CONSIDERING THE ROLE OF K-2 TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING PLACE VALUE IN MATHEMATICS INTERVENTION

A Dissertation

by

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This dissertation meets the standards for scope and quality of
Texas A&M University-Corpus Christi and is hereby approved.

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ABSTRACT

A significant number of students receive special education services for mathematics learning disabilities, however, many additional students need mathematics intervention to support their learning in the regular education classroom. A Response to Intervention (RtI) model has been identified as effective in addressing these students' needs; however, little is known about the skill set needed by the interventionist to provide the appropriate instruction for these students.

This study employed a qualitative, multiple case study approach to explore the relationship between teachers' mathematical knowledge for teaching (MKT) and their understanding of how students learn place value concepts. Additionally, the study explored the relationship between teachers' MKT and the instructional strategies they identified to support student understanding of place value concepts in an intervention setting. The population included four K-2 elementary school classroom teachers who also teach mathematics intervention in a South Texas school district. The response data from the participants was compared to Battista's (2012) Levels of Sophistication in Student Reasoning: Place Value and the Mathematical Knowledge for Teaching (MKT) framework proposed by Ball, Thames and Phelps (2008) and expounded upon by Bair and Rich (2011).

The findings of the study indicated that, for this sample of teachers, those with a greater mathematical knowledge for teaching (MKT) of place value concepts demonstrated a deeper understanding of the mathematics content. Additionally, the teachers with greater MKT of place value selected, with greater consistency, research-based effective instructional strategies to support students in a mathematics intervention setting. The instructional strategies identified

included a *concrete to pictorial to abstract* approach and the use of *systematic, explicit instruction.*

Implications of these findings suggest that K-2 elementary school mathematics teachers who also teach mathematics intervention should possess a greater mathematical knowledge for teaching (MKT) and would benefit from supplementary training in Response to Intervention instructional strategies. These findings also suggest that school leaders may need to consider the feasibility of training teachers to be content experts versus hiring mathematics specialists for intervention instruction. Further research on interventionists' mathematical knowledge for teaching (MKT) has the potential to inform teacher education and teacher professional learning.

DEDICATION

This dissertation is dedicated to my late mother, Hazel Earlene Juenke, and to my father, Henry Carl Juenke, Jr. for instilling within me the desire to excel in academics. I learned from their examples how to be a leader who leads with a firm but compassionate hand and how to love music, math and God. I learned and developed a desire to be a leader from my father. I saw from a distance how he quickly advanced through the ranks in the fire department from pipe and ladder man to chauffer to district chief to finally become the chief of the department. I learned a love of music from my mother which provided a foundation for my understanding of the mathematics in the written notes. I saw the math in the scales I was required to learn and perform and realized I loved both music and mathematics. The most important thing I learned from my mother was a love of God. She lived a Christian life consistently and compassionately before me and the world, and provided the human example for what it means to be a Christian. Without a foundation in God, I would be adrift without a purpose and without a life plan.

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CHAPTER I: INTRODUCTION

Background of the Problem

The number of students in the United States (U. S.) with a specific learning disability in reading and/or mathematics who receive services through special education has been consistently above 2 million since 1990-1991 but increased to almost 3 million students in 2000-2001 according to the U. S. Department of Education (2016). The number of students with a learning disability has decreased slightly since 2000 but has only decreased to 2.26 million as of the 2013-2014 reported data. This staggering number of students identified with reading or mathematics disabilities is a significant number of students who then often struggle academically in both content areas (Willcutt, Petrill & Wu, 2013). It is, however, important to note that this number accounts only for those students who are identified with learning disabilities and are receiving special education services and does not include students who struggle and often fail in the regular education reading or mathematics classrooms. Additionally, in a report by Kena et al. (2016) the percentage of fourth grade students in the U. S. who scored at or above the proficient level on mathematics performance reduced from 42% in 2013 to 40% in 2015 on the National Assessment of Educational Progress (NAEP). This large percentage of students who continue to perform below proficient levels lends credence to the need for increased attention at the K-2 mathematics level where foundational skills are introduced and developed for future mathematics learning. In an effort to address these concerns, a number of initiatives have been introduced over the past decade, including Response to Intervention (RtI).

A Response to Intervention (RtI) model has been recognized as an effective tool to help teachers identify students with mathematics difficulties before they fail (Fuchs & Fuchs, 2006). The National Center on Response to Intervention (2010) describes RtI as a multi-level prevention system with increasing levels of intervention. They note that each of the three levels or tiers include high-quality mathematics instruction within "core instruction" or regular class instruction. This core level is denoted as Tier 1 within the multi-level prevention system. Tier 2 includes high-quality mathematics instruction with the addition of small group intervention lessons to address foundational skills. Students who show little or no progress at Tier 2 continue to receive high-quality instruction during their regular mathematics classroom lessons, but are also provided more intensive, individualized intervention within Tier 3. The essential components of the RtI model include universal screening to identify students who might be at risk for reading or mathematics difficulties (O'Connor & Jenkins, 1999), progress monitoring to assess students' rate of improvement (Deno, 1985), responsiveness to intervention, and data-based decision making that includes data analysis of multiple data sources to determine appropriate supports for individual students (Fuchs, Fuchs, & Compton (2012).

Researchers in the field of mathematics intervention, Bryant, Bryant, Gersten, Scammacca, and Chavez (2008), noted that the research on early mathematics intervention identified number sense as an appropriate content and explicit instruction an effective instructional strategy to address the needs of struggling K-2 mathematics students. Additional research on mathematics intervention has revealed important aspects regarding mathematics learning. For example, Bryant et al., (2008) reported the following findings: 1) Students who have difficulties with early counting principles such as order of numbers will develop difficulties with later counting principles including counting on and simple addition or subtraction combinations; 2) Students who do not develop a conceptual understanding of place value will struggle with future mathematics skills, particularly multi-digit calculations; 3) The compounding factors of students not mastering counting principles coupled with difficulties with addition or subtraction combinations often result in students struggling with acquiring computational fluency (note: students who struggle with addition and subtraction combinations typically develop mathematics disabilities or difficulties); and 4) Systematic, explicit instruction is especially beneficial and necessary for effectively addressing the needs of struggling mathematics learners.

The specific role that student knowledge of place value plays in students' future mathematical ability has been well documented in the extant literature (Jordan, Glutting & Ramineni, 2009; Bryant et al., 2011; Gersten et al., 2012), however, teachers' mastery level of the mathematical knowledge for teaching (MKT) of place value has been given little attention in the literature. In one study of pre-service pre-kindergarten teachers, McClain (2003) noted that teachers would not be able to teach place value conceptually if they did not personally possess a conceptual understanding of place value.

Statement of Problem

Many elementary school mathematics teachers are ill-prepared to teach the content according to research findings by Ball (1990). Studies of teacher mathematical content knowledge during the late 1990's revealed that while many teachers possess the knowledge necessary to understand basic arithmetic and algorithms, they lack a conceptual understanding of this mathematics (Ball, 1990; Mewborn, 2001). As Ball, Hill and Bass (2005) reported, the mathematical knowledge of teachers impacts student performance unfortunately, that knowledge is lacking in depth.

Research suggests that effective teachers possess additional and specialized knowledge. Shulman (1987) called this knowledge *pedagogical content knowledge* (PCK), and defined it as "the ways of representing and formulating the subject that make it comprehensible to others . . .

[It] also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning" (p. 9). Foss & Kleinsasser (1996) considered this type of knowledge in their research on pre-service teacher beliefs about mathematics in which they paired *mathematical content knowledge* (MCK) with *pedagogical content knowledge* (PCK). For their study, they defined this knowledge as "knowing the content of a subject or discipline and being aware of the means by which the content is taught" (p. 430). Ball, Thames and Phelps (2008) referred to this type of knowledge as *mathematical knowledge for teaching* which they defined as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 4). These studies emphasized the importance of specialized training needed for teachers to acquire this additional knowledge and be effective classroom teachers. Additionally, the skill set of the teacher has been identified as the single variable accounting for the greatest variance in student achievement as compared to other school-related factors (Organization for Economic Co-Operation and Development [OECD], 2005).

If specialized training is necessary to elevate teachers to effectively teach grade level mathematics content, instruction on mathematics intervention will require or demand another level of specialized training. While intervention lessons and resources have been developed to help teachers and mathematics interventionists make better decisions regarding how to effectively support struggling mathematics learners through identification of the appropriate content (number sense) and instructional strategies (explicit instruction), there is an apparent absence in the literature regarding the skill set of the person providing the mathematics intervention.

Purpose of the Study

The purpose of this study is to discover what mathematical knowledge for teaching (MKT) of place value concepts one group of K-2 teachers use with their students in tier 2 intervention. Thus, this study will consider how teachers' MKT of place value impacts their understanding of student thinking of place value during intervention and how that informs the instructional strategies they identify to support student understanding of place value, specifically with tier 2 students who struggle with these concepts.

Research Questions

When evaluating and deciding upon appropriate research questions, Creswell (2014) recommends identifying a central research question or questions. He defines a central question as "a broad question that asks for an exploration of the central phenomenon/concept in a study" (p. 139) and suggests that researchers select one or two central research questions. To discern the central question(s), he recommends that researchers ask the question, "What is the broadest question that I can ask in the study?" (p. 139). The central phenomena/concepts identified by this researcher are (1) teachers' content knowledge of place value and, (2) the impact teachers' mathematical knowledge for teaching (MKT) place value has on their instructional decision-making.

Consequently, the following questions will inform this study:

1. *How do K-2 mathematics teachers describe their understanding of how students learn place value?*
2. *How does teachers' mathematical content knowledge (MKT) for teaching place value impact their instructional decisions with struggling mathematics students in tier 2 intervention?*

Research Method

The research method used to investigate the problem for this study was a qualitative, multiple case study methodology to allow for rich descriptions of each case (Yin, 2009). According to Herriott and Firestone (1983), a study is considered overall more robust and compelling when comparing results from multiple case studies. Face-to face interviews were utilized to discover teachers' understanding of place value content. Student work samples were provided during the interviews for teachers to explain their understanding of students' thinking and identify instructional strategies they would use to support these struggling mathematics students' reasoning about place value concepts. The research methodology is explained in greater detail in chapter 3.

Theoretical Perspective

The theoretical frameworks applied to the analysis of data included Levels of Sophistication in Student Reasoning: Place Value by John Battista (2012) and Mathematical Knowledge for Teaching (MKT) proposed by Ball, et al. (2008) and expounded upon by Bair and Rich (2011). Each of these theoretical frameworks are explained in detail in chapter 2.

Significance of the Study

This study is important in advancing the research on the elements that impact struggling students' ability to understand basic mathematics concepts. Identifying the necessary and appropriate content knowledge of the K-2 mathematics tier 2 intervention teacher highlights another significant variable that interacts with intervention outcomes of struggling mathematics students. This has the potential to inform not only the research on mathematics intervention but also inform the training K-2 teachers receive in pre-service and in-service settings. Equipping teachers with the appropriate content knowledge will allow them to be better prepared in meeting the learning needs of struggling mathematics students for both regular classroom instruction and intervention instruction.

Thus, the results of this study will be particularly important to K-2 educators who provide mathematics intervention instruction; school administrators who secure funding for K-2 mathematics intervention resources and professional development; university teacher preparation personnel who instruct pre-service teachers in K-2 intervention models and frameworks; parents of elementary school students who struggle with learning mathematics.

Limitations and Delimitation

Limitations of this study include variations in teacher years of experience, variations in teacher specialized training and potential lack of teacher motivation to participate in this study. The content knowledge and/or mathematical knowledge for teaching (MKT) determined by an online assessment and through teachers' evaluation of student work may not be reflective of their instruction in the intervention classroom. Teacher beliefs about mathematics instruction, teachers' personal experiences as students and teachers' perceptions about their own students' abilities could all impact teacher instruction in the intervention classroom. Additionally, the small sample size of the study limits the ability to generalize the results to other groups of teachers.

Delimitations of this study include a narrow focus on K-2 teachers' mathematical knowledge for teaching of place value in a tier 2 setting while other content may play a significant role. Also, the study will include only K-2 mathematics teachers who teach tier 2 mathematics intervention in one South Texas district.

Definitions

Direct Instruction – "a dynamic set of teacher-directed actions that center on academic content and activities in which the teacher communicates information directly to the students in ways that use instructional time deliberately and efficiently, involving the gradual release of learning responsibilities from the teacher to the learner…with scaffolds…from explicit teacher direction to independent guided practice" (Swanson, Harris & Graham, 2013, p. 493).

Explicit, Systematic Instruction – "…requires educators to clearly teach the steps involved in solving mathematical problems using a logical progression of skills…considers the scope and mathematical trajectories, such as the types of examples used for developing the foundational skills prior to introduction/re-teaching of grade level material and should include the following components: advance organizer; assessing background knowledge; modeling; guided practice; independent practice; maintenance" (National Center on Intensive Interventions, 2016, p. 3-4).

Intervention – "the systematic and explicit instruction provided to accelerate growth in an area of identified need…provided by both special and general educators…designed to improve performance relative to a specific, measurable goal…based on valid information about current performance, realistic implementation, and include ongoing student progress monitoring" (http://www.rtinetwork.org/glossary, p. 1).

Learning Mathematics for Teaching (LMT) – the assessment tool to identify teachers' MKT of place value developed by The University of Michigan and utilized to identify teachers selected to be interviewed for this study. This tool was developed under the supervision of Heather Hill, Deborah Lowenberg Ball, Stephen Schilling (2004) at The University of Michigan.

Mathematical Knowledge for Teaching (MKT)- - "mathematical knowledge that teachers need to carry out their work as teachers of mathematics" (Ball, et al., 2008, p. 4).

This includes subject matter knowledge (common, specialized and horizon knowledge of mathematics) and pedagogical content knowledge (knowledge of curriculum, content and students, and content and teaching).

Tier 1 - The scientifically-based instruction provided to all students by a qualified professional. Within this tier, students are periodically screened for baseline and "at-risk" purposes and are provided with differentiation and supplemental instruction as needed (http://www.rtinetwork.org).

Tier 2 - An increased level of instruction with varying levels of intensiveness and frequency based on students' rate of progress. This instruction is provided in addition to the scientifically-based instruction described in Tier 1. These interventions are provided in small group settings (http://www.rtinetwork.org).

Tier 3 - Individualized, intensive intervention instruction that targets students' individual content or skill deficits. This intervention is more intense and often provided with more frequency than a Tier 2 intervention and is provided in addition to Tier 1 instruction (http://www.rtinetwork.org).

Summary

This chapter included the background of the problem, a statement of the problem, and the purpose of the study, all necessary elements to provide a context and to explain the purpose of the study. Also, this chapter provided the research questions, significance of the study, limitations and delimitations of the study and a definition of key terms. Each of these components combined offer an introduction to the research study and provide necessary information to understand the research study undertaken.

CHAPTER II: REVIEW OF THE LITERATURE

Introduction

This study explores teachers' understanding of place value and their knowledge of struggling K-2 mathematics students' reasoning about place value concepts within a RtI framework. It begins with the literature on theories of how students learn and reason with place value concepts. The section continues with teachers' knowledge of place value then considers literature on RtI, and provides a review of other similar studies conducted on teachers' knowledge for teaching RtI and the methods that were used to carry out these studies. The final section looks at the role of the K-2 mathematics interventionist and highlights the importance of the interventionists' pedagogical content knowledge of mathematics. The thread tying each of these sections together is the emphasis on the content (place value) and instructional strategies that are appropriate for effectively working with students who struggle with mathematics in grades K-2. The theoretical and educational frameworks on which the study is built are explained and embedded within the review of literature. The theoretical framework explores the concept of Pedagogical Content Knowledge (PCK) and provides a model for considering teacher knowledge of the content and knowledge of student understanding. The educational framework explores students' reasoning about place value and the levels students traverse in their understanding of this content.

Review of Literature

The Role of Place Value in Early Math Acquisition

Elements of place value understanding and number development have been addressed through the following theories about how children count. Steffe, von Glasersfeld, Richards & Cobb (1983) asserted that children can be classified as being counters of a particular type based on the most sophisticated counting type they demonstrate. These counting types include: perceptual, figural, motor, verbal, and abstract. The first four types require a child to use a sensory-motor action such as moving counters or require a child to create a physical representation in order to give meaning to number words. According to Steffe, et al. (1983) children progress through each of the first four counting types and must internalize their actions before they can demonstrate abstract counting. They hypothesize that only after a child transitions to the abstract counting type can numbers or number words begin to be considered conceptually and exist independently from the previous sensory-motor action or physical representation.

As children manipulate numbers and understand the relationship between the written numeral, the number word and the spoken numeral, they require a system to organize their thinking and working with numbers (Sharma, 1993). Cobb and Wheatley (1988) studied how children began to form a conception of and construct meanings for *ten*. They applied the theory asserted by Steffe, et al. (1983) and discovered that children constructed meanings for *ten* that did not correspond with the counting types identified by these researchers. Instead, Cobb and Wheatley (1988) determined that children operated in one of two contexts: “(a) pragmatic, relational problem-solving and (b) academic, codified school arithmetic” (p. 1). They explained that when children demonstrated school arithmetic, *ten* was not related to *one*; *ten* was not made up of *ones* and *ten* could not be decomposed into *ones*. These researchers pointed out a distinct difference in their participants and those who participated in the research of Steffe, et al. (1983) who received two years of intense teaching involving reflection and opportunities to reorganize their thinking about problem-solving situations. Cobb and Wheatley selected participants who received what they refer to as "typical textbook instruction in which rules were taught for assigning value to digits based on their position (e.g., *ones* place, *tens* place)" (p. 1-2).

Steffe, Cobb and von Glasersfeld (1988) examined more deeply the abstract counting type and identified three increasingly sophisticated concepts of *ten* children develop when they reach this level. These concepts of *ten* include: *ten* as a numerical composite, *ten* as an abstract composite unit, and *ten* as an iterable unit. Children who understand *ten* as a numerical composite, do not recognize *ten* as a unit but see it as *ten ones* or as *ten* but do not see it as being both. Children who understand *ten* as an abstract composite unit rely on physical representations of *ten* to construct units of *ten*. When they count using *tens*, they are simply counting the number of physical objects representing a *ten* unit. Children who understand *ten* as an iterable unit are able to count by *ones* and *tens* without using physical representations. They are able to count by increments and decrements of *ten* mid-decade and recognize that their counting is *ten* more or *ten* less than the previous number. Cobb and Wheatley (1988) applied this principle and noted that in their study, understanding *ten* as an iterable unit was a requirement before children understood the positional principle and place value of the number system.

A study comparing the development of place value concepts of American students and Asian students yielded several notable findings for why Asian students had a better understanding of place value (Yang & Cobb, 1995). First, the counting strategies American mothers used with their children rarely included interpreting numbers in the teens as composites of *ten* and *ones*. The study also revealed that emphasis in American schools was focused on one-to-one correspondence and counting by *ones* up to twenty as initial learning activities. Additionally, it was noted that grouping by *tens* was taught immediately prior to the introduction of addition and subtraction of two digit numbers appearing to be introduced only to facilitate the procedural steps involved in two-digit addition and subtraction and disconnected to previous counting principles. Conversely, Chinese parents and teachers reported that early counting strategies for numbers less than *ten* was based on one-to-one correspondence and oral counting. This study noted that Chinese-based number word systems were more consistent with number meaning and place value than American-based number word systems. Chinese children were encouraged to form groups or collections of *ten* initially when working with teens. Chinese mothers reinforced the concept of teens as *ten* and some *ones* when interacting with their children. Instructional strategies noted in Chinese schools where students were adding numbers where the sum was greater than *ten* involved students making *ten* and adding the remainder. Learning activities at home and at school for the Chinese students studied revealed that Chinese children had greater opportunities to construct numerical relationships with *ten* and did not treat place value as a separate, distinct topic.

