s may have inspired the scholarly work that focused on ELLs within the realm of mathematical vocabulary which launched in the 1980s. The testing movement in the United States likely was, and continues to be, a source of inspiration for mathematical vocabulary scholarly work. This testing movement began in 1965 with the legislation of Title 1 of the Elementary and Secondary Education Act. The *Nation at Risk* report (National Commission on Excellence in Education, 1983) increased the emphasis of testing and led to high-stakes, mandatory assessments. The accountability associated with the high-stakes assessments already in place was increased (No Child Left Behind [NCLB], 2002). As assessments have become more rigorous over time, the reading demands have increased within the mathematics portions of these tests (Daro, Stancavage, Ortega, DeStafano, & Linn, 2007; Kovarik, 2010). This may also be responsible for the increase in scholarly activity pertaining to mathematical vocabulary in the 1990s to the present day.

Research Question Three

Research Question Three: *What is the relationship between the historical and current themes in mathematical vocabulary literature?* The themes of ELLs' Issues and Instructional Strategies also emerged as the current themes. Table 11 shows one way to examine the relationship between the historical themes and the current themes. The historical categories are listed in the first column. Note that the persisting categories throughout the time periods were Discourse, ELLs' Issues, Instructional Strategies for Mathematical Vocabulary, Mathematics as a Language, and Vocabulary Relationship to Problem Solving. The themes found in the historical analysis, Instructional Strategies and ELLs' Issues, remained in the current analysis..

Table 7

Relationship of Historical and Current Categories

	Number of works in time periods		
Category	1850s-present	2000s-present	2010s-present
Annotated Bibliography	1	0	0
Discourse	17	10	1
ELLs' Issues	15	9	2
Instructional Strategies for Mathematical Vocabulary	47	24	9
Mathematics as a Language	5	1	1
Readability of Mathematics Textbooks	11	0	0
Reading Inventory	1	0	0
Taxonomy of Mathematical Vocabulary	5	0	0
Vocabulary Knowledge and Mathematical Success	15	1	0
Vocabulary Lists	7	0	0
Vocabulary Relationship to Problem Solving	18	2	1

Figures 21 and 22 illustrate the themes and the categories from mathematical vocabulary literature. Figure 21 represents the time period from the 2000s to the present. Figure 22 signifies the time period from the 2010s to the present. It may be helpful to compare these to Figure 1, which displays the themes from the historical analysis of the time period spanning the 1850s to the present. The blue rounded rectangles are categories, and the red ovals show the categories which became the themes. Figures 21 and 22 do not have the green rounded rectangles shown in Figure 1. These categories do not exist in the current time period. Also note, Taxonomy and Readability of Mathematics Textbooks are not represented in Figures 21 or 22 for the same reason. Additionally, Mathematical Success is included on Figure 21 but not on Figure 22 due to the fact that this category does not exist in the current time period from 2010s to the present.

