Mathematics Education Research Journal
https://doi.org/10.1007/s13394-024-00489-x

ORIGINAL ARTICLE

Mathematics education researchers' practices in interdisciplinary collaborations: Embracing different ways of knowing

Elizabeth Suazo-Flores[1] · William S. Walker III[2] · Signe E. Kastberg[2] · Mahtob Aqazade[3] · Hanan Alyami[4]

Received: 17 May 2023 / Revised: 7 March 2024 / Accepted: 21 March 2024

Abstract

Mathematics education researchers (MERs) use practices unique to the mathematics education discipline to conduct their work. MERs' practices, i.e., ways of being, interacting, and operating, define the field of mathematics education, are initially learned in doctoral preparation programs, and are encouraged and sanctioned by conferences and publications. Disciplinary practices facilitate MERs' interactions within mathematics education. When working in interdisciplinary groups, differences in disciplinary ways of being, interacting, and operating can create challenges with completing research and other work. Since MERs' engagement in interdisciplinary collaborations is encouraged and can result in products contributing to the evolution of the mathematics education discipline, it is important to explore what practices MERs use in interdisciplinary collaborations. We interviewed four MERs who led international interdisciplinary collaborations and used qualitative content analysis to create descriptions of practices described by MERs in their collaborations. Five practices were common between the MERs in interdisciplinary collaborations. MERs conducted interdisciplinary work by using practices that allowed them to situate themselves and others in the group (i.e., being practices), develop ideas (i.e., interacting practices), work towards common goals, and use structures to get the work done (i.e., operating practices). We argue that MERs developed new practices to position themselves and others, interact with practitioners from other disciplines, and get interdisciplinary work done. This study contributes to the evolution of the mathematics education discipline by offering five practices that can orient MERs to conducting interdisciplinary work and discussing how MERs experience interdisciplinary collaborations beyond providing mathematics education expertise.

We confirm this manuscript is original and not under consideration for publication elsewhere. An earlier and shorter version of this manuscript was published in the 2022 proceedings of the Forty-third Annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. The authors report there are no competing interests to declare and nor funding sources to disclose. Please address all concerns to the corresponding author Elizabeth Suazo-Flores at Elizabeth.suazo@und.edu.

Extended author information available on the last page of the article

Published online: 19 April 2024

Keywords Mathematics education · Interdisciplinary collaborations · Practices · Disciplines

Introduction

Mathematics education researchers (MERs) are encouraged to work on interdisciplinary projects to advance the discipline of mathematics education (e.g., Bakker et al., 2021; Cai et al., 2020). Mathematics education is a discipline with specific objects of inquiry and practices that evolve over time (Williams et al., 2016). MERs conduct their work using practices learned in their doctoral preparation programs (e.g., Sherin, 2014), encouraged by funding agencies (e.g., National Science Foundation, 2023), and required for publication in peer-reviewed research journals (e.g., Andrade-Molina et al., 2020). Mathematics education practices support and constrain MERs when conducting interdisciplinary projects.

When working in interdisciplinary groups, MERs work with experts from different disciplines (e.g., mathematics, science education, engineering, biology). Differences in disciplinary-based practices can create challenges for MERs in interdisciplinary collaborations (e.g., Pimm, 2009; Roth, 2014). For example, MERs overcame differences in discipline-specific vocabulary that constrained interdisciplinary communication (Bruce et al., 2017). Also, interdisciplinary work can go unrecognized for faculty promotion purposes (Darragh, 2022; Goos & Bennison, 2018). Despite these challenges MERs continue to conduct interdisciplinary research, raising questions about practices MERs use in interdisciplinary collaborations. To support MER efforts to engage productively in interdisciplinary projects, we sought wisdom from those who have experienced such collaborations. Specifically, we wondered: What practices do MERs report using in interdisciplinary collaborations?

In this report, we start by conceptualizing "disciplines" and use this conceptualization to define mathematics education as a discipline. We then describe how the discipline of mathematics education has evolved through the interdisciplinary work of MERs. When collaborating with practitioners of other disciplines, MERs might need to adjust their practices. Subsequently, we introduce the concept of practices as lenses in our study. Given the calls in mathematics education for interdisciplinary collaborations, we assert that more insight into MERs experience is needed to support MERs seeking to engage in such partnerships. We then introduce the methods of our study. Based on a qualitative content analysis of four MERs' experiences leading interdisciplinary collaborations, we share five practices used by the participating MERs. Using existing literature about practices in interdisciplinary work and our findings, we argue that MERs developed new practices to position themselves and others, interacted with practitioners from other disciplines, and completed interdisciplinary work. We finish discussing how important it is for MERs to participate in interdisciplinary collaborations beyond only providing mathematics education expertise and how MERs need professional spaces to reflect on practices they can use in interdisciplinary collaborations.