Fuson and Briars (1990) developed a base-ten blocks teaching approach for use with first and second grade students on place value concepts of multi-digit addition and subtraction. They asserted that the English spoken system of number words is a "named-value system for the values of hundred, thousand, and higher" (p. 180) in which the number is stated followed by the value of that number. They offer an example to clarify: "five thousand seven hundred twelve, the "thousand" names the value of the "five" to clarify that it is not five ones (=five) but is five thousand" (p. 180). The researchers noted that the written system for numbers is a positional base-ten system in which each place or position of a written number represents a different value as designated by that relative position. In order to construct meaning of multi-digit numbers, the authors state that children must develop named-value and positional base-ten conceptual structures by recognizing these two different structures are used to represent the same value. The authors continue by stating that a lack of support for students to develop these conceptual structures often results in students learning multi-digit addition and subtraction as a sequence of steps or procedures to follow, relating little or no meaning to the value of the numbers they are adding or subtracting.

In a later study, Fuson, et al. (1997) highlighted the challenges students face with initial understanding of two-digit numbers due to the irregularities with the English spoken words for these numbers, particularly when compared to three-digit and larger numbers. The researchers discussed the word structure and written representation of eleven and twelve which seem to offer no hint as to the value of the numbers. Additionally, the authors reported that students' lack of understanding of two-digit numbers may result in them adding or subtracting two-digit numbers by operating on the digits as concatenated single digits, adding or subtracting the columns of numbers as if they were single digits. In this study, the researchers considered the conceptual frameworks students developed as they added and subtracted multi-digit numbers, specifically four- and five-digit numbers. The challenges noted when students initially encountered operations with two-digit numbers, had less impact when working with four-digit numbers and students were able to generalize their understanding back to two-digit addition and subtraction. According to the researchers, the regularity of the number structure of the hundreds and thousands allowed students to somehow view the tens with more regularity as they applied the same concepts to each place value representation.

Carpenter and Moser (1984) considered how students make sense of simple addition and subtraction problems by focusing on problem types: join, separate, combine (addition), combine (subtraction), and join missing addend. They reported that in order to solve these problem types, children used a combination of modeling and counting strategies. The modeling strategies involved students using physical objects or counting on their fingers to represent numbers in the word problems presented. The counting strategies included students counting on from the first number (for addition and subtraction) and counting on from the larger number. The other observed strategy students used to solve the math problems was number fact strategies or recall of number facts.

In a study looking specifically at how learning disabled students acquire place value concepts through different instructional approaches, Peterson, Mercer, Tregash & O'Shea (1987) found that students learn and retain initial place value concepts better when taught using a concrete, semi-concrete to abstract instructional sequence model. Students who were taught using only an abstract instructional approach did not acquire or retain place value skills at the same level. These researchers affirmed that utilizing concrete manipulatives first, then moving to pictorial representations of place value concepts prior to presenting students with abstract mathematical problems positively affects acquisition and retention of place value concepts.

In American schools, elementary students are taught to understand and represent place value concepts through the use of proportional and non-proportional models (Reys, Lindquist, Lambdin, & Smith, 2015). Proportional models utilize materials that are ten times the size of the material used to represent one and each subsequent place value is ten times the previous place. Non-proportional models do not have this quality but are used in daily activities including money, counters and beads. According to Van De Walle, Karp and Bay-Williams (2016), non-proportional models should not be used when introducing place value and should be reserved for use with students who have a better understanding of the base-ten numeration system.

Multiple models exist about how students learn and understand place value, however, each model is similar with specific levels of place value understanding. One model by Ross

(1989) describes student understanding of two-digit numerals as students consider written numbers and groups of objects representing the number. In stage one, whole numeral, students recognize the two-digit number as representing the whole of the group of objects but do not assign a value to the number. Students in stage two, positional property, recognize that the numeral on the right is in the *ones* place and the numeral on the left is in the *tens* place but there is no recognition of the value of the digits together nor is there a connection of the numeral that represents the group of objects. Students in stage three, face value, verbally label the numeral in the *tens* place as being *tens*, however, there is no understanding that it represents that many groups of *tens*. In stage four, construction zone, students recognize that the left numeral represents sets of *ten* objects and the right numeral represents what is left. Stage five, understanding, is characterized by the students' ability to represent the two-digit number as *tens* and *ones*, labeling the groups of *ten* with the number in the *tens* place and labeling the individual objects with the number in the *ones* place.

Levels of Sophistication in Student Reasoning: Place Value

John Battista's (2012) Levels of Sophistication in Student Reasoning: Place Value offers a description for students' reasoning across multiple place value levels (level 0 – level 6) within his framework of cognition-based assessment for place value (Table 1). While these major levels provide an overview of students' reasoning, a deeper, more detailed view of these levels is necessary for teachers to be able to identify student misconceptions and to maximize learning of place value concepts. Battista (2012) offers sublevels that compartmentalize place value concepts into manageable chunks that can be mastered in a relatively short amount of time.

The levels are grouped in levels 0 through 3 which provides a framework for how students' reason with place value concepts of individual numbers while levels 4 through 6 are grouped to provide a framework for how students' reason with place value concepts in traditional computational algorithms. As a point of clarification, level 0 and level 6 do not have sublevels, however, all other levels have multiple sublevels. Each sublevel provides a description of students' incremental reasoning of place value concepts and mastery at each sublevel is a prerequisite for movement to the next level with understanding.

Table 1. Place Value Levels

Place-Value Levels	
Level 0	Student has difficulties counting by ones.
Level 1	Student operates on numbers as collections of ones (no skip-counting by place value).
Level 2	Student operates on numbers by skip-counting by place value (e.g., counts by tens).
Level 3	Student operates on numbers by combining and separating place value parts (e.g., adds tens parts without counting).
Level 4	Student understands place value in expanded algorithms.
Level 5	Student understands place value in traditional algorithms.
Level 6	Student generalizes place value understanding to larger numbers, numbers less than 1, and exponential notation.

At level 0, a student has difficulty counting by ones and often counts sets of objects inaccurately by omitting objects or counting objects multiple times. At level 1, students think of numbers as collections of 'ones' and do not think of numbers as having place value beyond 'ones'. Level 1 has three sub-levels (1.1, 1.2 and 1.3) and students at sub-level 1.1 are unable to consider groups of ‘tens’ and therefore are unable to make sense of computational algorithms. Students at sub-level 1.2 can group objects by ‘tens’ but when counting the total number of objects, they count by ‘ones’. At sub-level 1.3, students operate on ‘ones’ and ‘tens’ separately, however, they do not understand the relationship between the two (i.e. students do not understand that ten ‘ones’ is equal to one ‘ten’). Students at this sub-level may be able to correctly represent a number with base-ten manipulatives, however, they do so with only rote understanding.

Students at level 2 are able to skip count using place value and can count by ‘ones’ and by ‘tens’ and understand that each ‘ten’ is a collection of ten ‘ones’. Level 2 has two sub-levels (2.1 and 2.2) and at sub-level 2.1, students are able to skip count by ‘tens’ only by starting with a multiple of ten. At this sub-level, students can add and subtract 2-digit numbers, however, operate on the ‘tens’ and ‘ones’ independently of each other. At sub-level 2.2, students can count by ‘tens’ starting in mid-decades, demonstrating their understanding and ability to maintain the relationship between ‘tens’ and ‘ones’ throughout their counting series. Students at level 3 can operate on numbers through successful application of their understanding of place value parts. They are able to compose and decompose numbers into ‘hundreds’, ‘tens’ and ‘ones’ correctly.

Level 3 has three sub-levels (3.1, 3.2 and 3.3) and at sub-level 3.1, students can use multiples of ‘tens”’ language when composing and decomposing 2-digit numbers into ‘tens’ and ‘ones’ (e.g. - 2 ‘tens’ is translated into ‘twenty’). At sub-level 3.2, students can use ‘tens’ language when operating on numbers (e.g. – 2 ‘tens’ plus 5 ‘tens’ is equal to 7 ‘tens’). At sub-level 3.3, students begin integrating previous sub-levels (2.1, 2.2, 3.1 and 3.2) and can shift among sub-levels easily and with understanding of the relationship of ‘ones’ to ‘tens’ to ‘hundreds’.

Students at level 4 apply their understanding of place value of individual numbers to make sense of expanded computational algorithms. At level 4.1, students demonstrate understanding of place value for expanded addition and subtraction algorithms by decomposing numbers into ‘ones’, ‘tens’ and ‘hundreds’ and operate on those parts independently. Students may operate on the decomposed number in any order (i.e. – ‘hundreds’ before ‘ones’, etc.) and are not restricted to a specific order. Students at level 4.2 apply their understanding of place value concepts of individual numbers to expanded multiplication and division algorithms. For students to reason proficiently with multiplication and division expanded algorithms, they must be able to multiply multiples of 10 mentally. Students at level 5 apply their understanding of place value of individual numbers to reason with place value in traditional algorithms. They understand place value parts of three digit numbers and can move flexibly between different place value parts of multiple numbers as necessary to regroup with ‘carrying’ or ‘borrowing’ performed in standard algorithms. Students demonstrate their understanding through use of correct mathematical language and through explicit demonstration of the steps utilized in performing the algorithm.

Students at level 5.1 demonstrate their understanding of place value concepts of individual numbers and of expanded addition and subtraction algorithms to reason with traditional addition and subtraction algorithms. Students at level 5.2 demonstrate their understanding of place value concepts of individual numbers and of expanded addition and subtraction algorithms to reason with traditional multiplication and division algorithms.

At level 6, students can generalize their understanding of place value concepts described in levels 0 through 5 and apply this understanding to larger numbers beyond 'hundreds', numbers less than 1 and exponential notation. Students understand the relationship between adjacent place values for all numbers (e.g. - each is a multiple of 10, moving 1 place to the right or left) and this recognition allows for the understanding of exponents used in representing a number with exponential notation.

Teachers' Knowledge of Place Value

Teachers' knowledge of mathematics is germane to their ability to use appropriate instructional materials, to evaluate students' performance and to make informed decisions about how to sequence and emphasize specific mathematics concepts (Ball, et al., 2005). Specific research on teachers' content knowledge of place value is limited, however, there is some early research on teachers' mathematical content knowledge. In a report for the National Center for Research on Teacher Education, Ball (1988) noted that researchers were previously unable to link teachers' mathematics content knowledge with classroom instruction. This early research on teacher content knowledge resulted in a simplistic list of characteristics or qualities of teachers others' deemed effective (Medley, 1979). Research on teacher content knowledge later evolved to consider how teachers understand mathematical concepts and procedures (Ball & McDiarmid, 1989). Research on teachers' pedagogical content knowledge emerged from Shulman's (1987) work on teachers' knowledge and teaching which spurred research on teachers' mathematical pedagogical content knowledge (Ball & Bass, 2003; Ball, et al., 2008; Hill, Ball & Schilling, 2008).

A study that considered teacher content knowledge of place value utilizing the Learning Mathematics for Teaching (2006), Elementary Place Value Content Knowledge assessment compared pre- and post-tests to determine teacher pedagogical content knowledge (Kulhanek, 2013). The focus of this mixed methods study was to determine whether professional development of place value instruction would increase teacher PCK of place value. The researcher utilized only the portion of the assessment that addressed second-grade place value skills and found that pre-test results yielded passing rates ranging from 25% to 62.5%, however, after 12 hours of professional development, the passing rates on the post-test increased and ranged from 50% to 100%.

Another study that employed the use of the Learning Mathematics for Teaching (2006) considered the relationship between math intervention teachers' pedagogical content knowledge (PCK) and their students' gains in a math intervention setting (Waller, 2012). This quantitative study compared mathematics intervention teachers' years of experience, hours of professional development training, student contact hours, and the teachers' scores on the Learning Mathematics for Teaching (LMT) (2006). This version of the LMT assessed teachers' knowledge of mathematics across multiple mathematics domains with place value being part of one of the domains. The findings revealed a positive correlation between student's achievement gains on a standardized mathematics assessment and the number of contact hours of mathematics instruction provided by the mathematics interventionist. Additional findings indicated an increase in mathematics interventionist's MKT scores following professional development that spanned one school year.

Another study utilized error analysis of coded data to consider pre-service elementary school teachers' pedagogical content knowledge of mathematics of whole number and operations, place value and fractions (Matthews & Ding, 2011). The findings suggested that even after specific college course-work on these concepts, many pre-service teachers continue to exhibit difficulties with these concepts. Data analysis was discussed in terms of "robust categories", however, exact percentages were not reported due to overlap of categories.

A study by Cady, Hopkins and Price (2014) employed a qualitative methodology to attempt to impact early childhood teachers' understanding of the complexities of place value. The researchers analyzed observations of students' work with manipulatives, written teacher reflections of lessons and oral arguments within class discussions. This qualitative study considered the responses of pre-service and in-service teachers who attended a two-day workshop on place value learning. The researchers created a base-five number system using symbols instead of numerals to allow the teacher participants to experience learning place value with a different place value system. They hypothesized that these teachers would make connections in their learning to their own students' learning of place value of the base-ten number system and would develop a deeper understanding for how students learn place value. The researchers concluded they met the requirements of their hypothesis.

Fuller (1996) compared novice (pre-service) and experienced (in-service) elementary teachers' pedagogical content knowledge of whole numbers (including place value) and operations, fractions and geometry. Data was collected via a survey in which each teacher evaluated a series of mathematical tasks students completed. The teachers were asked to explain their understanding of student thinking and identify the instructional strategies they would use with each student. The results of this qualitative study revealed that more than half of the novice teachers responded with procedural instructional strategies while approximately three-fourths of the experienced teachers responded with instructional strategies that included some evidence of greater conceptual understanding, particularly of whole number operations.

Response to Intervention History

Response to Intervention (RtI) was introduced initially as an approach for educators to support students who struggle in reading. The knowledge gained from supporting students in reading was applied to mathematics content and what follows is the journey of how RtI became an effective approach for working with struggling math learners. Beginning work in RtI dates to the 1960's with the initial focus on reading, however, it did not gain much traction until the reauthorization of the Individuals with Disabilities Education Act (IDEA, 2004). Other seminal work in RtI includes the research of Bergan (1977) and Deno and Mirkin (1977) focusing on behavior and reading respectively that included an intervention plan and the use of progress monitoring. Researchers began considering how what was learned from using an RtI model with students who had behavioral concerns and reading difficulties could be applied to mathematics.

The work of Fuchs et al. (2005) at Vanderbilt University provided the groundbreaking research on RtI in mathematics, focusing on small group tutoring for students identified as at-risk for mathematics difficulties. These researchers applied strategies already identified for reading RtI to mathematics content utilizing a three-tiered approach. According to The National Center on Response to Intervention (2010), this approach includes Tier 1 defined as high-quality core instruction. Tier 2 also includes high-quality core instruction but is coupled with small group intervention lessons. Tier 3 also includes high-quality core instruction but is paired with intensive, individualized instruction.

The field of mathematics intervention has benefitted immensely through consideration and application of research in reading intervention according to Bryant et al. (2008). Lembke, Hampton and Beyers (2012) presented a list of reading principles from reading researchers Roccomini and Witzel (as cited in Lembke, et al., 2012) that they suggested could apply to mathematics. These principles included: a belief that all students can learn; use of a universal screener; progress monitoring of student performance; and use of research-based instruction and interventions.

In an effort to effectively address students with mathematics difficulties, Lembke and Foegen (2009) emphasized early intervention for mathematics as a necessary component of successful mathematics development for students with mathematics difficulties. The broad topic of number concepts, namely number sense (of which place value is a component), was identified as a critical area for students to become successfully proficient in mathematics. Successive research efforts by Jordan, et al. (2009), Bryant et al. (2011), Gersten et al. (2012) and other leaders in the field of mathematics intervention have provided great insight into the screening of students for intervention as well as appropriate mathematics content and instructional strategies to be utilized in effective intervention. The content identified for screening, number sense and its component parts, became the basis for content taught in RtI mathematics instruction.

RtI Mathematics Content

The research on early mathematics intervention has narrowed in on a focus of number sense, defined as "moving from the initial development of basic counting techniques to more sophisticated understanding of the size of numbers, number relationships, patterns, operations, and place value" (National Council of Teachers of Mathematics, 2000, p. 79). The National Council of Teachers of Mathematics (NCTM) identified key elements of number sense to be counting, number knowledge, number transformation, and estimation.

To determine students' current mathematics ability level, Lembke and Foegen (2009) reviewed multiple assessments that could be used as screeners to identify students in need of intervention. After considering the body of research on mathematics intervention content and screening, the researchers narrowed their focus to four early numeracy skills or indicators: "quantity discrimination, quantity array, missing number, and number identification" (Lembke & Foegen, 2009, p. 14). These researchers defined quantity discrimination as the ability to compare two numbers and determine which was the larger number. Quantity array was defined as the ability to correctly identify the number represented by a set of dots arranged in arrays. Missing number was defined as the ability to correctly identify a missing number in a series of numbers represented in a pattern. Number identification was defined by the ability to correctly identify a number when presented with the number in numeral form. This research focused specifically and discriminatingly on identifying indicators of early numeracy that when monitored and assessed repeatedly over time, would provide valid and reliable information regarding improving low-achieving students' mathematics performance. Jordan et al. (2009) reported that number sense had a significant and positive impact on variances in student academic performance in first and third grades. Bryant et al. (2011) studied the effects of a numeracy intervention with a group of first grade students who exhibited mathematics difficulties. The screening instrument assessed magnitude comparison, number sequences, place value and addition/subtraction combinations. The research findings indicated an overwhelming sample of students who received tier 2 small group intervention instruction responded more favorably on an early numeracy evaluation than the control group who did not receive this intervention. Gersten et al. (2012) reviewed the current screening tools available for early primary grades in mathematics and noted the consistency across screening measures of focusing specifically on number sense and the component parts of number sense.

RtI Instructional Approach

Having identified the content to be addressed in a mathematics RtI lesson, it is necessary to consider the pedagogy associated with effective instruction in RtI. Examples of effective instructional approaches include small group instruction, individual feedback during lesson, and direct or explicit instruction of skill. In fact, Zheng, Flynn and Swanson (2012), emphasized that direct and explicit instruction combined was a critical instructional component associated with strong positive results. They pointed out that their synthesis of effective instructional strategies for students with learning disabilities was consistent with other researchers' syntheses of effective instructional strategies in that explicit instruction was a highly effective strategy. As reported by Bryant et al. (2008a), systematic, explicit instruction was especially beneficial and necessary for effectively addressing the needs of struggling mathematics learners. Gersten et al. (2009) in their publication, *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools (NCEE 2009-4060),* identified and evaluated eight recommendations to assist teachers, principals and school leaders in addressing the needs of struggling mathematics students through a Response to Intervention model. The top three recommendations and their corresponding level of evidence from research are: 1) Screen all students for potential mathematics difficulties; 2) Focus intervention on whole numbers, grades K-5 and rational numbers, grades 4-8; 3) Use systematic, explicit instruction.

Considering the research presented within this literature review, a model for framing and understanding K-2 elementary school mathematics teachers' mathematical knowledge for teaching of place value concepts in intervention lessons was identified. Teachers' responses to student work samples of place value concepts during the face-to-face interviews were compared with the model of Mathematical Knowledge for Teaching (MKT) proposed by Ball, et al. (2008) (Figure 1).