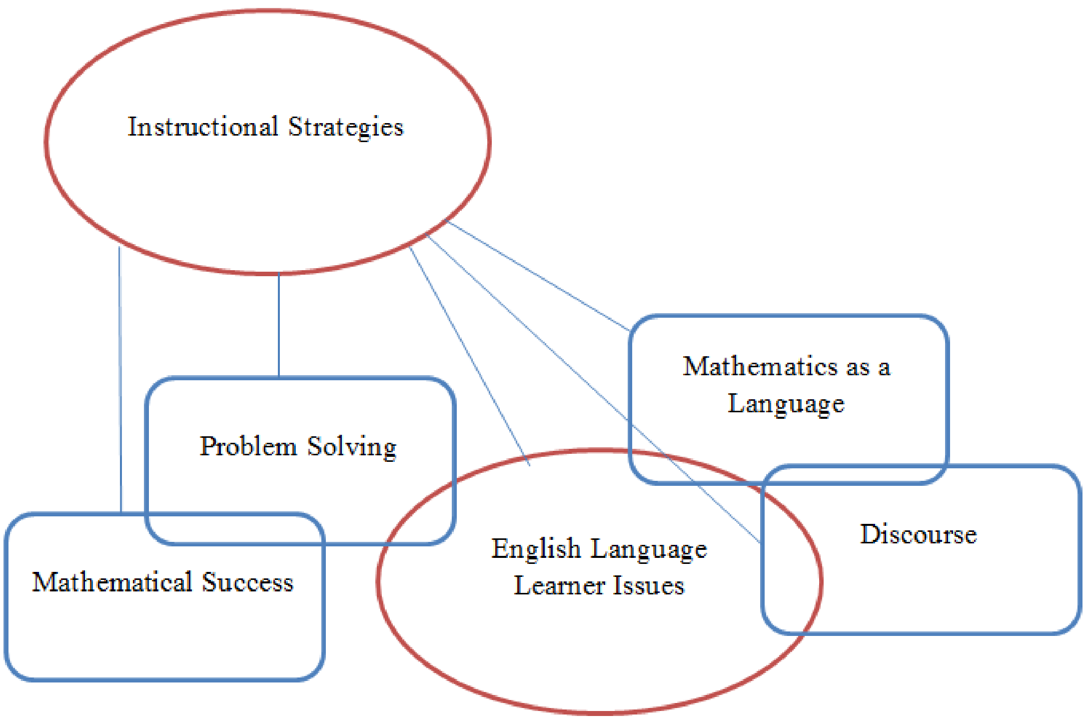

Figure 22. Themes from 2000s to Present

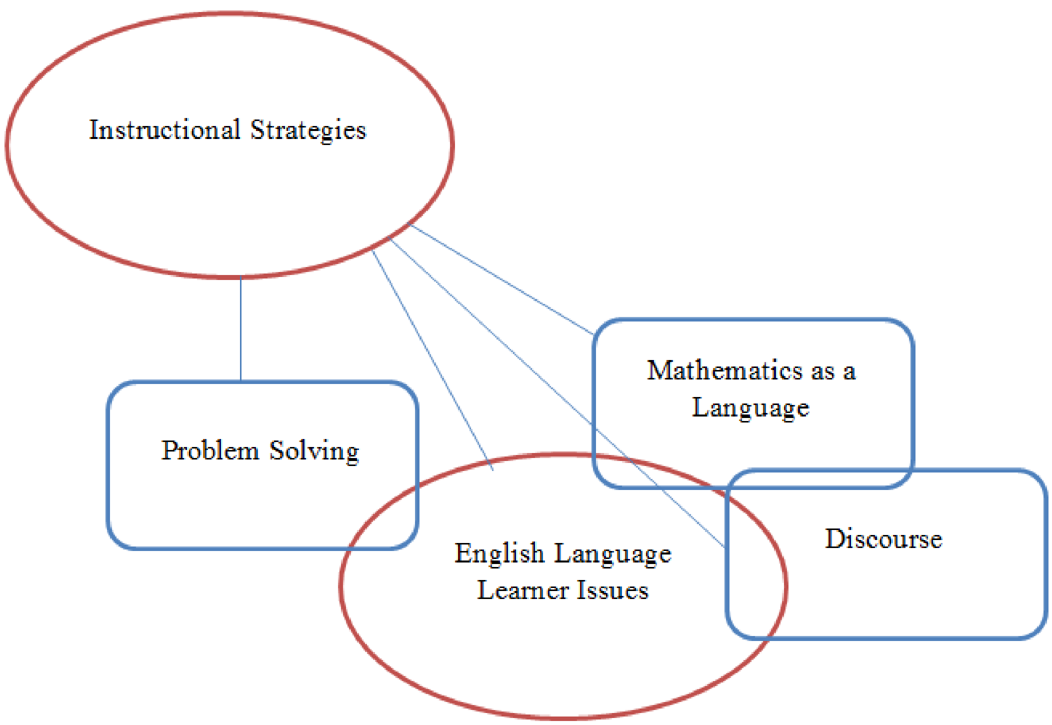

Figure 23. Themes from 2010s to Present

Case Study

The analysis of the interviews of four current mathematical education professors resulted in two themes, Discourse and Mathematical Vocabulary Instruction. The categories which emerged were Discourse, Environment to Support Discourse, Proposed Research, and Lack of Discourse. All four participants were emphatic that communication be a strong part of mathematical learning in general, and they specifically advocated for strengthening students' understanding and use of mathematical vocabulary.