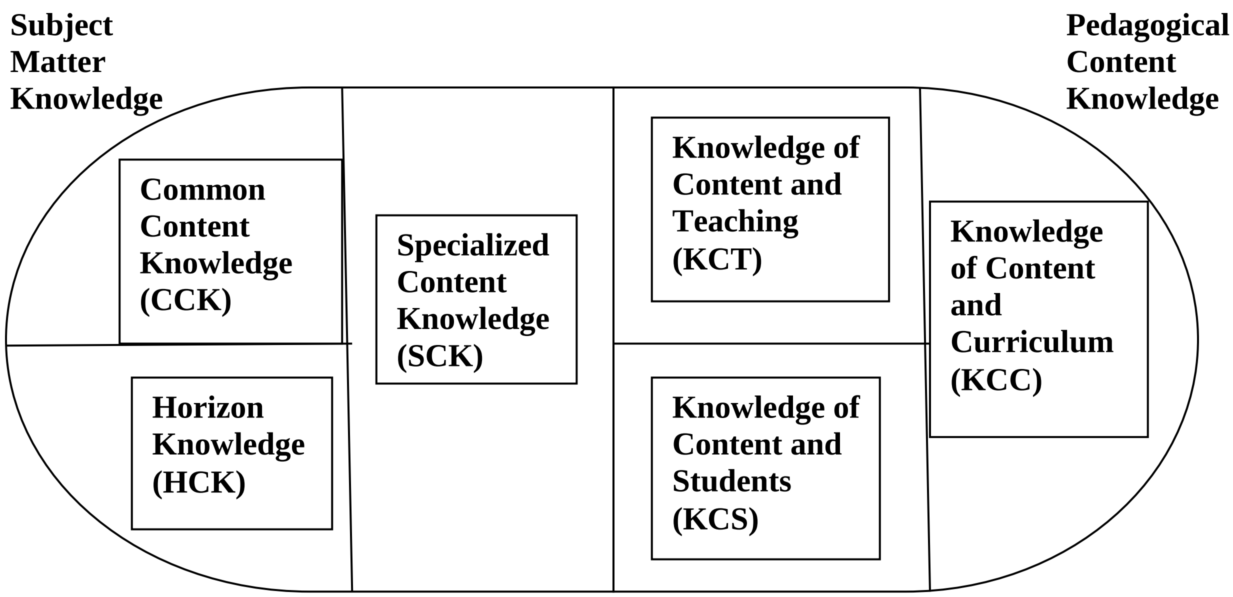


Figure 1. Mathematical Knowledge for Teaching (MKT) "egg" model.

Much research has been conducted by Ball and colleagues considering and studying the mathematical knowledge teachers need for teaching (Ball & Bass, 2003; Hill, Rowan & Ball, 2005). This model has evolved from an original teacher practice-based theory of MKT (Ball & Hill, 2003) to now include "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball, et al., 2008). An adapted version of the "egg" model defining each component part of MKT has been offered by Bair and Rich (2011) and provides a clear description of each part.

The "egg" model suggests that there are two larger halves of MKT made up of Subject Matter Knowledge and Pedagogical Content Knowledge. Subject Matter Knowledge includes Common Content Knowledge (CCK), Horizon Knowledge (HCK) and Specialized Content Knowledge (SCK). Pedagogical Content Knowledge includes Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS) and Knowledge of Content and

Curriculum (KCC). The specific component parts that will guide this study include both Specialized Content Knowledge (SCK) and Knowledge of Content and Students (KCS), considering teachers' knowledge of place value and teachers' knowledge of student thinking about place value. Hill, et al. (2005) note that PCK provides a foundation for how teachers identify and create the mathematical tasks they use, how they explain these tasks and facilitate productive classroom discourse. They stress this includes teachers' understanding of both how students respond and how teachers check for student understanding of the task coupled with how teachers quickly analyze student misconceptions and provide meaningful feedback. According to Bair and Rich (2011), SCK includes the ability of the teacher to explain or "unpack" the mathematics in a manner that supports students' understanding of the content and be able to respond to students' questions about mathematics productively. They further define KCS as teachers' ability to understand students' thinking about mathematics and the ability to relate that understanding to clear, correct mathematical thinking. The other components of the "egg" model included CCK, defined as basic mathematical knowledge an adult would hold if they were mathematically literate. The understanding of how mathematics topics are related across grade level spans is how they define HCK. The interaction between the mathematical content a teacher possesses and their knowledge of pedagogical matters that might impact student learning or understanding is KCT. Teachers' knowledge of how the mathematical content is related to the tools (e.g. textbook, manipulatives, teacher guides, etc.) used in mathematics teaching is KCC (Bair & Rich, 2011).

The Mathematics Interventionist

To better understand the issues impacting student understanding of mathematics in a RtI lesson, it is important to consider the role the mathematics interventionist. Research has identified both the content (place value which is a component of number sense) and the instructional strategy to be utilized within a RtI mathematics lesson (systematic, explicit instruction) but not much is known about the role of the mathematics interventionist. While intervention lessons and resources have been developed to help teachers and mathematics interventionists make better decisions regarding how to effectively support struggling mathematics learners, there is an apparent absence in the literature regarding the skill set of the person providing the mathematics intervention. It is true that frequently the classroom teacher serves as the mathematics interventionist, however, the content and instructional strategies for intervention as reported in this paper comprise a completely different skill set than the skill set for the mathematics classroom teacher.

In a report by the National Joint Committee on Learning Disabilities (2005), the importance of the qualifications of the interventionist was discussed in terms of potential changes in licensure, higher education accreditation, certification, and training. The report stated "the specialists providing more intensive interventions will be expected to master a variety of scientific, research-based methods and materials" (p. 7), and indicated this would be a challenge. When asked whether it would be important to consider the skill set of the mathematics interventionist, Diane Bryant, a recognized mathematics RtI researcher at The Meadows Center for Preventing Educational Risk at The University of Texas at Austin, stated "It's a good point to focus on the interventionist" (D. Bryant, personal communication, November 9, 2015). Also, when posed the same question, Ben Clarke, a recognized mathematics RtI researcher at The University of Oregon, replied "I believe as the field advances and understands more about the nature of interventions, more work will focus on critical variables that interact with intervention outcomes. And you've identified a huge one - the content knowledge of the teacher" (B. Clarke, personal communication, November 9, 2015).

Fuchs, et al., (2012), discussed a next generation RtI in which they proposed the term "smart RtI" to mimic current technology that is more effective and efficient such as "smart phones". They explained that by utilizing researched-based tools and knowledge it is possible to design "more effective and efficient multi-level prevention" (p. 264) to better meet the needs of struggling students. They proposed that the person providing the intervention be a specialist and state that it is not only naïve but also poor policy to expect the generalist to be cross-trained to meet the demands of their 20 to 28 students in addition to the students who are the more "difficult to teach children" (p. 270). Fuchs and Fuchs continued by stating that "effective educators at this level will be instructional experts" (p. 271) who are knowledgeable about the curricula and about effective instructional strategies appropriate for these students but the researchers did not elaborate further on the specific skill set of these specialists.

Summary

A thorough search of the current literature reveals much research exists on the content and instructional strategies effective for use when working with K-2 students who struggle with mathematics, however, little research exists on the skill set of the teacher who works with these students. Kahan, Cooper, and Bethea (2003), indicate that strong mathematical content knowledge (CK) seems to be ''a factor in recognizing and seizing teachable moments'' (p. 245), but they stress that this knowledge of the content alone does not provide rich mathematical experiences for students. They indicate that what is required is pedagogical content knowledge (PCK), which includes teachers' understanding of the mathematics content, understanding of students' reasoning, and understanding of effective instructional strategies. If teacher pedagogical content knowledge is necessary for rich mathematical experiences in the regular classroom, it would seem this would be essential for working with students in a mathematics intervention setting. And, if teacher content knowledge or PCK is a critical variable for math intervention as Clarke (2015) suggested, it would seem that additional research could provide invaluable data for the field of mathematics intervention. Specifically related to this study, what teachers know about place value concepts, what they know and understand about how students reason with place value concepts, and what they know about the instructional strategies that are effective for an intervention setting begs for additional attention in the research of mathematics intervention.

CHAPTER III: METHODOLOGY

Introduction

This chapter includes the research design selected for the study and the rationale for selecting this design. Additionally, an overview of the study is described and details about the setting and participants are discussed. Next, the procedures followed in the study are detailed and the instrumentation tools utilized in creating data are expressed. A description for how the data was collected is presented, followed by the interview protocol used during the face-to-face interviews. The next section offers an explanation and discussion for how the data was analyzed. The final sections in this chapter provide an understanding for how trustworthiness and rigor were established and maintained in the study, and how potential researcher bias was identified and managed.

Research Design

This study employed a multiple case study qualitative methodology using face-to-face interviews to investigate teachers' understanding of place value and their knowledge of struggling mathematics students within a RtI framework. A multiple case study was chosen because the evidence collected from multiple case studies has the potential to make the study more robust and compelling (Yin, 2009). Also, according to Yin (2009), the logic underlying a multiple case study should be to either replicate the initial case study or contrast the initial case study. This study considered teachers who have achieved a range of scores on an online assessment of their mathematical knowledge for teaching (MKT) of place value and their understanding of students' thinking of place value concepts coupled with their instructional recommendations for these struggling math learners. Each teacher is a separate case in the study, with the exception of the two teachers with the middle scores, since each represents differing ranges of scores on the MKT assessment. The cases were deliberately selected because they represent contrasting situations and a direct replication was not being sought (Yin, 2009; Eilbert & Lafronza, 2005; Hanna, 2005). It was supposed by the researcher that the contrasting situations of score ranges would support the working hypothesis and allow for a start toward theoretical replication. The intent was to allow for comparison of teacher understanding for how students learn place value concepts, coupled with the selection of mathematical instructional strategies they select for use with K-2 students in a mathematics intervention setting. Construct validity was a great consideration therefore, the construct being studied was defined in terms of specific concepts (in this case, the theoretical frameworks), and the operational measures that match the concept are identified within related research findings (Yin, 2009). By comparing the conditions under which an event occurs (identified in the frameworks) with the conditions under which the event does not occur, probable results can be anticipated for comparisons of new or additional cases (Yin, 2009).

The student work samples included in the interview protocol were representative of students who may struggle with place value concepts at various levels of Battista's (2012) framework and were similar to work samples of struggling tier 2 math students. The researcher's working hypothesis was that teachers with less knowledge of MKT of place value would respond with a more limited understanding of student thinking and suggest or recommend limited instructional strategies than teachers with a higher score. Therefore, this multiple case study began with the initial case of the K-2 math teacher who achieved the highest score on the Elementary Place Value Content Knowledge (LMT, 2006) assessment and each subsequent case was contrasted and compared with this case (high score, middle score, low score). A case study approach allowed the researcher to discover the teachers' understanding of the complex construct of place value through interviews and an online assessment. It also afforded the researcher the opportunity to observe the nuances included in the events when the teachers explained or described their use of a place value component or strategy. This approach provided a richer picture of what was happening in the classroom from the teachers' perspective and offered explanations that might not otherwise emerge from the study (Frechtling, 2010).

Overview of the Study

This study investigated the content knowledge and mathematical knowledge for teaching (MKT) of place value concepts of K-2 teachers in one south Texas district. The study included interviews of 4 teachers who received a range of scores on the Elementary Place Value Content Knowledge (LMT, 2006) assessment tool. This assessment tool was used specifically and uniquely to identify the teachers to be selected for interviews. However, teacher scores on the assessment tool were discussed in the study as representing a range of scores and those ranges were compared with instructional strategies teachers identified in the interview questions. Teachers were interviewed first, on their understanding of place value and their understanding of student thinking about place value which corresponds with research question 1. They were also interviewed on how their mathematical knowledge for teaching (MKT) place value informs the instructional strategies they identified to support student understanding, specifically with tier 2 students who struggle with these concepts which corresponds with research question 2. Teachers were asked to explain student thinking and to suggest or recommend instructional strategies for tier 2 intervention that would support student understanding of these place value concepts. The work samples in interview questions 3-6 included student work from Battista's (2012) book on place value and align to the levels and sub-levels of student reasoning of place value described and explained earlier. Additionally, teachers were asked to provide artifacts of their work with tier 2 students on place value concepts. These artifacts included de-identified student work samples and teacher instructional tools that exhibit their work with tier 2 math students on place value concepts. Collecting artifacts that demonstrate the instructional strategies teachers use with tier 2 mathematics students allowed for triangulation of the interview data with the artifacts and with the range of scores on the assessment.

The main goal of this study was to discover what K-2 elementary mathematics teachers know about place value content, what they know about mathematical knowledge for teaching (MKT) of place value, and consider the nature of the relationship (if any) between the two. Comparisons were made to the levels in Battista's (2012) Cognition-Based framework of student reasoning of place value concepts and to the "egg" model (Ball et al., 2008: Bair & Rich, 2011) component parts of teacher Specialized Content Knowledge (SCK) and teacher Knowledge of Content and Student (KCS). The four participants were selected based on their score on an online assessment of mathematical knowledge for teaching. One participant was selected because she achieved a perfect score on the assessment and another participant was selected because she achieved the lowest score on the assessment. All other participants' scores hovered around the mean so two of these were selected for interviews. Initially, in the IRB protocol, up to six participants were suggested for the study but since eight scores were clustered around the mean score, it was determined that two of these participants would be a representative sample of this group.

Setting and Participants

The population for the study consisted of four K-2 mathematics teachers who teach tier 2 mathematics intervention within a school district in south Texas. All teachers selected for this study worked in an elementary school where an established mathematics intervention (RtI) program was in place. All of the teachers selected for this study were female, white, and have teaching experience ranging from 10 years to 31 years. All of the teachers' classroom experience has been in grades K, 1, or 2 their entire teaching careers. One of the teacher's experience in education included serving as a math educator for five years and as a curriculum director for five years for a math museum. Three of the teachers hold a master's degree and one holds a bachelor's degree. None of the teachers claimed to have had recent (within the past year) professional development on place value concepts. A more detailed description of each participant is included in chapter 4.

Procedures

Upon IRB approval, the researcher initiated contact with the superintendent and the math/science curriculum coordinator for a local South Texas school district. Following approval from these district officials, the researcher contacted the elementary principals of the two schools for the study. Dates were scheduled to meet with K-2 teachers in after school meetings to discuss the research study and explain voluntary participant roles. A separate meeting was scheduled at each school to accommodate teachers and to allow for a more personalized explanation of the study. Following these meetings, all K-2 elementary teachers received an email with a link to the online assessment of place value pedagogical content knowledge. Teachers were given a ten-day window of time to complete the assessment.

Upon completion of the assessment, the researcher scored the assessments and selected 4 teachers (Teachers A, B, C, and D) to participate in the face-to-face interviews. These teachers achieved scores that exhibit a range of scores from higher to lower on the assessment instrument. Each teacher was contacted via email to request participation in the interviews. Teachers were interviewed on their home campus in either their classroom or in a room provided by the school (teacher choice) or another location selected by the teacher. Rapport was established with each teacher prior to beginning the interview. The interviews were recorded via an electronic recording device and handwritten transcription. The recorded interviews were transcribed for analysis. Teachers were asked to provide artifacts that exhibit or relate to their teaching of place value concepts. The data from the interviews was triangulated with the artifacts and teachers' score ranges on the online assessment.

Instrumentation

Tools utilized in this study included the online version of the Learning Mathematics for Teaching (2006) Elementary Place Value Content Knowledge assessment from The University of Michigan (See Appendix A). This tool assesses teachers' mathematical knowledge for teaching of place value. The assessment, in the original form, included 18 question items with multiple parts which addressed place value concepts taught at grade K through grade 5. Since this study focused on K-2 place value content knowledge, it was necessary to reduce the assessment to include only those items taught at these grade levels. This resulted in an assessment comprised of six questions, each with multiple parts. The modified version of the assessment was presented to a group of three professors of mathematics who are experts in mathematics education for review and approval. This assessment was used to identify teachers for the interview part of this study whose scores represented a range of pedagogical content knowledge of mathematics (higher or lower).

An interview protocol was developed by this researcher to provide insight into teachers' knowledge and understanding of place value concepts and the instructional approaches these teachers would recommend for tier 2 students who struggle with these concepts. The semi-structured interviews included questions about teachers' understanding for how students learn place value concepts. Also included in the semi-structured interviews were student work samples of mathematics problems on place value concepts that included some level of misunderstanding or breakdown in student thinking, representative of students in tier 2 intervention. Teachers were asked to explain students' thinking and suggest instructional approaches to address individual student responses.

Data Collection

Data collection strategies included an online assessment of teacher MKT of place value, face-to-face semi-structured interviews and collection of teacher artifacts. Teachers were selected for interviews to include a range of scores on the online assessment so that the level of pedagogical content knowledge could be considered. Teacher scores on the online assessment were not disclosed or specified in the study but rather were discussed in general terms ("higher" or "lower" scores) and to indicate extreme scores, where they existed. Rapport was established with teachers prior to beginning the interview. Interviews were captured via handwritten notes and audio recordings that were transcribed. Each interview lasted approximately one hour. Audio recording was used to maintain accuracy of the data collected. Three of the interviews were conducted in teacher classrooms at the school and one interview was conducted in the cafeteria, on the stage behind the curtains, due to the teacher's classroom being used for district benchmark testing.

Teachers were asked to provide artifacts that exhibit their work with tier 2 students on place value concepts. This allowed for data triangulation and should decrease concern for construct validity of the study by considering multiple measures of the same concept or phenomenon. Construct validity is established when the following conditions are met: the construct being studied is defined in terms of specific concepts, and the operational measures that match the concept are identified (Yin, 2009). For this study, the construct being studied is K-2 elementary school teachers' mathematics pedagogical content knowledge of place value and the operational measures that match the concept are research studies identified in the literature review that have similar findings.

Interview Protocol

The interview questions were carefully crafted to consider both the interviewee and the question content following Michael Patton's (2002) advice on the development of good interview questions. Patton (2002) recommends beginning with a problem statement, moving to the answer, then applying the procedures in the methodology which may in turn reshape or refine the question. The problem identified by this researcher is that K-2 students continue to struggle with understanding place value concepts even when provided tier 2 intervention instruction. The proposed solution by this researcher supposes that increased teacher MKT of place value will result in changes in tier 2 intervention instructional strategies consistent with increased teacher pedagogical content knowledge as discussed in the research provided in this study. This study did not consider student performance but focused only on identifying the teacher MKT of place value that one group of K-2 math teachers possess for tier 2 instruction. Interview protocol questions 1 and 2 were designed to answer research question 1 and interview protocol questions 3 through 6 were designed to answer research question 2.

Research Question 1

How do K-2 mathematics teachers describe their understanding of how students learn place value?

The following interview questions were asked to answer research question 1:

1. How would you describe your understanding of how students begin to learn place value?

a. How would you describe the progressions students make in their learning of place value?

b. How would you describe some of the challenges and struggles students may encounter in learning place value?

2. What are some of the instructional strategies or tools you use to facilitate students' learning of place value concepts with students in tier 2 intervention instruction? (Prompt with the following)

    a. What strategies or tools do you use to help students learn to count?

    b. What strategies or tools do you use to help students learn to add?

Research Question 2

How does teacher mathematical knowledge for teaching (MKT) place value impact their instructional decisions with struggling math students in tier 2 intervention?