The analysis of the participants' scholarly works determined how the participants situated themselves into the field of mathematical vocabulary. The participants were active contributors to the field, with 67 works among them. Not all were directly related to mathematical vocabulary, but they included topics that addressed the teaching and learning of mathematics. Two themes arose to describe their works, Mathematical Literacy and Discourse. The categories that were identified were Assessment, Discourse, ELLs' Considerations, Mathematical Literacy, Proof, Using Literature, and Outliers. Figure 18 displays the relationship among the categories and themes which emerged from the analysis. A majority of the works attributed to the participants were not included in the references to protect the anonymity required in a qualitative study.

The participants identified scholarly works that were foundational to their own. An analysis uncovered the commonalities and uniqueness among the participants. All four participants identified works and scholars who valued communication as a vehicle for developing mathematical understandings of both concepts and vocabulary. Figure 20 shows the relationships between the participants and shared foundational works. Participant 1 strongly identified with literacy scholars and their works. Participants 2 and 4 just as strongly situated themselves with scholars focused on ELLs. Participants 2 and 3 named scholars and scholarly works that centered on mathematics conceptual learning. As a group, the participants noted scholars and scholarly works that dealt with vocabulary, literacy, ELLs' issues pertaining to learning mathematical vocabulary and its unique register, and mathematics concepts and content. These works were largely not included in the references to protect the anonymity of the participants.

Research Question Two asked, *What commonalities arise from interviewing experts about mathematical vocabulary?* The commonalities which emerged from the holistic analysis of the case study were Discourse, Mathematical Literacy, ELL Considerations, and a central focus on Mathematical Vocabulary Instruction (Creswell, 2007). This question is best answered by an analysis of the participants' interviews, the participants own scholarly works, and the scholars and scholarly works the participants identified as foundational. Table 12 organizes the results of each portion of the case study. Four trends surfaced during the analysis of the participants' interviews, the participants' scholarly works, and the works the participants identified as foundational: Discourse, Mathematical Literacy, ELL Considerations, and ultimately Mathematical Vocabulary Instruction as it is the central hub that connects all of these ideas.

Table 8

Relationships within the Case Study

Participants' interviews	Participants' scholarly works	Participants' foundational works
Themes	Themes	Categories
Discourse	Discourse	Vocabulary
Mathematical Vocabulary Instruction	Mathematical Literacy	Literacy
		ELL Considerations
Categories	Categories	

Discourse	Assessment	Mathematics Content and Concepts
Environment to Support discourse	Discourse	
Lack of Discourse	ELL Considerations	
Proposed Research	Mathematical Literacy	
	Proof	
	Use of Literature	
	Outliers	

Historical Analysis Combined with Case Study Results

The results of the case study highlighted the views and works of four current mathematics education professors. Their body of works also serves as part of the makeup of the literature and research on the history of mathematical vocabulary.. The shared categories between the historical analysis and the case study were Instructional Strategies, English language learner Issues, and Discourse. It may be noted that these results are closely aligned to the current themes and categories from the historical analysis. This is expected due to the fact that the participants contributed works to the current time period from which the current categories and themes were derived.

Implications

The purpose of this study was to fill a gap in the body of research concerning the understanding of mathematical vocabulary. The historical analysis identified trends, major topics and ideas, the emergence and development of those ideas, and gaps in the research that have not been explored, which define the field. In addition, the case study determined the views of current mathematics education vocabulary experts, how they interpreted and made meaning from the history of mathematics vocabulary research and writing, and how they situated themselves within the field.