The following interview questions were asked to answer research question 2:

The student work below represents a range in students' application of place value understanding to common algorithms. If this work was from students in your tier 2 intervention group, how would you…

3. Explain your understanding of the students' thinking below. What evidence would indicate to you that a student understands place value concepts?

a. $$\begin{array}{r} 12 \\ \underline{+\ 15} \\ 27 \end{array}$$

b. $$\begin{array}{r} 10 \\ \underline{+\ 23} \\ 3 \\ \underline{30} \\ 33 \end{array}$$

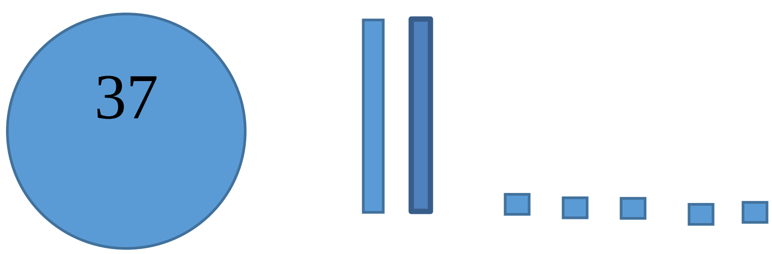
c. $$\begin{array}{r} 17 \\ \underline{-\ \ 9} \\ 7 \\ \underline{1} \\ 8 \end{array}$$

4. Considering your understanding of place value concepts, what instructional strategies would you use to determine whether or not a tier 2 intervention student was relying on rote memory or if they understood the use of "tens" language?

    46 = _____ tens _____ ones.

Students in tier 2 mathematics intervention who struggle with understanding place value concepts exhibit some of the following challenges in applying place value concepts. Given student responses to the following problems, what instructional strategies/tools would you use to support K-2, tier 2 student's understanding of place value?

5. Jon has 30 checkers. How many stacks of ten checkers can he make?
    a. The student is unable to correctly count the 30 checkers.
    b. The student correctly counts the 30 checkers but cannot determine that there are 3 stacks of ten.
6. There are 37 squares under the circle. There are also 2 ten-strips of squares and 5 single squares. How many squares are there altogether?



a. The student ignores the squares under the circle and counts the 2 ten-strips and single squares by "ones" to get 25 total squares.

b. The student counts "37, 47, 57, 58, 59, 60, 61, 62" and states "there are 62 total squares".

Data Analysis

The transcripts of all teachers were coded in the first cycle coding utilizing Dedoose Version 7.0.23 (2016), an online data analysis software. This first cycle coding utilized attribute coding, which Saldaña (2016) asserts "provides essential participant information and contexts for analysis and interpretation" (p. 83). He indicates that coded data can be categorized to develop themes/concepts which in turn may become assertions or theories. The first cycle codes were applied to all 4 interview transcripts and organized by interview questions. The codes that emerged from this first cycle coding included teacher professional development, place value learning, progressions in learning place value, manipulatives and strategies utilized, challenges in learning place value concepts, learning to count, learning to add or subtract, questions I would ask, and understanding of student thinking. The data were coded to correspond to each interview question and were organized by research question.

The second-cycle coding was utilized as a more advanced approach to analyzing and organizing the data and develop a sense of thematic or categorical organization of the first-cycle codes. Pattern coding was utilized during this cycle which is a strategy for grouping the data into smaller themes or concepts. Saldaña (2016) indicates pattern coding is appropriate when a researcher is laying the groundwork for cross-case analysis, looking for common themes. This researcher analyzed the data for each participant then provided a cross-case analysis by comparing participants B, C and D to participant A. Participant A received the highest score on the place value assessment of teacher PCK of place value concepts. As noted earlier, (Yin, 2009), the logic underlying a multiple case study should be to either replicate the initial case study or contrast the initial case study. This researcher elected to contrast the initial case study to test the working hypothesis that teachers with less knowledge of MKT of place value would respond with a more limited understanding of student thinking and suggest or recommend limited instructional strategies than teachers with a higher score. Participants were assigned a pseudonym for purposes of discussion and to preserve the anonymity of the participants. Creswell (2013) notes that it is important to mask the identity of participants because the research results might "unwittingly present a harmful picture of the participants or the site" (p. 60). The first letter of each pseudonym corresponded to the letter of the alphabet presented above. Participant A was referred to as Anna, B was Beth, C was Carol, and D was Diane. Pseudonyms were used to maintain confidentiality.

Initially, the analysis of data for all interview questions was to be compared with both the Levels of Sophistication in Student Reasoning: Place Value (Battista, 2012) and with the Mathematical Knowledge for Teaching (MKT) model proposed by Ball, et al. (2008) and expounded upon by Bair and Rich (2011). After the first-cycle coding of data, this researcher determined that the first two interview questions about teacher knowledge of place value would be more appropriately compared to Battista's (2012) levels. The remaining four questions where participants evaluated student work samples on place value concepts were more appropriately compared to the Mathematical Knowledge for Teaching (MKT) model by Ball, et al. (2008) and Bair and Rich (2011).

Trustworthiness and Rigor

The researcher utilized member checks throughout the interview process to allow teachers to confirm or correct the researcher's understanding of their responses to the interview questions. Additionally, the questions asked during the interview were open-ended to allow teachers to explain their understanding of the student work samples and to identify the instructional approach they would employ. Informal questions were asked throughout the interviews to clarify and relate responses to the research questions of this study. In addition to member checking during the interviews, this researcher contacted participants via email to clarify or provide detail to the transcribed interviews. Coding included triangulation of the interview responses with artifacts and with individual teacher scores on the online assessment of place value concepts. The data was coded with first cycle and second cycle coding methods which were peer reviewed with colleagues in math and science education.

Potential Researcher Bias

The researcher was previously employed as the mathematics specialist for RtI in a large school district and was later promoted to director of the RtI department. Having worked closely with classroom teachers and having provided professional development to these teachers on mathematics RtI, I acknowledge a personal bias. However, I did not previously consider teacher content knowledge of place value concepts and my work was limited to training teachers in workshop settings and modelling intervention instruction in their classrooms. I did not observe teachers implementing intervention lessons in my role as RtI mathematics specialist or as director, and did not have the opportunity to evaluate their mathematical knowledge in any manner. To manage my personal bias, I did not make use of leading questions during the interviews and during analysis of data, I focused on the specific quotes provided in the interviews. Additionally, codes established for coding of first- and second-cycle coding was peer reviewed by other math and science educators.

Summary

This chapter provided the details for how the study was conducted, how data were collected and how data were analyzed. First- and Second-Cycle coding was utilized to establish coherence allowing the identified themes to be woven together during the analyses. Specific interview questions were aligned to specific research questions to permit the researcher, when writing up the analyses, to present to findings of the study in a clear and lucid manner. The data were triangulated to establish a convincing narrative that supports the researcher's working hypothesis.

CHAPTER IV: ANALYSIS OF DATA & RESULTS

Introduction

This chapter provides a description of the analyses of the data. This section begins with a deeper look at the setting and sample then continues with a description of the teachers' classrooms followed by the presentation of data and a cross-case analysis. Each participant's responses were considered with respect to the research questions, Battista's (2016) framework for student reasoning about place value, and Ball et al.'s (2008) model for teacher mathematical knowledge for teaching (MKT). The analyses of data are presented and discussed regarding teachers' MKT of place value concepts and how that knowledge may inform the instructional decisions they make with K-2 students in a mathematics intervention setting.

Setting

The school district selected for this study has two elementary campuses for grades PK-3 and an additional campus for upper elementary grades. The district has had an established mathematics intervention program in place for the past five years and reported that individual teachers provided mathematics interventions independently for longer periods beyond the five years. The school district is on the outskirts of a larger city and boasts an average 87% passing rate on the grades 3-8 mathematics state exam. Each of the PK-3 campuses have student enrollments close to 600. The race demographic data for elementary 1 is 65.7% Hispanic and 30.1% White while the race demographic data for elementary 2 is 50.5% Hispanic and 45.5% White. The total enrollment for the district is approximately 4,000 students.

The interviews took place on 4 successive days at the corresponding elementary campuses during the week of February 20-24, 2017. Each interview was approximately one hour in length. Follow up member checking occurred through emails with each participant to allow teachers to respond when it was convenient for them. Three of the interviews were held in the teachers' classrooms while one was in the cafeteria, on the stage behind the curtains. The teachers were given a choice to have the interviews in their rooms or in a conference room provided by the principal. The one interview which was conducted on the stage occurred on a day when that teacher's classroom was being utilized for district benchmark testing. This teacher selected the stage for her interview although other alternatives were considered.

Sample

The sample was a purposeful sample and included 4 grade K-2 teachers who teach mathematics intervention to their students. Purposeful sampling allows the researcher to discover an understanding gained through the specific insight of the selected participants (Merriam & Tisdell, 2015). Purposeful sampling means that the researcher selects the participants and the research site because they can "purposefully inform an understanding of the research problem" (Saldaña, 2016, p. 156). The participants each have a personal story to tell about their experiences with mathematics intervention students and their understanding of how students learn place value concepts. These teachers achieved a range of scores from high to low on the place value assessment they took prior to being selected for the interview phase of the study. One teacher was selected for achieving the highest score, another was selected for achieving the lowest score, and the other two were selected because their scores reflected average or mean scores.

Anna

I arrived at the elementary campus a few minutes prior to the final bell and release of students for the school day. Only a few minutes after the bell, I was greeted by Anna, a first-grade teacher, who led me through the maze of the campus to her classroom. Anna has been an educator for 18 years which includes 8 years as a classroom teacher, 5 years as a math educator in a children's museum and 5 years as the director of curriculum and instruction for that museum. She has taught first, second and third grades during her 8 years in the classroom. Upon arrival in Anna's classroom she welcomed me to her campus took a seat behind a kidney-shaped table and offered me a seat across the table from her. We spent a few minutes discussing her day and the school year up to now. Student work samples were displayed on multiple bulletin boards and Anna invited me to look at some of these work samples at different times during the interview. As I looked around the classroom, I asked Anna to explain to me the set-up of her room. She showed me the bins where she kept her math manipulatives (Figure 2) and the SMART Board she uses for math instruction. Figure 3 below shows a student demonstrating Anna's SMART Board.

Figure 2. Anna's math manipulatives bins. Carl Juenke ©.

Anna's classroom was arranged so that students could sit on the floor in front of the SMART Board and interact with her and each other easily. She indicated that she uses this technology in math instruction frequently and felt lucky to have it in her room.

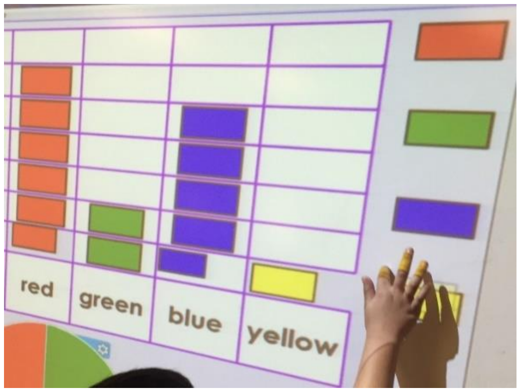


Figure 3. Students demonstrating Anna's SMART board. Carl Juenke ©.

Anna was curious about the research study and asked multiple questions to clarify her understanding of the interview and her role. I reviewed the consent form with her and answered her questions and she signed the form (a copy of the form had been provided to her at the initial teacher meeting when I explained my research study). I assured her that her responses would remain confidential and only I would have access to those responses but that I might include some quotes to provide clarity and detail to the description of the study results. Anna was curious about her results on the online place value assessment that qualified her for the interview part of my study. I explained that I was hoping to have a range of scores on the assessment so I might consider whether teachers who possess a greater pedagogical content knowledge of place value concepts make different, and potentially better and more informed instructional decisions with K-2 students in intervention lessons. I did not disclose to Anna her actual score, however, she achieved a perfect score on the assessment and was the only teacher who did so. After establishing rapport, we began the interview.

Anna indicated that she had not received any professional development on place value concepts in the past five years. She said that she previously was a math educator for five years for a children's museum in a large metropolitan city and received numerous training opportunities while in that position. That training included a week-long math workshop by a nationally recognized math educator, Pre-K math training by a “renowned early-level mathematics teacher”, and project partnership trainings with universities and school districts. Anna had the opportunity while serving as a math educator to present her work at state and national math conferences. She indicated that after five years in the role of math educator, she was promoted to the director of curriculum and instruction and was able to “oversee all of our (the children’s museum) outreach programs and curriculum production including literacy, math, science and STEM (science, technology, engineering and mathematics) curriculum.” She stated that serving in this role allowed her to see the “big picture” of how the curriculum fit together between and among the different content areas.

Beth

I arrived at the elementary school where Beth teaches second grade a few minutes before the final bell so I waited in the school office until students were dismissed for the day. Beth has taught second grade for all her 10 years in education except for half of one year when she taught kindergarten. Beth came to the office and greeted me with a firm handshake and we discussed whether she would like to be interviewed in the conference room or her classroom. She said that she preferred her classroom so she led me through the hallway to her classroom. Beth’s classroom was filled with tools to help students learn math concepts including charts for math centers and a hundreds pocket chart for practicing counting (Figure 4). Desks were arranged in a horseshoe shape to bring all students closer to the teacher and a computer center was located across one side of the classroom (Figure 5).

Figure 4. A hundreds pocket chart in Beth's classroom. Carl Juenke ©.

Figure 5. Configuration of desks in Beth's classroom. Carl Juenke ©.

I asked Beth about her day and she stated that it was just another busy school day. We briefly discussed her students and after establishing rapport I explained the informed consent form (all teachers were provided a personal copy of this at the initial teacher meeting). I thanked her for participating in my research and she said she was glad to participate and hoped that what she had to say would be helpful. I assured her that any response she provided would be helpful as I wanted to know what teachers understand about place value concepts. I did not disclose Beth's score on the place value assessment she took to qualify her for the study, however, Beth achieved a score equal to the mean score for all teachers who took the assessment. I explained that her answers would be confidential and that I would be the only person with access to her comments. I told her that I might possibly use a few quotes in my analyses of the data to explain or clarify my analyses but that she would not be identified personally.

Beth discussed her recent professional development and indicated she attended GT (gifted and talented) training every year and she recalled attending an academy training for reading and then one for math that was offered by the school district. She said she attended all math training the district provided but did not recall any training specifically on place value concepts.

Carol

When I arrived at Carol's school where she teaches kindergarten, I had to wait in a line of cars filled with parents who were all waiting to pick up their children. I was finally able to make it into the parking lot of the school and find an empty parking space. The bell had just rung for students to be dismissed so I made my way to the school office, maneuvering between lines of students who were heading to the front door exit. I reached the office and told the staff that I had an appointment with Carol and an office worker led me through the hallways which were still teeming with students who were anxiously scurrying to the exits. Carol has taught elementary grades for 31 years: first grade for 4 years, second grade for 5 years and kindergarten for 22 years. We arrived in Carol's room and I waited for her to return from escorting her students, some to their school buses and others to their waiting parents. I took the opportunity to look at the bulletin boards and math strategies and tools that were posted around the room. In the back of the room were cubbies that contained plastic bins full of counters, base-ten blocks, snap cubes, and an assortment of other math manipulatives (Figure 6). Across the room was a chart for counting and on the white board was a tool she uses for explaining and practicing place value skills (Figure 7).

Figure 6. Bin storage for Carol's math manipulatives. Carl Juenke ©.

Figure 7. Carol's whiteboard with base-ten examples. Carl Juenke ©.

Carol entered her room and greeted me with a big smile and I asked her where she would like to be interviewed. She stated that she preferred her classroom if that was okay with me. I asked her about her day and how the week was going. She seemed a little winded from escorting her students out of the building so I wanted to give her time to catch her breath and relax. I asked her where she wanted us to sit and she directed me to a small student chair at a table near her desk. Carol sat in her chair beside her desk and we began to discuss the research study. I thanked her for agreeing to participate in the interview part of my research. She had multiple questions about how she performed on the test and kept stating "I just don't know if I am going to be able to help you much in your research." I assured her that she achieved a score that was important to my research and that after looking at her classroom walls and available math manipulatives I was sure that she would benefit my research. I did not disclose Carol's score on the online assessment of place value concepts that all teachers took to qualify for the interview part of my study, however, she received a score just below the mean score of all teachers. I explained the informed consent form which she had received at the initial teacher meeting. I told her that her answers would be confidential with only me having access to her responses. I also let her know that if it was appropriate and necessary I might use some quotes to substantiate or clarify my analyses of the data but that she would not be identified personally.

I asked Carol about her recent professional development and she indicated that she had attended a math academy for PK-2 teachers for a week several years ago and attended GT (gifted and talented) teacher training each year. She stated that she did not recall receiving any professional development specifically on place value in the past five years other than what was discussed in the math academy.

Diane

I arrived at the elementary school where Diane teaches second grade about 15 minutes before the final bell. I wanted to arrive earlier than the previous day because I wanted to make sure I avoided the normal but hectic traffic flow of parents picking up their children. Diane has taught elementary grades for 13 years: third and fourth grades for one year each, second grade for 6 years and first grade for 5 years. Only a few minutes after I entered the school office, Diane walked into the office and greeted me. I thanked her for agreeing to participate in my research and asked where she would like to be interviewed. She indicated that her classroom was being used for benchmark testing so we decided to go to the conference room. When we arrived at the conference room, we discovered that a meeting was being held in that room so Diane suggested another room that might be vacant. We tried that room but found other teachers using the room. Diane suggested we use that stage in the cafeteria. I asked if there might be another room we might use that would be more private but she said the stage would be fine because we could go behind the closed curtains and we would have privacy. When we arrived in the cafeteria, it was empty and quiet so we made our way up a few stairs and went behind the heavy velvet-lined stage curtains and discovered a small table with several chairs where we sat for the interview.

I asked Diane about her day and her week and she said that it had been very busy. She asked about the place value assessment she took which qualified her for an interview. She wanted to know if the interview was going to be a math test, like the place value assessment. She stated that the assessment was very difficult and she was curious about her score. I did not disclose Diane’s score from the online place value assessment, however, she achieved the lowest score on the assessment. I assured her that the interview was not a test like the place value assessment and that it was about what she knows about place value concepts and the instructional decisions she makes when looking at student work samples. She seemed more comfortable knowing this and she said “Okay, let’s go!” I reminded her that this was a research study and reviewed the informed consent form with her (all teachers received a personal copy of this at the teacher meetings I conducted previously). She signed the form and we began the interview. I assured Diane that her responses would be confidential and I would be the only person with access to her responses. I told her that if it was necessary to support or explain my analyses of data that I might use some of her quotes but that I would not identify her personally. Unfortunately, I was unable to see Diane’s classroom personally, however, she provided pictures via email of a mathematics station and mathematics instructional tools she uses with her students (Figure 8 and Figure 9).

Figure 8. Mathematics station in Diane’s room. Carl Juenke ©.

Figure 9. Example of a math tool in Diane's classroom. Carl Juenke ©.

I asked Diane about her recent professional development and she said she had attended a K-2 math academy she thought was presented by the Texas Education Agency TEA) several years ago. She stated that she couldn't recall attending any professional development recently or even within the past 5 years on place value concepts.

Analysis of Data

During the initial coding phase of the data, numerous themes emerged that were too lengthy and cumbersome to include. There was overlap among the first-cycle coding themes so it was necessary to not only clarify but to also narrow these themes. This resulted in the data being coded with a second-phase coding to first, combine and refine the themes so that they reflected clear comparisons of the data to the student levels of reasoning about place value (Battista, 2012). This process involved analyzing teacher responses to the different levels of reasoning about place value concepts (Table 1). As the data were analyzed, it was determined that it was more appropriate to compare only interview questions one and two to the levels of reasoning about place value (Battista, 2012). Where appropriate, direct quotes were provided from transcripts of teacher interviews to support and substantiate the analysis. Also, when appropriate, pictures of teacher identified instructional tools or manipulatives were included to provide the reader with a visual of the tools described. The pictures of the artifacts teachers provided were included as evidence of teacher claims about their instruction.

Table 1. Place Value Levels

Place-Value Levels: Zoomed Out to Major Levels	
Level 0	Student has difficulties counting by ones.
Level 1	Student operates on numbers as collections of ones (no skip-counting by place value).
Level 2	Student operates on numbers by skip-counting by place value (e.g., counts by tens).
Level 3	Student operates on numbers by combining and separating place value parts (e.g., adds tens parts without counting).
Level 4	Student understands place value in expanded algorithms.
Level 5	Student understands place value in traditional algorithms.
Level 6	Student generalizes place value understanding to larger numbers, numbers less than 1, and exponential notation.

Secondly, the data for questions three through six were compared to the “egg” model (Figure 1) of mathematical knowledge for teaching (Ball, et al., 2008; Bair & Rich, 2011). These specific questions focused on teacher pedagogical knowledge of mathematics while the first two questions focused on teacher content knowledge of place value. Interviews, photographic artifacts, teachers' range of score on the place value assessment and researcher observations were utilized to explore teacher understanding of place value concepts. Additionally, these data were analyzed to determine the tools and instructional strategies teachers use when working with students who struggle in mathematics. Each case study participant's responses were compared to the participant's responses who achieved the highest score on the place value assessment. The themes that emerged from the second-cycle coding included: how students learn place value, challenges struggling students encounter, instructional tools used by participants, strategies to understand student thinking, and instructional strategies.

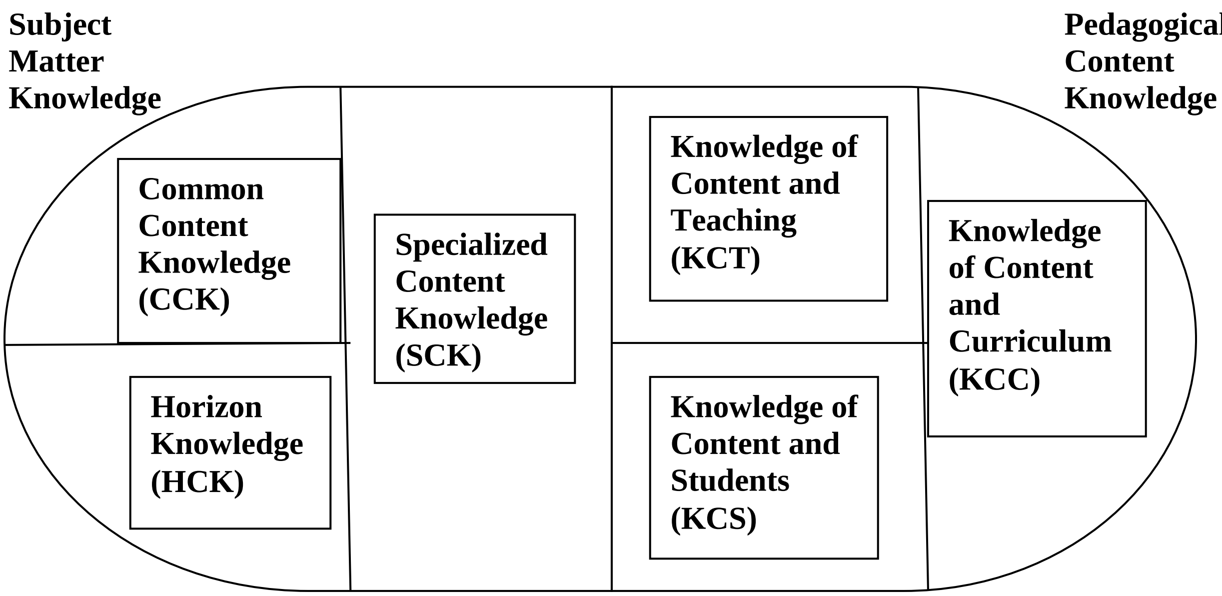


Figure 1. Mathematical Knowledge for Teaching (MKT) "egg" model.

The first two interview questions were designed to elicit responses to answer research question 1 and the remaining four questions were designed to elicit responses to answer research question 2. Each research question appears before the analysis of data for each group of interview questions.

Research Question 1

How do K-2 mathematics teachers describe their understanding of how students learn place value?

The following interview questions were asked to answer research question 1:

1. How would you describe your understanding of how students begin to learn place value?
    a. How would you describe the progressions students make in their learning of place value?
    b. How would you describe some of the challenges and struggles students may encounter in learning place value?
2. What are some of the instructional strategies or tools you use to facilitate students' learning of place value concepts with students in tier 2 intervention instruction? (Prompt with the following)
    a. What strategies or tools do you use to help students learn to count?
    b. What strategies or tools do you use to help students learn to add?

How Students Learn Place Value

Anna

Anna described her understanding of how students learn place value concepts as initially learning through number songs or chants such as choral recitation of numbers (e.g. 1-10). This understanding or reasoning about numbers is consistent with Level 0 on Battista's (2012) levels of reasoning about place value. Students at this level have difficulty counting by ones. Anna described this learning as being on a continuum or through a series of progressions, beginning with these rote counting methods and moving to counting objects such as counters or cereal pieces. This is a strategy for helping a student progress through Level 1 and move from correctly counting groups of objects by ones to being able to compose numbers as groups of tens and ones.

Anna indicated that students explore the written numeral later and begin with small numbers. Anna's explanation of how student learn place value was inexplicably connected to the use of tools and strategies to foster this understanding. She listed specific tools such as "hundreds charts", "base-ten blocks" and "number lines" to help students learn about numbers as she described how students learn place value concepts. These tools can also be used to reinforce students' reasoning about place value concepts in Level 1. Anna indicated that she attempts to make connections between and among these tools for students to reinforce the meaning of a number. She offered the example "here is a 45, what does the four mean? What does the 5 mean?" and used the hundreds chart to discuss how to find a number that is "ten more than or ten less than" to reinforce the patterns in numbers. Students who can understand numbers in this way are beginning to operate at Level 2 because they "understand the connection between counting by ones…and counting by tens" (Battista, 2012, p. 20).

Anna stated that when students begin to encounter larger numbers they look for patterns in the digits of the numbers, discovering that the numbers 0 through 9 appear repeatedly, but appear in different places in a number. She indicated that students recognize when these numerals appear in different places in a number, they have a different value depending on the place the number represents (ones place or tens place or hundreds place). Anna described how students' understanding of number progresses, recognizing how the numbers look different or similar with numerals in different digits and how that translates into students' understanding what place value the number represents. Students who have developed a strong understanding of place value and can compose and decompose numbers by their place value parts display place value understanding consistent with Level 3. "Students at this level decompose numbers into their place-value parts – ones, tens, hundreds, and so on. These place value parts are combined or separated directly, without counting" (Battista, 2012, p. 24).

Beth

Beth, who achieved the mean score on the online place value assessment, described her understanding of how students learn place value by providing examples for how she teaches these concepts. She indicated she starts with base-ten blocks and explains the value of each (which is the ones, tens, hundreds) to students. She reported that when students know and understand the value of each base-ten piece she shows them how to make equivalent values with each (10 ones = 1 tens, 10 tens = 1 hundreds). Beth translates this understanding to written problems and has students use a hundreds-tens-ones (HTO) chart to record their number in the correct place value location. She commented that when students have learned the standard form to write a number she has them write the number in expanded form (453 is 4 hundreds, 5 tens, 2 ones). Students who understand place value concepts and "operate on collections of tens using tens language" (Battista, 2012, p. 30) have achieved level 3 reasoning. When students can add or subtract numbers in expanded form, they are operating with level 4 reasoning of place value concepts.

When teaching students counting strategies, Beth said she tries to keep students from counting with their fingers and uses number lines and dot counting. Beth provided an example for clarification "five plus three…starting with the largest number five and hitting (the air) six, seven, eight' for plus three" which she uses instead of having students count with three fingers. Beth did not articulate her understanding for how students learn place value concepts as being in steps or progressions or levels as Anna described.

Carol

Carol described her understanding about how students learn place value to be “a series of student milestones”. She stated that students who struggle in mathematics do not understand that “the number 10 has two separate parts and the one and zero mean something different.” She pointed out that until students “get the concept of going from 9 to 10 because the 9 only involved the ones place and now the 10 involves the ones and tens places” they do not really understand place value. This understanding is consistent with Level 1 student thinking of numbers being composed only of a collection of ones. Carol stated that students demonstrate an understanding of place value when they recognize 10 is a “bundle” of 10 ones.

Carol explained that when she is teaching students to count they begin with rote counting by mimicking or repeating the numbers she calls out. She said she also uses songs to help students learn and memorize counting numbers. Carol stated that students who struggle with place value concepts such as counting require her to expand her instruction to include counting strategies such as using one-to-one correspondence of touching the paper (1 is one touch, 2 is two touches, etc.). Demonstrating counting skills is necessary for a student to progress to Level 1, with first counting by ones, then tens.

Diane

Diane offered limited responses to all interview questions. She also achieved the lowest score on the online place value assessment of teacher pedagogical knowledge of place value. She stated students understand place value concepts beginning with concrete objects such as base-ten blocks. She said that when students demonstrate an understanding of the difference between ones and tens they can perform two-digit addition and subtraction problems. Students who demonstrate an understanding of the relationship between ones and tens operate on Level 2

Challenges Struggling Students Encounter

Anna

Anna reported that when students who struggle with mathematics begin to count objects they frequently count the same object multiple times. A strategy Anna employs with these students is to arrange the objects in a straight line which alleviates the repetitive counting of the same objects. Students who struggle with counting objects by ones are reasoning at Level 0. Anna commented that when struggling mathematics students interact with numbers beyond single digit numbers they often have trouble keeping the numerals in the correct place value space. She indicated that this is particularly evident when students begin to add or subtract two- or three-digit numbers. Anna noted that struggling students also have difficulty determining how to write a number they read in a story problem or a number that is spoken to them. She commented students are unsure which numeral belongs in which place value location and this results in them having difficulty solving word problems. This challenge is consistent with a student who is reasoning at Level 1 because they attempt to perform addition or subtraction operations by operating on the ones and tens separately. Also common with a student reasoning at Level 1 is solving algorithms accurately but performing the operation rotely, still seeing the numbers as collections of ones and tens. An additional challenge Anna relayed was that struggling students frequently overgeneralize mathematics rules, particularly if a teacher teaches them "a cute rule" which she says "might not be mathematically correct in every situation". Other rules Anna pointed out that are confusing to students who struggle with mathematics included "the bigger number has to go on top when you subtract two numbers."

Beth

Beth reported challenges in second grade place value learning of students not being able to move between different forms of a number. She said students understand the expanded form of a number but have difficulty knowing what that number looks like in standard form or vice versa. She noted that the vocabulary or spoken number causes difficulties for her struggling mathematics students because they don't readily understand "fifty" is the same as "five tens" or "50" and will frequently just write "5". Students who struggle with these concepts operate somewhere in Level 1 since they do not have a firm grasp of the relationship between tens and ones.

Beth also reported that another challenge her struggling mathematics students have is not knowing their addition facts (plus zero, plus one, plus two, plus three). Her goal is for students in second grade to have these facts committed to memory to alleviate the need for students counting with their fingers. Beth did not discuss or demonstrate an understanding that students who struggle with relating "fifty" to "five tens" or "50" have not progressed beyond Level 1 and need instruction at that level to progress to the next level, according to Battista (2012).

Carol

Carol mentioned that many struggling mathematics students have difficulty arranging a group of numbers in chronological order. The next challenge she described was understanding the relationship between ones and tens. Students who struggle with these concepts are operating at Battista's (2009) Level 1 and do not consider a two-digit number as being composed of ones and tens. Additionally, Carol pointed out that students who struggle in mathematics often forget that when they count and reach 10, each successive number up to 19 only changes in the ones place. She stated that these students sometimes have difficulty with recognizing and stating the value of a written numeral.

A challenge Carol noted which was not addressed by other participants was the concept of regrouping in subtraction problems. Carol stated that students frequently do not understand what happens in the ones or tens places to two-digit numbers when you regroup 10 ones. This observation by Carol relating to students' confusion and reasoning with these concepts is consistent with a student operating at Battista's (2012) Level 1 since this level involves students' understanding the relationship between the ones and tens places. Carol, like Anna, was able to articulate various stages or levels students must progress through and master in order to move on to understanding more complex place value concepts.

Diane

Diane did not elaborate on specific challenges students face when learning place value concepts but stated that students have difficulty learning the relationship between the ones and tens places with two-digit numbers. When asked, Diane responded to every question with very short answers even when asked to provide more detail, making it difficult to know whether she possessed more knowledge on her understanding for how students learn place value concepts or whether she was offering all that she knew about the topic. Because she was prompted multiple times on each question, this researcher operated on the assumption that Diane conveyed and exhausted the extent of her knowledge within her brief responses.

Instructional Tools Used by Participants

Anna

A common and frequent tool Anna said she utilizes with students who struggle with math is base-ten blocks. She claims that with this manipulative (Figure 10) students can "visually see what a number and quantity look like" stating that larger numbers seem difficult for these students to understand without a visual representation. She uses "a lot of repetition" to help students see and recognize patterns in numbers and employs the use of a SMART Board to help with counting strategies (Figure 11).

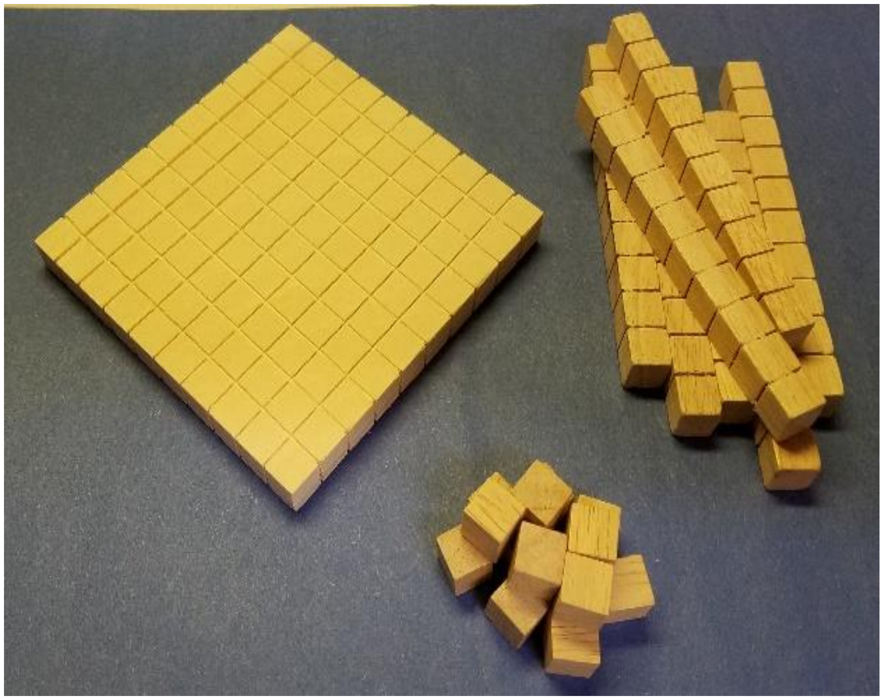


Figure 10. Base-ten blocks instructional tool used by Anna. Carl Juenke ©.

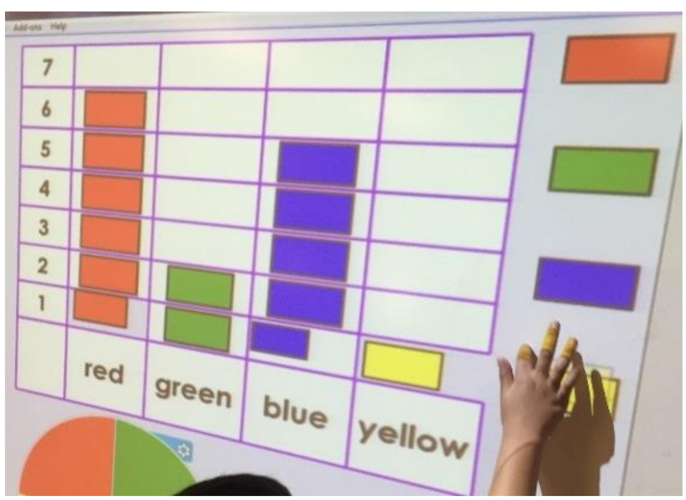


Figure 11. Students demonstrating the SMART board in Anna’s classroom. Carl Juenke ©.

Anna stated that as students begin to see the patterns in numbers with these strategies, then she can use a number line to help them with recognition of the size of numbers, understanding larger and smaller numbers, etc. (Figure 12). Anna reported that the number line also reinforces the physical arrangement of a number, indicating “moving from 10 to 11 often baffles students and causes them to falter when counting.” She commented that the number line allows students to see the order of numbers and helps to “alleviate that mistake” of not knowing how to progress from 10 to 11.

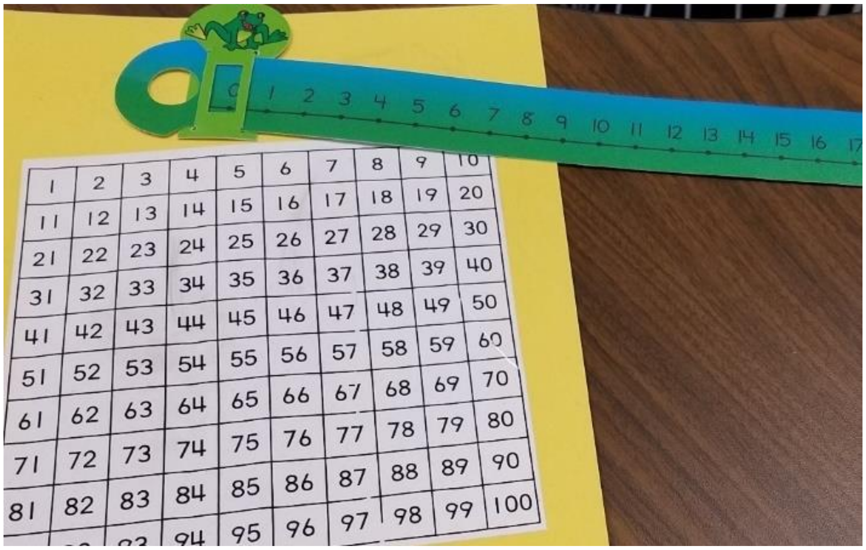


Figure 12. Number line and hundreds chart Anna uses as instructional tools. Carl Juenke ©.

Another tool that Anna stated she uses with her struggling students is a “little picture window” which is simply a piece of paper with a small rectangle cut out (Figure 13).

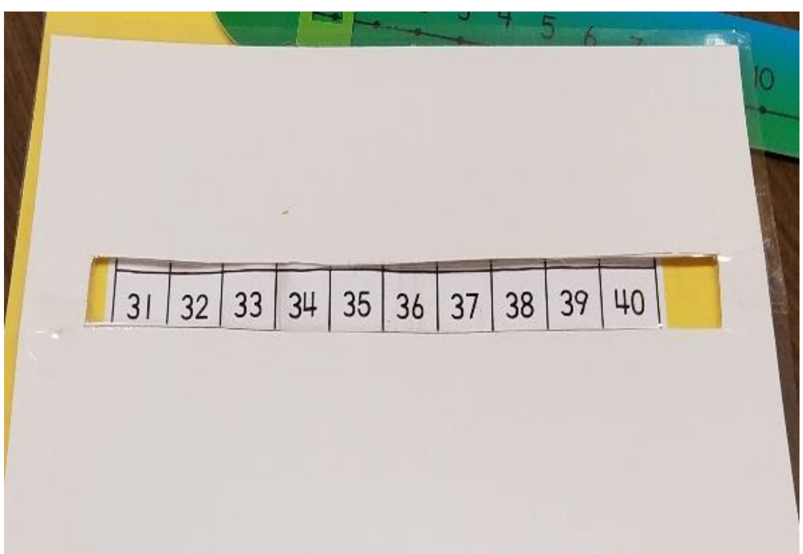


Figure 13. Example of "picture window" Anna uses to isolate numbers. Carl Juenke ©.