The results of this study fulfilled the purpose and offer a rich and well-defined foundation upon which future research and writings can be situated. The field of mathematical vocabulary is now defined, organized, and categorized. This is of value to educators and benefits scholars. Areas in the field where little to no studies have been performed were identified. This research shed light on her other areas that have lain dormant for decades and perhaps could or should be revisited under current conditions. For example, scholars and educators who have an interest in issues pertaining to ELLs now have a defined category of works which has been organized and summarized. This will enable future researchers to more readily conduct a literature review and make decisions about how to position their own research into the field. Consider the category of Reading Inventory; this category consists of one work but could be revisited and/or expanded upon. The category of Instructional Strategies, which is of great interest to practitioners, is also limited as it consists of a body of work with few supporting studies.

Limitations

The limitations of this study rest in two main areas: for the historical analysis, the ability to locate relevant primary sources of mathematical vocabulary scholarly works; for the case study, the limited field of potential participants. Although the researcher conducted an exhaustive search, it is possible that works appropriate for use in this study were not located. The inclusion of the unknown works may have led to different results in the historical analysis. The case study was limited to four participants due in part to a narrow field of potential participants and also to the purposeful exclusion of those who had coauthored or co-presented.

Chapter Summary

Historical analysis is a well-established branch of educational research which employs a range of research strategies. Rury (2006) stated:

> Like qualitative researchers in other disciplines, historians often must be opportunistic in constructing interpretations, drawing from the sources they can find. A critical difference is that historians use documents and other artifacts to develop such explanations, and this is what is most challenging about their research. (p. 323)

This study constructed interpretations from an examination of the mathematical vocabulary scholarly works in the United States. The researcher studied the body of work by time period and then by category. The interpretation of these led to overarching themes of Instructional Strategies and English language learner Issues. These themes and the categories that aligned to the themes, as well as those that did not, may be of interest to both scholars and practitioners. The implications for further research involve not only what was studied, but also that which was not. The rich body of works that constitute the historical analysis serves as a strong foundation for scholars desiring to conduct research in this field. Just as important is the lack of studies found as a result of this study. This provides an abundant source of inspiration to be exploited by those desiring to expand the understanding of mathematical vocabulary. Practitioners may find this study beneficial when looking for instructional strategies they wish to employ in the classroom. Practitioners may also find the scholarly works addressing ELLs helpful to support instruction in the classroom. This study has provided clarity to the field by defining the major categories of works that allow readers to dive deeper into their areas of interest within the field of mathematical vocabulary.

The case study portion of this research explored a bounded case over time with multiple sources of data which resulted in a case-based theme (Creswell, 2007; Denzin & Lincoln, 2005; Stake, 2005). The theme centered on Mathematical Vocabulary Instruction and included the categories of Discourse, Mathematical Literacy, and ELL considerations. The participants in this study were generous with their time and openly shared their insight and educated opinions with the researcher, which greatly enhanced the analysis.

A consideration of the historical analysis with the case study brings clarity to the current realities of the field of mathematical vocabulary works. Instructional Strategies, ELL Issues, and Discourse emerged as the commonality in the marriage of the historical analysis with the case study.

Recommendations

As a result of this study, the researcher recommends the following areas to be studied.

- The field is open to explore the effectiveness of the instructional strategies that have been suggested in scholarly work; these could include word walls, graphic organizers, word parts, analogies, and open sentences.
- Research the effectiveness of instructional strategies using a variety of student subpopulations.
- Replicate studies of the readability of current textbooks available.
- Research the discourse patterns in mathematics classrooms.
- Revisit the Reading Inventory and expand to additional grade levels.
- Research the vocabulary demands on current high-stakes assessments.
- Replicate this study and expand it to include a historical analysis of mathematical vocabulary works from English speaking countries.
- Research how practitioners situate themselves in the field of mathematical vocabulary.
- Research teacher preparatory class syllabi for the inclusion of mathematical vocabulary, ELL issues, discourse, and mathematical literacy

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