Anna indicated this helps student who are easily distracted or who struggle with identifying where to focus when solving a mathematics problem. An additional tool Anna reported using was snap cubes (for numbers less than twenty), to allow students to visually see a number and to compare with other numbers (Figure 14).

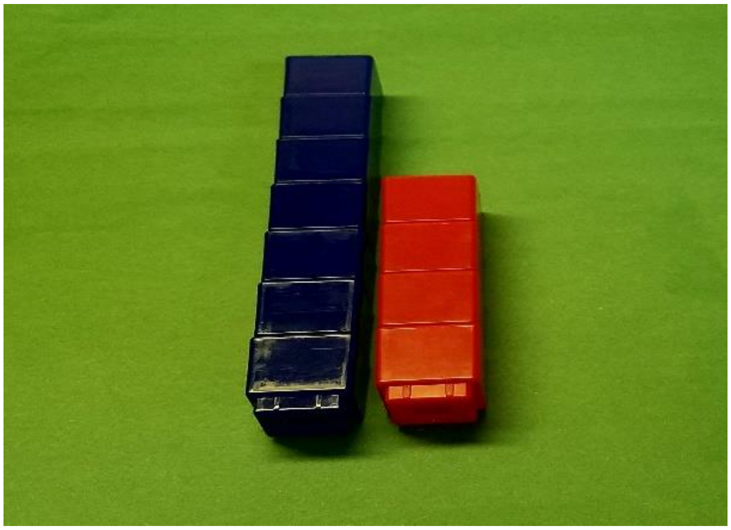


Figure 14. Snap cubes Anna uses to show the size of a number. Carl Juenke ©.

Beth

Beth said she uses base-ten blocks, dot counting, simple counters, number lines, hundreds charts and student journals as tools to teach and reinforce place value concepts (Figures 15 and 16). She emphasized that children in second grade must know their addition math facts of 'plus 0', 'plus 1', 'plus 2', and 'plus 3'. Beth indicated that the math journals provide students with an additional resource for reminding themselves about place value concepts discussed in classroom math lessons. She stated this frees her up to help other students because she can refer a student to their personal math journal to find the answers to many of their questions.

Figure 15. A hundreds pocket chart Beth uses to teach place value concepts. Carl Juenke ©.

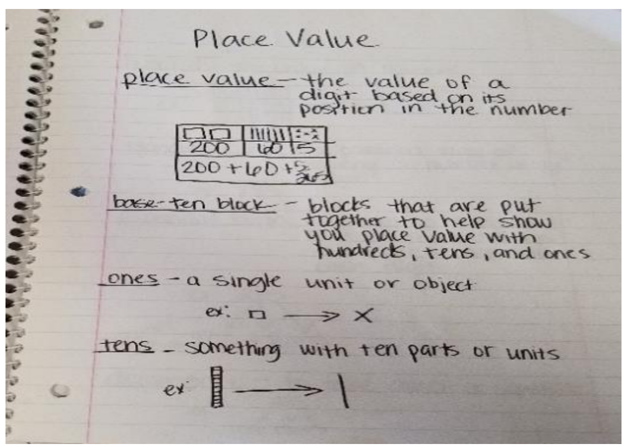


Figure 16. Mathematics journal Beth uses to reinforce concepts. Carl Juenke ©.

Carol

Carol provided great detail about the tools and strategies she uses to teach place value concepts. She described her use of both five-frames and ten-frames to demonstrate the relationship between a single-digit number and ten. She explained how she emphasizes that an empty space on the five-frame or ten-frame means that the number is that many less than either 5 or 10, or that they need that many more to equal 5 or 10, respectively. She modeled how she uses straws as a method for reinforcing place value, telling students that she cannot bundle the straws until she has 10. She described her questioning strategies for helping students recognize a complete or tied bundle of straws is equal to 10 and any loose straws are ones. She places the straws of single straws as ones and bundles of 10 straws as tens and eventually she reported they are able to bundle 10 bundles of tens into hundreds. These straws are placed in the correct pocket within a plastic pocket chart with places labeled "ones", "tens", and "hundreds" (Figure 17).

Figure 17. Hundreds, tens and ones pocket chart for teaching place value. Carl Juenke ©.

Carol explained she ties the bundle of 10 straws and then records the exchange between 10 ones and a ten to her whiteboard representation with the numbers recorded on small yellow circles and the exchange represented with arrows between the tens and ones places (Figure 18). Carol uses the context of a sports team and explains to students they must have 10 players in to make a whole team and in order for a group of straws to be bundled or tied together. She reinforces place value by reminding students that "only tens can play with tens". Carol indicated she uses a lot of repetition, particularly with her struggling students.

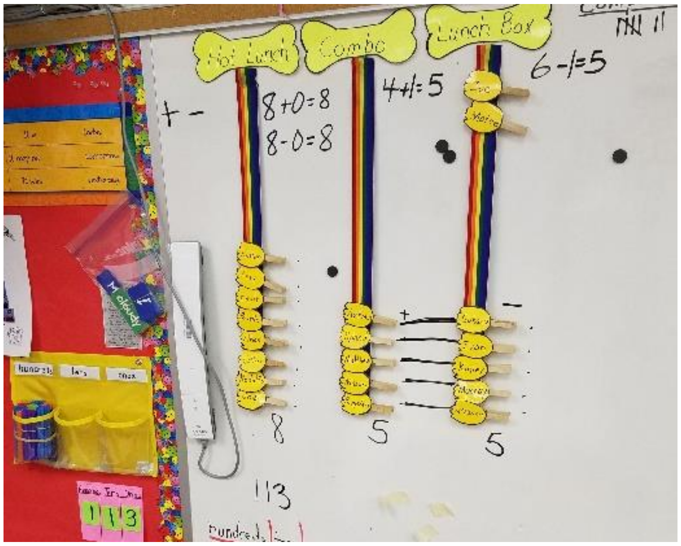


Figure 18. Whiteboard representation of place value concepts. Carl Juenke ©.

Other instructional tools Carol mentioned were counters which could be anything from buttons to bottle caps to commercially made plastic counters. She reported she seizes on every opportunity to reinforce counting and place value concepts which included lunch count (how many students brought their lunch versus how many students will buy a lunch) because these numbers change frequently enough for students to be able to compare smaller numbers. She also indicated that these smaller numbers make for great simple addition and/or subtraction problems. An additional strategy Carol reported was the use of skip-counting or counting by multiples of 10 to help students know the order of tens. She indicated this is particularly important and helpful to her struggling learners who have difficulty knowing the name of the number that follows 19 or 29 or 39, etc.

Diane

Diane reported that the instructional tools she utilizes for teaching place value concepts such as counting include base-ten blocks, unit blocks, number lines and ten-frames (Figure 19).

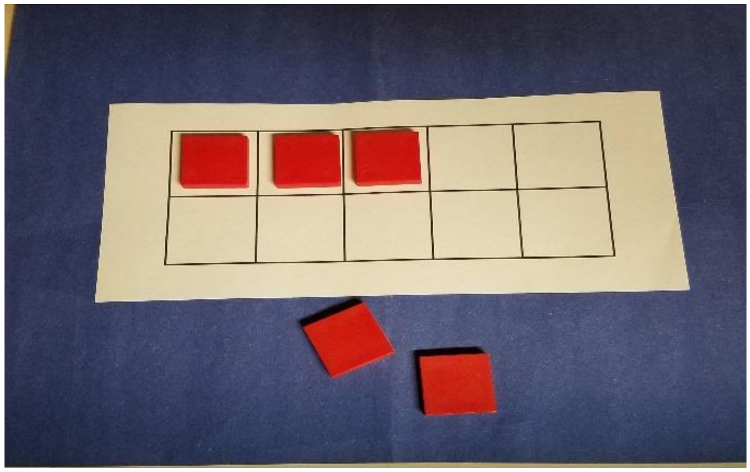


Figure 19. Tens-frame Diane uses for teaching place value concepts. Carl Juenke ©.

She stated she also uses expanded models and greater than and less than models. Diane explained that students are first introduced to place value concepts with concrete models or manipulatives then they progress to a pictorial model or drawing. An additional strategy Diane mentioned was the use of multiple Styrofoam plates to represent and demonstrate part-whole relationships for solving simple addition or subtraction problems.

Research Question 2

> *How does teacher mathematical knowledge for teaching (MKT) place value impact their instructional decisions with struggling math students in tier 2 intervention?*

Teacher responses to questions 3, 4, 5 and 6 during the interviews were about specific student work samples representative of student misconceptions or struggles with place value concepts. The teachers were asked to first evaluate the student work and develop a sense of understanding about the students' thinking. Teachers were then prompted to provide strategies they would employ with a struggling math student about the place value concepts in the math problems. Their responses and the instructional tools and/or strategies they identified were evaluated and analyzed to determine whether they qualified as evidence of applications of teacher specialized content knowledge (SCK) or teacher knowledge of student and content (KSC).

The following interview questions were asked to answer research question 2:

The student work below represents a range in students' application of place value understanding to common algorithms. If this work was from students in your tier 2 intervention group, how would you…

3. Explain your understanding of the students' thinking below. What evidence would indicate to you that a student understands place value concepts?

a.
$$\begin{array}{r} 12 \\ +\ 15 \\ \hline 27 \end{array}$$

b.
$$\begin{array}{r} 10 \\ +\ 23 \\ \hline 3 \\ \underline{30} \\ 33 \end{array}$$

c.
$$\begin{array}{r} 17 \\ -\ \ 9 \\ \hline 7 \\ \underline{1} \\ 8 \end{array}$$

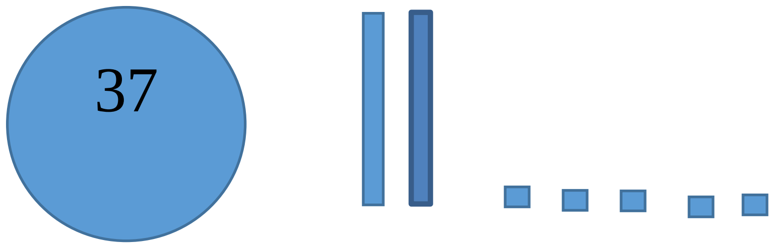
4. Considering your understanding of place value concepts, what instructional strategies would you use to determine whether or not a tier 2 intervention student was relying on rote memory or if they understood the use of "tens" language?

   46 = _____ tens _____ ones.

Students in tier 2 mathematics intervention who struggle with understanding place value concepts exhibit some of the following challenges in applying place value concepts. Given student responses to the following problems, what instructional strategies/tools would you use to support K-2, tier 2 student's understanding of place value?

5. Jon has 30 checkers. How many stacks of ten checkers can he make?
   a. The student is unable to correctly count the 30 checkers.
   b. The student correctly counts the 30 checkers but cannot determine that there are 3 stacks of ten.

6. There are 37 squares under the circle. There are also 2 ten-strips of squares and 5 single squares. How many squares are there altogether?



   c. The student ignores the squares under the circle and counts the 2 ten-strips and single squares by "ones" to get 25 total squares.

   d. The student counts "37, 47, 57, 58, 59, 60, 61, 62" and states "there are 62 total squares".

Strategies to Understand Student Thinking

Anna

Anna discussed possible questions she might ask of students such as "Tell me what you did first?" And stated she would listen to student's thinking to uncover student misconceptions, acknowledging students who struggle in mathematics may be at many different levels in their understanding of place value concepts. In her reasoning about student work sample 3 (a), Anna stated she would want to know the students' thinking about how they added the 2 and 5 commenting "sometimes they might say the 2 and count on 5 more or they might count 2 and count 5 or they might say the 5 and count on 2." She said that most teachers, including her, teach addition of two-digit numbers procedurally by beginning with the ones place and then moving to the tens place but she would accept a child adding the tens place first, then the ones as she noted might have been employed by the student in interview question 3 (a). Anna indicated she would question students if they used a different strategy with questions such as "Does this always work for all numbers?' Other questions Anna suggested include "What did you do next?” and “What were you thinking?” Anna’s verbal thoughts represent an understanding of knowledge of content and student (KCS) for how students learn to add described by Hill et al. (2008) as “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content” (p. 375). While analyzing student work samples provided in the interview, Anna thoughtfully considered the student work and verbally processed her thinking of what the student did in the sample problems. She commented that the student used a different strategy that resulted in the correct answer but she would need to question the student to understand their reasoning in interview questions 3 (b) and 3 (c) and determine if the strategies would work with other numbers.

Anna’s flexibility in accepting student’s use of alternative strategies and the type of probing questions she suggested indicate she has a deep understanding of her content. This is an example of teacher pedagogical content knowledge described by Hill, et al. (2005) of how a teacher checks for understanding and creates the mathematical tasks to facilitate productive classroom discourse.

Beth

Beth described strategies she would use to get at student understanding of place value concepts by beginning with reminding students of the difference between ones and tens place value representations. She said she would ask a student to explain their understanding of tens and ones. If a student struggled she would have them focus on the ones place and when the student demonstrated an understanding of the ones place she would progress to the tens place. Additionally, Beth indicated she would question a student about their strategies for solving a problem if the strategy was not clear to her. Questions she said she would ask of students for interview question 3 (a), (b) and (c) included “What did you do with those two numbers?” If the student used an alternative strategy she would ask them “Was that strategy easier?” and if so, “Why was it easier and why did they choose that approach?” Beth commented these types of questions would help her better understand a students’ reasoning.

Beth’s descriptions of how she would get at student understanding of place value concepts demonstrated some understanding of the content but were limited to “What did you do?” and “Why did you do it?” When compared to Anna’s responses, Beth’s responses lacked the level of depth of knowledge about place value concepts.

Carol

Carol described how she would gain an understanding of a students’ grasp of place value concepts by asking questions to prompt a student to explain their thinking. Her questions were limited to “how” and “why” questions for interview question 3 (a), (b) and (c) such as “How did you get that answer?” and “Why did you do what you did?” Carol said she would ask a student who worked a problem like interview question 3 (a) “How did you get the 7?” She said “hopefully they will explain that the 2 and the 5 are in the ones place and there is a 1 in the tens place for each number, and they added that to get the 2.”

Other ways Carol reported getting at student understanding involved using counters or other objects to represent a number then asking the student to model a math problem where they take away a certain number of counters and discover how many counters remain. Carol also described how she would uncover student thinking by modelling a math problem using straws that are tied in bundles of 10. The specific problem she described was a subtraction problem that involved regrouping a bundle of 10 to add to the ones place. Carol indicated that she used this bundling strategy for multiple classroom activities such as determining the date on the calendar and discussing the lunch count for how many students will buy their lunch versus how many brought their lunch. She added that approaches like this help her better understand students' thinking and help her identify students' misconceptions about place value concepts.

Carol demonstrated some understanding for how to better understand student thinking about place value concepts but included only "how" and "why" questions and an approach of listening for student understanding as she modelled math problems. These strategies are not incorrect but do not demonstrate a clear understanding of student reasoning about place value concepts and lack the depth of understanding for how students think about, know, or learn this particular content demonstrated by Anna's responses.

Diane

Diane described strategies to understand student thinking about place value concepts by asking questions about interview question 3 (b) such as "Were you wanting to add the ones and the tens separately?" and "Why did you combine them together?" She stated she uses concrete manipulatives like base-ten blocks, place value charts and place value mats to help students solidify their understanding of ones and tens.

Diane's responses to all the interview questions were brief. She displayed some understanding of place value concepts but on a basic level. It was difficult to determine if she had a deeper understanding of place value concepts because of her limited responses. Diane's questioning strategies for understanding student thinking were superficial and focused on the procedures students utilized rather than conceptual understanding. Her responses lacked evidence that she could anticipate student actions based on either knowledge of the content and student (KCS) or specialized content knowledge (SCK) about place value concepts.

Instructional Strategies

Anna

Anna described specific instructional strategies she uses to teach place value content. One such strategy is intended to help students better understand place value of two-digit numbers, particularly the teens because these numbers do not follow the same pattern as twenties and thirties with the names of these numbers (eleven, twelve, etc.). The strategy involves students using their fist to punch numbers in the air to physically feel the different place values for ones and tens. Instead of having students say "eleven, twelve" Anna has students say "Ten-one, ten-two" to reinforce the place value positions of these numbers and to help students remember that these numbers contain ones and tens.

Another strategy Anna uses to help students understand the relationship between tens and ones is to build a two-digit number such as 46 with base-ten blocks (interview question 4). She has students count aloud, holding up base-ten rods that represent tens, "ten, twenty, thirty, forty, fifty" and acknowledges they went too far with the last ten ("fifty") and directs students to count ones to reach the number 46. Anna stated she understands that students need to be able to translate their verbal understanding to a written representation. A strategy she uses to help students recognize the written numbers is using number cards or flash cards with numbers written on them. She scrambles the cards and asks students to arrange them in correct chronological order. She uses tools such as hundreds charts with struggling math students to help them discover the correct order of the numbers. She stated she may need to prompt some students by reminding them of the chronological order for one-digit numbers and explaining how that order is repeated in two-digit numbers for the ones place. These strategies are examples of what Bair and Rich (2011) describe as Knowledge of Content and Student (KCS) when a teacher understands students' thinking and is able to relate it to clear, correct mathematical thinking.

Anna contemplated strategies she would use to support a struggling student's understanding of place value concepts in interview question 5 (a) and (b) when a student is unable to make a stack of 30 checkers. She described some scaffolding strategies she uses with her struggling math students by asking students to use counters to make 10 multiple times to understand how 10 ones becomes a ten. Anna indicated she responds to student reasoning that is incorrect by having them create concrete models of numbers to visually see the tens and ones of a two-digit number. An additional strategy Anna uses for struggling math students is to be sure students understand one-to-one correspondence. She uses counters and has students touch each counter as they count them, then repeats the strategy but she touches the counter as the student counts aloud. Anna reported she uses concrete models such as base-ten blocks to help students physically see a number but acknowledges students must be able to move fluently among different representations of a number (concrete, representational, abstract) to the written numeral. She also mentioned that if a student is unable to work a problem involving larger numbers she often will use smaller numbers to help them understand the concept and gradually build up to using larger numbers.

Other tools Anna stated she uses with students who struggle with mathematics are hundreds charts to help students add two-digit numbers. She places a card with the first number written on it such as 25 in the correct location on a hundreds chart and prompts the student to add the next number such as 37 by moving ahead by 3 tens and then 7 ones. She acknowledged students would need to understand how to move ahead by multiples of ten on a hundreds chart and that, if a student was able to correctly use this strategy, they could apply it to other similar problems.

The ability to correctly scaffold place value concepts to very basic levels as described indicates that Anna understands her content well enough to “unpack” the mathematics. This ability to “unpack” the mathematics, coupled with the strategies Anna identified to respond to students’ mathematical questions or misconceptions productively is representative of how Bair and Rich (2011) describe Specialized Content Knowledge (SCK).

Beth

Beth reported that some instructional strategies she has used with struggling math students to understand place value concepts included decomposing a two-digit number from standard form to expanded form. The example in interview question 4 prompted Beth to describe how she would work with a struggling learner to understand the relationship between tens and ones of the number 46. She stated she would ask a student to explain it in terms of “How many ones?” and “How many tens?” She indicated she also has utilized a “tens” and “ones” or “TO” chart to separate 46 into the correct place value locations on the chart. She said she would also use base-ten blocks to ask students to use units (ones) to make 46, then have them regroup and trade 10 ones for a rod (tens). Beth stated she would then ask students “How many ones are left over?”

While looking at the student work sample provided in interview question 5 (a) and (b), Beth identified scaffolding strategies she would use to uncover student thinking about why the student was unable to make a stack of 30 checkers. She indicated she would use a hundreds chart with a struggling student to correctly count by ones and then progress to counting by fives. Another strategy Beth reported using for struggling students was to use counters to make groups of five and have a student practice counting groups of five counters by fives. She said would gradually have the student progress to counting by tens using the same instructional strategies and tools. Beth stated she has used various questioning strategies to help her better understand student thinking about place value concepts. The questions she suggested were focused on "why" a student did what they did in a particular problem.

Beth's responses about scaffolding mathematics problems was limited as compared to Anna's responses. She demonstrated a good but not a deep understanding of place value concepts through her limited "unpacking" of the mathematics presented in the student work samples provided during the interview. She demonstrated a knowledge of the content but did not provide an example for how she would help the student make connections in their learning of these skills. Helping students make these connections by relating the students' thinking to clear, correct mathematical thinking is necessary to establish a clear understanding of the mathematics and to qualify as teacher knowledge of content and student (KCS).

Carol

Carol described instructional strategies she has utilized to teach place value concepts for interview question 4 to include strategies such as the bundling of straws in groups of 10, a strategy she has applied to multiple classroom activities. She also reported using ten-frames and a TO or "tens" and "ones" chart to reinforce student thinking about place value.

Carol pointed out that she scaffolds difficult problems about place value by "going back to counting" using plastic counters. She indicated she would have students put the counters in a row or straight line to prevent them from counting a plastic counter twice. Carol said she has students touch the counters as they begin to count, then she allows them to move the counters as they progress in their ability to count correctly. She stated that once a student can count by ones correctly then she would have them count by fives and eventually by tens.

Some of the questioning strategies Carol reported using with questions similar to interview question 4 include “How many ones are represented in the number?” and “How many tens are represented in the number?” She indicated that she has used manipulatives with these questioning strategies as a concrete representation of a number to reinforce ow many ones and how many tens are in the number. Carol also said she uses ten-frames to help students understand how to make 10 and hundreds charts to help students count by tens. These were strategies Carol reported she would use to support a struggling student attempt to solve questions similar to interview question 5 (a) and (b). Her explanation of counting by tens included students counting mid-decade by tens (e.g. 17, 27, 37, etc.).

Carol’s scaffolding strategies and questioning strategies demonstrate some understanding of place value concepts but not the deep level demonstrated by Anna. Carol reported using one manipulative frequently which may result in all students learning correct applications for this manipulative but also may result in some students struggling with math concepts if they do not understand the various applications of this manipulative. Carol provided responses that were consistent with a limited level of pedagogical content knowledge as compared to Anna’s responses. Both Beth and Carol demonstrated common content knowledge of place value concepts but did not provide evidence of specialized content knowledge or specific knowledge of content and students that would result in producing productive mathematical classroom discourse.

Diane

Diane reported that she utilizes place value mats as a frequent instructional tool to help students understand place value concepts. She described how she has used them with students and stated she would ask students *prompting* or *guiding questions* such as “How many ones are you going to take?" and "How many tens are you going to take?" These are strategies Diane stated she would use with struggling learners for interview question 4.

Strategies Diane indicated she would use to support struggling math students when a student is unable to make a stack of 30 checkers as in interview question 5 (a) and (b) included scaffolding instruction by having students count aloud by ones. If a student was not able to count by ones aloud, Diane stated she would have them count concrete items to determine if a student was able to count correctly. Another scaffolding strategy Diane said she used was taking strips of yarn to make circles and have a student count ten concrete items (e.g. checkers) and place ten items in each circle to make the number 30. She indicated this would allow her to determine whether or not a student had one-to-one correspondence and whether or not their ability to count to 30 was just memorization of a chant. She also stated she has students represent numbers in different formats to reinforce their understanding of the number and the place value parts.

Diane described a limited number of instructional strategies she has used to help students learn place value concepts. When prompted to elaborate on her responses during the interview, she said multiple times "That's it" indicating she was not able to provide any additional response or provide more detail to her comments. As compared to Anna who provided great detail and elaboration, Diane's responses were constrained and lacked a deep understanding of place value concepts. Diane did not demonstrate how she would support student thinking about place value concepts or respond to student questions about mathematics productively, both necessary to illustrate specialized content knowledge (SCK). She also did not provide evidence of the depth of content knowledge that was necessary to exemplify an understanding about student thinking or reasoning of the interconnectedness of single skills that translate into larger place value concepts (KCS) (Ball, et al., 2008).

Results and Summary of Analysis

Research Question 1

> *How do K-2 mathematics teachers describe their understanding of how students learn place value?*

The themes that emerged from this section of the research included: how students learn place value; challenges struggling students encounter, and instructional tools used by participants. Responses by teachers in each of these themes demonstrated inconsistencies for how they believe students learn place value. Teacher responses were more consistent when describing challenges students encounter when learning place value concepts and with the instructional tools they identify for use with struggling math learners. However, some of the tools or manipulatives they identified appear to be grade-level specific (e.g. five- and ten-frames only discussed by kindergarten teachers).

The Case of the High Score

The responses of Anna, the teacher who achieved the highest score (100%) on the assessment of mathematical knowledge for teaching (MKT) provided a rich description of how students learn place value concepts that reflected an understanding consistent with how Battista (2009) describes student reasoning in his levels of reasoning on place value concepts. The examples she provided included a series of progressions students traverse as they begin to reason with place value concepts. She offered examples for how students reason with understanding place value concepts at Level 0, Level 1, Level 2, and Level 3. The examples included the use of oral chants which is indicative of students who struggle within level 1 when initially learning to count by *ones*. This teacher explained that students require rote counting methods in order to progress to counting objects such as plastic counters. The next step she identified included counting smaller groups of objects to compose a larger number, for example in order to *make five* or *make ten* but noted that students continue to operate on the number as a collection of *ones*. This teachers' explanations for how students learn place value concepts included the use of manipulatives or mathematical tools as part of the *concrete-representational-abstract* model she described for how students' mathematical learning progresses. She identified additional mathematics tools such as hundreds charts, base-ten blocks, and number lines as being helpful tools to reinforce students' thinking about place value concepts that correspond and are consistent with reinforcing student reasoning of place value concepts in level 1.

This teacher with the highest MKT score gave additional examples for how students reason with place value concepts within level 2 through use of the tools previously noted. She stated that within level 2, students are able to count utilizing skip-counting strategies (e.g. by tens) using hundreds charts. She reported that students at this stage are able to begin to recognize patterns within the digits of two- and three-digit numbers, discovering that the numbers 0 through 9 appear repeatedly but in different place value locations. These comments indicated a recognition that this progression of learning and reasoning with place value concepts is a significant understanding for how students move along a progression that must be attained prior to moving to the next level of understanding of place value. She noticed that when students make these connections with numbers and place value, they are able to compose and decompose numbers into their place value parts and recognize important mathematical ideas (e.g. so many *ones*, so many *tens*, etc.). Anna's responses demonstrated a clear delineated progression of reasoning with place value concepts consistent with Battista's (2012) levels 0 through 3

The Case(s) of the Middle Score

Two teachers were selected for the interview section of the study who achieved MKT scores consistent with the average and/or mean score. One of the teachers made use of the word "milestones" when describing her understanding of how students learn and reason with place value concepts, however, did not expound upon the terminology. Beth and Carol both gravitated toward the use of manipulatives or math tools within their explanation of how they understood student reasoning and learning of place value concepts. Each teacher identified grade-level specific challenges students encounter with place value concepts rather than explaining the required order of reasoning with place value to progress to another level of reasoning. All references to levels of reasoning with place value concepts as described by Battista (2012) were absent from their explanations and no evidence of learning place value concepts within a sequence was verbalized. The teachers who scored a middle MKT score did not verbalize or provide any evidence that students learn place value concepts within a series of levels in which mastery at one level must be attained before progressing to the next level. The focus for these teachers was more on how they teach place value concepts rather than on an understanding for how students learn place value concepts.

Beth and Carol both described student behaviors as they reason with place value but each immediately went to identifying and selecting manipulatives to help students learn or understand place value. This lack of verbalization for why they were selecting these manipulatives, even after prompts to explain why, seemed to indicate their focus was not on how students learn but rather what their role was as teacher, without considering whether or not it would support the students' current thinking. One of the teachers who achieved the middle MKT score described challenges students face when learning specific place value concepts but again, offered little processing for how student behavior linked with teacher behavior (instruction).

The Case of the Low Score

The teacher who achieved the low score on the MKT assessment responded to questions about how she believes students learn place value concepts by describing a mathematical model for teaching of moving from *concrete to representational to abstract*. Diane did not describe student learning of place value concepts in levels or stages. She did not describe student behavior when learning or reasoning with place value concepts. Her focus was on how she would teach the concepts and did not indicate a consideration for how students learn or where students were in their thinking about place value. There was minimal evidence provided by this teachers' responses to make connections to Battista's (2011) model for student reasoning with place value concepts.

The analysis of data collected compared teachers' achieved score on the assessment of teacher mathematical knowledge for teaching (MKT) to their understanding for how students learn and reason with place value concepts. The responses teachers provided coupled with the instructional tools they identified provided evidence of their level of student understanding. The findings support the first part of this researcher's working hypothesis that teachers with less knowledge of MKT of place value (based on their score on the assessment of teacher MKT) would respond with a more limited understanding of student thinking than teachers with a higher score.

Research Question 2

> *How does teacher mathematical knowledge for teaching (MKT) place value impact their instructional decisions with struggling math students in tier 2 intervention?*

The instructional decisions teachers identified based on their evaluation of student work samples for this portion of the study were compared with the model of Mathematical Knowledge for Teaching (MKT) proposed initially by Ball, et al. (2008) and expounded upon by Bair and Rich (2011). The themes identified with the analysis of teacher responses included: strategies to understand student thinking, and instructional strategies. There was some overlap in teacher responses with teachers identifying specific tools they would use when identifying strategies to understand student thinking.

Teachers demonstrated consistency in their identification of strategies to understand student thinking about place value concepts with questioning strategies. The level of depth or sophistication of questions, however, seemed to diminish in a manner consistent with the teachers' MKT score of place value concepts. There was consistency with the instructional strategies teachers identified for use with struggling mathematics students in that each reported using manipulatives or other tools coupled with questioning strategies to support students' learning of place value concepts. Again, the depth and rigor of questions asked decreased in a parallel manner based on each teacher's achieved MKT score.

The Case of the High Score

Anna, who achieved the highest score on the assessment of teacher MKT, described instructional decisions she uses with struggling math students that reflected a deep understanding of the mathematics concepts she teaches. She indicated she would be open to allow students to solve mathematics problems using alternative methods as long as their thinking was mathematically correct. She suggested multiple probing questions she would ask students to discover their understanding and reasoning about place value concepts. These are consistent with Hill's, et al. (2005) description of how a teacher checks for student understanding and promotes productive mathematical discourse. Anna also described multiple instructional strategies and tools she uses with struggling students that reflected a deeper understanding of students' thinking that allowed her to relate the students' thinking to clear and correct mathematical thinking (Bair & Rich, 2011).

Anna provided examples of instructional tools she uses with struggling math students that included scaffolding strategies. The ability to scaffold mathematical concepts into component parts was an indication she knows and understands place value concepts at a deeper level. This ability to scaffold place value concepts as she explained is consistent with how Bair and Rich (2011) describe specialized content knowledge (SCK). Their description includes an ability to "unpack" the mathematics taught so that struggling math students have access to the content.

The Case(s) of the Middle Score

The two teachers' who achieved the middle score(s) responses were less about checking for student understanding to promote productive mathematical discourse (Hill, et al., 2005) but were more about "how" or "why" a student made certain mathematical decisions. The teachers reported they would listen for student thinking to determine what instructional strategies to utilize, however, their responses indicated they had already pre-determined the instructional strategy to use prior to hearing students' explanations of their understanding. Some of their questioning strategies included "What did you do?" and "Why did you do that?" These types of questions seemed to lack the depth of understanding for how to best prompt a student to describe their reasoning about place value concepts and were not consistent with uncovering a deeper understanding of students' thinking (Bair & Rich, 2011).

Beth offered scaffolding strategies she would use with struggling math students similar to Anna's examples. However, the examples Beth gave were less sophisticated than Anna's and lacked the depth found in the strategies Anna provided. Carol also described questioning strategies she would use with struggling math students which were consistent with a limited level of pedagogical knowledge about place value concepts but did not reflect a strong connection between mathematical content and knowledge of student (Bair & Rich, 2011).

The Case of the Low Score

Diane, who achieved the lowest score on the assessment of teacher MKT, gave only partial responses when describing the instructional strategies she would utilize with struggling math students on place value concepts. She also suggested she would use questioning strategies to understand student thinking but the questions she listed were questions about the procedures a student might be using with place value concepts.

Since this teacher provided such abbreviated responses, it was difficult to determine if she possessed a deeper level of understanding about place value and effective instructional strategies as identified by Hill, et al. (2005), however, within her responses, there was no evidence to indicate this level of understanding.

The data analysis compared teachers' achieved score on the assessment of teacher mathematical knowledge for teaching (MKT) to the description of the instructional strategies and tools they would utilize. The findings support the second part of this researcher's working hypothesis that teachers with less knowledge of MKT of place value (based on their score on the assessment of teacher MKT) would suggest or recommend more limited instructional strategies than teachers with higher scores.

CHAPTER V: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Introduction

This study explored the nature of the relationship between the mathematical knowledge for teaching (MKT) of place value a group of K-2 elementary mathematics teachers possess and the instructional decisions they make with students in a mathematics intervention setting. The previous chapters included the background of the problem, a statement of the problem, purpose of the study, significance of the study and a review of the literature. Also included was a discussion of the frameworks applied to the study, the methodology and procedures utilized and a presentation of the data with an analysis addressing each of the research questions. This chapter includes a summary of the study with conclusions and implications drawn from the study. The chapter ends with recommendations for future research and for practice.

Summary of the Study

This qualitative study compared multiple case studies of teachers' responses for how they believe students learn place value concepts with Battista's (2012) Levels of Sophistication in Student Reasoning: Place Value. Teachers were selected for the interview portion of this study based on their score on the Learning Mathematics for Teaching (2006) Elementary Place Value Content Knowledge assessment. This tool provides a measure of teachers' mathematical knowledge for teaching (MKT). Additionally, comparisons among the three cases were made to the "egg" model of mathematical knowledge for teaching (Ball, et al., 2008; Bair & Rich, 2011) based on their score range and the instructional decisions they make to support student thinking in a K-2 mathematics intervention setting.

Teachers' score ranges (high score, middle score, low score) were utilized in this study for comparison purposes and each of the three cases (high score, middle scores, low score) was considered with respect to the theoretical frameworks. The response data from teachers who achieved lower scores of mathematical knowledge for teaching (MKT) place value concepts were compared to the responses of the teacher with greater MKT of place value concepts. The themes that emerged from this analysis included: how students learn place value; challenges struggling students encounter, and instructional tools used by participants. What follows is a summary of the participants' responses that addressed research question 1.

Research Question 1

> *How do K-2 mathematics teachers describe their understanding of how students learn place value?*

When explaining how students learn place value, the study revealed inconsistent thinking among teachers about how students learn and reason with place value concepts. The case of the teacher with the high score on the MKT assessment of place value provided a clear description of how students learn place value concepts in steps or stages with only one other teacher commenting that students learn these concepts as "milestones". This teacher's descriptions of the stages for how students learn place value concepts came closest to mirroring Battista's (2012) levels 0-3. These levels explain how students' reason with place value concepts of individual numbers.

The case of the teacher(s) with the middle score revealed a leaning toward the instructional strategies each had acquired through multiple years of teaching place value concepts. There was little evidence that these teachers reflect on student thinking and understanding in their explanation for how they believe students learn place value concepts. Their focus was on how they would teach the concepts without considering if it would support students' current thinking about place value

When explaining the challenges struggling mathematics students encounter learning place value, the results of the study showed a more consistent response among teachers. All teachers reported that students struggle with understanding the relationship between *ones* and *tens* in some manner. The teachers with higher MKT scores responded in greater detail and provided evidence of their claims through student work examples consistent with Battista's (2012) levels 0 and 1.

The case of the teacher with the lowest MKT uncovered a lack of connections between how she believes students learn place value concepts and progressions in that learning (Battista, 2012). This teachers' responses also indicated a stronger connection between her instruction and the content being taught rather than on how students learn or where a student might be in their understanding of place value concepts.

Teachers' responses about instructional tools also demonstrated some consistency with tools they use with struggling math students. All teachers indicated they use *base-ten blocks* and some form of physical objects (e.g. plastic counters) to count and to represent a number's physical size (e.g. snap cubes). Most of the teachers reported using *number lines* and *hundreds charts* to teach and reinforce place value and order of numbers. Only the two kindergarten teachers indicated they use *five- and ten-frames* to demonstrate the difference in one-digit and two-digit numbers. The tools identified seemed to be grade-level specific in some instances such as *five- and ten-frames* and corresponded to differing levels of Battista's (2012) framework for how students' reason with place value concepts.

The instructional decisions teachers identified based on their analysis of student work samples of place value concepts were compared with the model of Mathematical Knowledge for Teaching (MKT) proposed by Ball, et al. (2008). The teachers responded with examples for how they would work with struggling K-2 students in an intervention or RtI setting and do not necessarily reflect their teaching of core instruction (regular classroom instruction). The themes identified with this analysis included: strategies to understand student thinking, and instructional strategies. The following is a summary of teachers' responses to address research question 2.

Research Question 2

> *How does teacher mathematical knowledge for teaching (MKT) place value impact their instructional decisions with struggling math students in tier 2 intervention?*

Each of the case studies (high score, middle score, low score) included consistent use of questioning as teachers discussed the strategies they would use to understand student thinking. The level of sophistication of questioning strategies described and the understanding of place value demonstrated by this group of teachers decreased in a manner consistent with their MKT of place value (score on the assessment). Additionally, teachers' responses for the instructional strategies they would use indicated some consistency with the use of manipulatives coupled with strategic questioning. Each of these cases was compared to the "egg" model of mathematical knowledge for teaching (Ball, et al., 2008; Bair & Rich, 2011).

The case of the teacher with the highest score on the MKT assessment of place value offered interpretations of student work samples and identified instructional strategies and tools consistent with Bair and Rich's (2011) definition of a teacher with Specialized Content Knowledge (SCK). The instructional strategies provided by this teacher indicated an ability to "unpack" the mathematics to provide meaning for students, also a component of SCK. This teacher described probing questions to uncover student thinking that were reflective of a deep understanding of place value concepts and knowledge of how students think about these concepts. The ability to pair a deep understanding of mathematics with student thinking about the mathematics is how Bair and Rich (2011) describe Knowledge of Student and Content (KSC).

The case of the teachers who achieved the middle MKT score indicated a propensity toward “how” or “why” a student made a mathematical decision rather than on checking for student thinking to promote productive mathematical discourse, a necessary component identified by Bair and Rich (2011). Additionally, there was a lack of connections between student thinking and place value concepts being taught as indicated by these questioning strategies.

The responses provided in the case of the teacher with the low MKT score revealed a limited understanding of connections with student thinking and place value concepts (Bair & Rich, 2011). The truncated responses provided by this teacher provided minimal opportunities for uncovering how to attain a deeper level of understanding by students for place value concepts. This case as compared to the other cases indicate teachers with a lower MKT of place value seem to lack the ability to help students make connections in their own understanding and reasoning with place value.

Discussion

The results of the study indicated that, for this sample of teachers, the case of the high MKT score of place value concepts demonstrates a deeper understanding of the mathematics content. Additionally, the teachers with greater MKT of place value identify, with greater consistency, instructional strategies that research indicate are more effective with students who struggle with mathematics understanding, particularly those in an intervention setting as identified within the frameworks discussed.

The teacher with the highest score on the Learning Mathematics for Teaching (2006) Elementary Place Value Content Knowledge assessment had additional experience and training above and beyond the training elementary mathematics teachers receive. The knowledge gained and required for this additional experience was not explored, however, the additional professional development received was extensive. The trainings described by this teacher far exceeded the professional development trainings described by the other teachers in this study.

All of the teachers noted that when learning counting strategies, students initially rely on physical objects to count, similar to the findings of Steffe, et al. (1988). Also, teachers in the study described teaching strategies for reinforcing counting by *tens* through the use of hundreds charts which Cobb and Wheatley (1988) reported was a requirement before children understand the position of numbers and their place values.

Most of the teachers acknowledged the challenges students encounter when learning two-digit numbers due to the spoken words being inconsistent with the written number (e.g. 11, 12, 13, etc.). This matches challenges reported by Fuson, et at. (1997) of initial understanding for two-digit numbers due to the irregularities with the English spoken words for these numbers.

The teacher with the highest MKT score on place value and the teacher with the lowest MKT score both described some version of a *concrete to representational to abstract* approach when teaching students about place value concepts in a mathematics intervention lesson. This approach is described by Peterson, et al. (1987) as being effective for helping students learn and retain initial place value concepts.

The findings of this research indicated that teachers with a greater MKT of place value concepts identify more consistently effective instructional strategies to support struggling mathematics students' understanding of place value concepts in an intervention setting. Other studies from the existing research that considered teacher MKT or pedagogical content knowledge (PCK) of place value focused more on increasing teachers' PCK of place value by applying a treatment (professional development). Two of these studies (Kulhanek, 2013; Waller, 2012), both dissertation studies, employed the Learning Mathematics for Teaching (2006) assessment, however, they selected different items from the assessment for their studies.

The focus of this study was on the MKT of a group of K-2 elementary mathematics teachers and the relationship between their MKT and the instructional strategies they identify for working with struggling mathematics students in an intervention or RtI setting. Research on RtI (Bryant, et al., 2008a; Zheng, et al., 2012) suggests that the use of systematic, explicit instruction is an effective instructional strategy for use with students in mathematics intervention lessons and while none of the teachers in this study used these words to describe their instruction, some evidence was present of this approach. For example, the teacher with the greatest MKT demonstrated an understanding of learning trajectories of mathematics and where place value fit within the progression of learning. This teacher provided clear examples of scaffolding instruction for place value concepts that indicated an understanding or foundational skills needed prior to learning other grade level content.

Also, teachers' descriptions of beginning steps in teaching students counting strategies indicated they have students count by *ones* up to twenty and focus on one-to-one correspondence. This finding is consistent with a study comparing the development of place value concepts in Asian schools and American schools (Yang & Cobb, 1995). They found that American schools focus on one-to-one correspondence and counting by *ones* up to twenty as initial learning activities and they reported this was a weakness in content sequencing.

It was, however, surprising to note that none of the participants in this study identified the specific instructional strategy of *explicit instruction* (by name) as being effective with students in an intervention setting. This instructional strategy has been reported extensively in the research about Response to Intervention (Jordan, et al., 2009; Bryant et al., 2011; Gersten et al., 2012) as an effective instructional strategy to use in mathematics intervention instruction.

The findings of this research suggest that teachers who provide K-2 mathematics intervention lesson should possess a greater MKT of specific mathematics content appropriate for supporting students' learning in RtI or intervention settings. The mathematics content knowledge addressed by a report to the National Joint Committee on Learning Disabilities (2005) indicates the interventionist "will be expected to master a variety of scientific, research-based methods and materials" (p.7). Another report addressing the content knowledge of the interventionist stated the intervention specialist, in order to be effective, "will be instructional experts" (p. 271) who are knowledgeable about the curricula and about effective instructional strategies appropriate for students in intervention settings.

Conclusions

The findings of this qualitative study supported the researcher's initial belief that teachers with greater mathematical knowledge for teaching (MKT) would demonstrate a deeper understanding of instructional strategies that support struggling mathematics students learning in intervention settings. The conditions under which these comparisons were made for each of the three cases, high score, middle score, and low score, were considered with respect to the frameworks of Battista's levels of reasoning about place value (2012) and the "egg" model of mathematical knowledge for teaching (Ball, et al., 2008; Bair & Rich, 2011). Additional research results of other studies reported here supports this assertion and underscores the importance of the pedagogical content knowledge or MKT of the person providing mathematics intervention, whether it be a mathematics specialist or classroom teacher.

Implications

Implications for Teachers and School Districts

The results of this study suggest K-2 elementary mathematics teachers who are responsible for teaching mathematics intervention should possess a greater mathematical knowledge for teaching (MKT). The teachers in this study with greater MKT identified more consistently, effective instructional strategies for supporting students in mathematics intervention lessons. However, there was limited knowledge demonstrated about specific instructional strategies identified in the literature review as being effective for addressing the needs of struggling mathematics learners in RtI, such as systematic, explicit instruction (Bryant, et al., 2008a). It might be helpful for teachers to seek out specific professional development on RtI instruction when seeking out professional learning opportunities. It might also be beneficial for school district leaders and curriculum coordinators to provide this type of training for their teachers either by enlisting local RtI mathematics experts or bringing in outside RtI mathematics experts. School district leaders might also consider whether it is feasible or even desirable for all elementary teachers to be content experts in mathematics, especially considering these teachers are usually responsible for teaching multiple content areas in addition to mathematics. School district leaders may want to consider whether it is more beneficial to hire additional staff, who have this level of expertise, to provide mathematics intervention for students.

Implications for Response to Intervention (RtI) Research

The review of literature for this study revealed limited research on the pedagogical content knowledge of mathematics interventionists. The larger body of RtI research may benefit from additional consideration of the impact of mathematics pedagogical content knowledge (PCK) or mathematical knowledge for teaching (MKT) on intervention instruction and student academic success. If research can demonstrate a significant relationship between elementary mathematics teachers' MKT of specific content (appropriate for RtI) and student academic success, this may inform questions school districts face including teacher professional development and staff hiring decisions. Additionally, this may inform specific qualifications and skills mathematics interventionists should possess in order to be effective.

Limitations

The teachers selected for this study teach different grade levels and the place value content taught at each grade level differs based on the mathematics curriculum expectations at that grade level. This could have influenced the place value content knowledge each individual teacher possesses. Also, while no teacher reported recent professional development (within the past 5 years) on place value concepts, one teacher had previous professional development and work experiences, unique to her, that could have informed her understanding of place value. Additionally, the sample size and qualitative methodology applied prevent the findings to be generalizable to other groups of teachers.

Recommendations for Practice

The findings of this study indicate that a teacher with a greater level of mathematical knowledge for teaching (MKT) will exhibit specific behaviors that promote students' mathematical thinking in an intervention setting. Those teacher behaviors observed in this study are listed below.

- The teacher displays a greater understanding for "how" students reason and understand mathematical concepts.

- The teacher understands there are progressions of learning students follow in reasoning with mathematics concepts.
- The teacher understands students' thinking and can relate it to clear, correct mathematical thinking.
- The teacher demonstrates the ability to scaffold concepts to "unpack" the mathematics.
- The teacher responds to students' mathematical questions or misconceptions productively.
- The teacher identifies appropriate manipulatives to support mathematics learning,
- The teacher considers student's thinking and reasoning about mathematics concepts prior to identifying an appropriate instructional strategy.
- The teacher identifies appropriate instructional strategies to support mathematics learning and understanding.
- The teacher checks for students' understanding and promotes productive discourse based on students' responses.

Recommendations for Future Research

The following recommendations for future research are proposed based on the findings of this study.

- It is recommended that future research on mathematics RtI consider the relationship between the mathematics interventionists' MKT and student academic performance.
- It is also recommended that future research on mathematics RtI consider the feasibility of training all elementary mathematics teachers to be content experts or whether it is more beneficial for mathematics intervention experts/specialists to provide mathematics

intervention for students. This research should consider the benefits of various models of mathematics instruction (push-in, pull-out or other models) on student academic success.

- It is also recommended that future research on mathematics RtI explore additional options for how elementary classroom teachers' MKT may be increased. This research might consider what teacher professional training might include (e.g. mathematics content, instructional strategies, use of tools/manipulatives, etc.).
- It is also recommended that future research on mathematics RtI consider how new research on MKT of elementary mathematics teachers informs the field of mathematics education. This might include evaluating and informing teacher training at the university level or at the district level.

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APPENDIX A

Learning Mathematics for Teaching (2006) Elementary Place Value Content Knowledge

1. Ms. Wilson's class is working in groups to decompose 391 into hundreds, tens, ones, and tenths. As she walks around, she sees groups have arrived at very different answers. Which of the following ways to represent 391 should she accept as correct? (Mark YES, NO, or I'M NOT SURE for each choice.)

	Yes	No	I'm not sure
a) 3 hundreds + 90 tens + 1 one	1	2	3
b) 2 hundreds + 19 tens + 1 one	1	2	3
c) 3 hundreds + 9 tens + 10 tenths	1	2	3
d) 39 tens + 1 one	1	2	3

2. Mr. Siegel and Mrs. Valencia were scoring their students' work on the practice state mathematics exam. One question on the exam asked:

> Write the number that is halfway between 1.1 and 1.11.

Mr. Siegel and Mrs. Valencia were interested to see the different answers students wrote. What should the teachers accept as correct? (Mark ONE answer.)

a) 1.05

b) 1.055

c) 1.105

d) 1.115

3. Teachers often offer students "rules of thumb" to help them remember particular mathematical ideas or procedures. Sometimes, however, these handy memory devices are not actually true, or they are not true for all numbers. For each of the following, decide whether it is true all of the time or not. (Mark TRUE FOR ALL NUMBERS, NOT ALWAYS TRUE, or I'M NOT SURE.)

	True for all numbers	Not always true	I'm not sure
a) If the first of two numbers is smaller than a second, and you add the same number to both, then the first sum is smaller than the second.	1	2	3
b) Multiplying a number makes it larger.	1	2	3
c) A negative number plus another negative number equals a negative number.	1	2	3
d) To multiply any number by 10, add a zero to the right of the number.	1	2	3

4. As Mrs. Boyle was teaching subtraction one day, she noticed a few students subtracted in the following way:

$$\begin{array}{r} {}^{13} \\ 6\cancel{3} \\ -\,{}^{3}\cancel{2}8 \\ \hline 35 \end{array}$$

What were these students **<u>most likely</u>** doing? (Mark ONE answer.)

a) The students "subtracted up," by taking 3 away from 8, and then tried to compensate for this mistake.

b) The students compensated by subtracting 30 from 63, then dealt with the 8 and 3 in a second step.

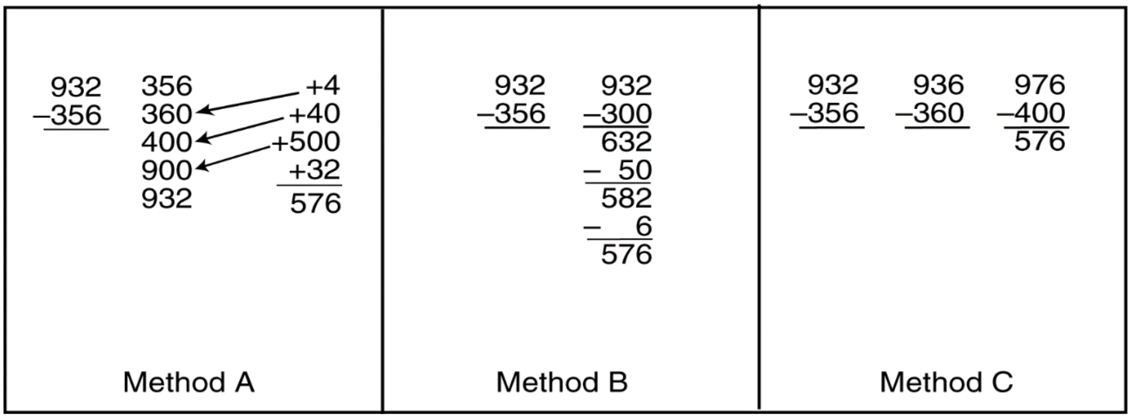
c) The students made a mistake with the standard procedure, crossing out the 2 rather than the 6.

d) The students added ten to both 63 and 28, then subtracted.

5. Ms. Lawrence is making up word problems for her students. She wants to write a word problem for $3 \div \frac{1}{2}$. Which word problem(s) can she include? (Mark YES, NO, or I'M NOT SURE for each problem.)

	Yes	No	I'm not sure
a) Melissa has 3 pizzas and she wants to give half of them to her friend. How much pizza will her friend get?	1	2	3
b) Dan has 3 cups of chocolate chips. He wants to bake cookies, and each batch requires $\frac{1}{2}$ cup of chocolate chips. How many batches of cookies can Dan make if he uses all of the chocolate chips?	1	2	3
c) Three friends each have half of a cookie. How many cookies would they have if they put them all together?	1	2	3
d) Jacquie has collected three cans of pennies for her fund-raiser. If she is halfway to her goal, how many cans of pennies had she set as the goal?	1	2	3

6. Imagine that you are working with your class on subtracting large numbers. Among your students' papers, you notice that some have displayed their work in the ways:



Which of these students is using a method that could be used to subtract any two whole numbers? (Mark ONE answer.)

a) A only

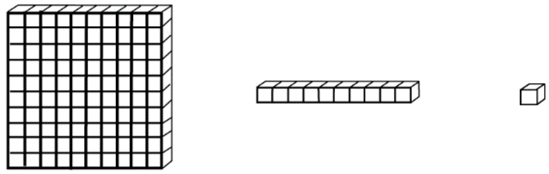
b) B only

c) A and B

d) B and C

e) A, B, and C

7. Mrs. Kwon decides to try teaching decimals using base ten blocks. She has three kinds of base ten blocks available to her:



When teaching place value with whole numbers, the use of the blocks seems simple. But for decimals, it seems more complex, and she asks Mrs. Carroll next door what she thinks the values of the blocks should be. How should Mrs. Carroll reply? (Mark ONE answer.)

a) Ones "cubes" become wholes; tens "rods" become tenths; hundreds "flats" become hundredths.

b) Hundreds "flats" become wholes; tens "rods" become tenths, and ones "cubes" become hundredths.

c) Either use of the blocks will work.

d) Neither use of the blocks will work.

8. Nathaniel suggested the following idea for doing the problem:

$$\begin{array}{r} 0.23 \\ \underline{\times\ \ 95} \end{array}$$

> First I ignore the decimal point and do the multiplication, which gives me 2185. Then I use estimation to place the decimal point. I know that 0.23 is about 1/4 and 95 is about 100 and 1/4 of 100 is 25, so my answer would be 21.85.

Which of the following is most appropriate to say about Nathaniel's approach? (Mark ONE answer.)

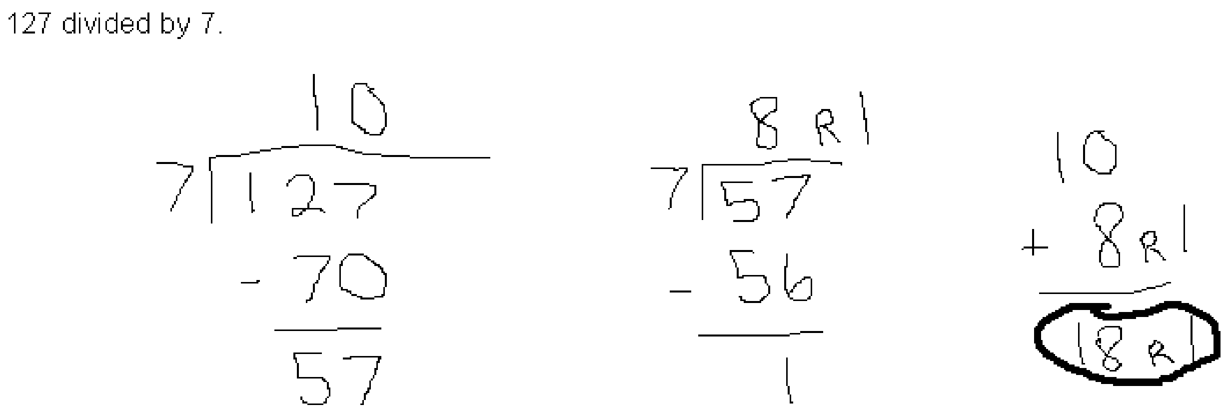
a) It happens to work in this case, but will not work for most problems.

b) It only works if one of the numbers is a whole number.

c) It works for any numbers, but some examples are harder to estimate.

d) It works equally well for all problems.

9. Ms. Barber was reviewing her students' division homework and saw that Chad used the following non-standard approach to divide 127 by 7:



What is true about Chad's approach?

a) His approach is not mathematically valid; it is a coincidence that his answer is correct.

b) His approach is not mathematically valid because he subtracted 70 from 127 instead of subtracting 7 from 12.

c) His approach is mathematically valid, but could be inefficient with large dividends.

d) His approach is mathematically valid, but only works with single-digit divisors.

10. Mrs. Jamieson was looking for a good problem to give her class that would produce many solutions, but not infinitely many solutions. Which of the following would work? (Mark INFINITE, NOT INFINITE, or I'M NOT SURE.)

	Infinitely many solutions	Not infinitely many solutions	I'm not sure
a) Find fractions between 0 and 1.	1	2	3
b) I have pennies, nickels, and dimes in my pocket. Suppose I pull out three coins. What amounts of money might I have?	1	2	3
c) If Joseph has three times as many cookies as Mary, how many cookies could they have altogether?	1	2	3

11. Mr. Lewis was surprised when one of his students came up with a new procedure for subtraction (pictured below), and he wondered whether it would always work. He showed it to Ms. Braun, next door, and asked her what she thought.

$$\begin{array}{r} 37 \\ -19 \\ \hline -2 \\ 20 \\ \hline 18 \end{array}$$

How do you think Ms. Braun should respond? (Mark ONE answer.)

a) She should tell Mr. Lewis the procedure works for this problem but would not work for all numbers.

b) She should tell him this does not make sense mathematically.

c) She should let Mr. Lewis know that this would work for all numbers.

d) She should say that this procedure only works in special cases.

12. Which of the following is the <u>best</u> explanation for why the conventional long division algorithm works, as in the following example? (Circle ONE answer.)

```
    111 R 29
37)4136
   37
    43
    37
     66
     37
     29
```

a) It works because you divide 37 into smaller parts of 4136 (the dividend) to make the problem easier to solve.

b) It works because you subtract multiples of powers of ten times 37 (the divisor) from 4136 (the dividend) until you have less than 37 left.

c) It works because if you multiply 111 (the quotient) by 37 (the divisor), and add in 29, you get 4136 (the dividend).

d) It works because you subtract 37's (the divisor) from 4136 (the dividend) until you have less than 37 left.

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learning mathematics for teaching project

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