Mathematics education as a discipline and MERs in interdisciplinary work

Disciplines are domains of knowledge whose borders are formed by specialization or key domains of inquiry that differ from other disciplines and forms of discourse specific to that inquiry (Williams et al., 2016). Disciplines were created to have specific objects of interest and approaches to conducting work, which involve practitioners operating within the community of their discipline (e.g., Pimm, 2009; Roth, 2014). Members of a discipline often develop expertise specific to the discipline and have limited interactions and collaborations with practitioners from other disciplines (Roth, 2014; Williams & Roth, 2019). Disciplines constrain and empower practitioners because disciplinary activities support the production and reproduction of disciplinary knowledge. Although practitioners are positioned as implementers of their discipline's discourse and approaches, they are more than that; practitioners' views and interests serve as filters for taking action when conducting their disciplinary work (Williams, 2016).

Disciplines evolve and become more specialized through practitioner work (Williams et al., 2016). Mathematics education is one such discipline. Initially, mathematics education drew from psychology and mathematics (Kilpatrick, 2014; Lester & Lambdin, 1998; Stinson & Walshaw, 2017) to conduct studies of mathematics learning, teaching, and curriculum. More recently, mathematics education research has drawn from disciplines including neuroscience (e.g., Norton et al., 2019), science education (e.g., de Freitas & Palmer, 2016), and anthropology (e.g., González et al., 2001; Gutiérrez, 2013) to address complex questions. Some of the evolution of questions in mathematics education resulted in MERs' engagement with experts from other disciplines. For instance, to address questions about the origins of undergraduate students' mathematics anxiety, Norton et al. (2019) collaborated with colleagues from neuroscience. Similarly, Krummheuer et al. (2013) collaborated with researchers from psychoanalysis to explore children's mathematical creativity. Teams of researchers have also collaborated to gain insights regarding phenomena of interest across disciplines. For example, MERs interested in spatial reasoning collaborated with colleagues from psychology, mathematics, cognitive science, and philosophy (Bruce et al., 2017). Interdisciplinary collaborations between MERs and mathematicians have improved the teaching of mathematics content and preparation of mathematics teachers (Darragh, 2022; Movshovitz-Hadar & Kleiner, 2009). For example, in Goos and Bennison (2018), mathematicians and mathematics teacher educators collaborated to co-develop and co-teach content and pedagogy courses for preservice teachers. They found that without talking about content and pedagogy at the same time, students "had not made the connections [between them]…and the maths they were going to be teaching at school" (p. 264). Thus, MERs' collaborations have allowed them to tackle more complex issues in mathematics learning, teaching, and curriculum.

We have described how mathematics education has evolved as a discipline in terms of research questions MERs pose. The objects and themes of inquiry coupled with discourse and techniques emerged through collaboration with colleagues from other disciplines (Bakker et al., 2021), yet sustaining interdisciplinary collaborations can be challenging (Bruce et al., 2017; Roth, 2014). Reports from empirical studies

describe how MERs interdisciplinary collaborations have been constrained. For instance, Darragh (2022) and Goos and Bennison (2018) reported that interdisciplinary collaborations need to align with institutional reward structures. In addition, collaboration across disciplinary boundaries has been constrained by geographic and physical space (e.g., Goos & Bennison, 2018).

Although there are interdisciplinary studies involving MERs (e.g., Bruce et al., 2017; Darragh, 2022; Goos & Bennison, 2018; Norton et al., 2019; Ramful & Narod, 2014; Stephan et al., 2016), how MERs engage in and experience such collaborations is understudied. MERs use approaches, or what we will call practices, to conduct their work. Investigating practices used by MERs in interdisciplinary collaborations will provide additional insights into how MERs can develop and participate in interdisciplinary collaborations (Bakker et al., 2021; Cai et al., 2020). In the next section, we introduce our conceptual framework: disciplinary practices.

Conceptual framework: Disciplinary practices

Practices are a defining characteristic of disciplines and disciplinary work (Williams et al., 2016). They are methods or techniques that become tacit, spontaneous, and automatic ways of operating or knowing (Schön, 1983; Wenger, 1998). Goodwin (1994) asserted that practitioners of disciplines negotiate and build "socially organized ways of seeing and understanding events" (p. 606) that we call practices. The author explored practices in archaeology and law and described how practitioners use disciplinary discourse and techniques to shape and study events. Goodwin's work highlights how practices are learned over time, and practitioners hold each other accountable for properly executing disciplinary practices.

In mathematics education, researchers typically develop practices initially through their doctoral programs. For example, Sherin (2014) described her graduate study experiences as the process of becoming specialized in seeing mathematics classrooms as objects of study using the framework of noticing. Sherin's (2014) experiences and ongoing work in noticing provide evidence of how mathematics education research practices learned in graduate school are developed through conducting and publishing research. During or after graduate studies, MERs engage in the mathematics education research community to publish their work to achieve advancement in rank (Andrade-Molina et al., 2020). Mathematics education journal requirements and rankings influence the production, dissemination, and socialization of mathematics education research practices.

In earlier work, we defined practices as "established or emergent ways of being, operating, and interacting with others within a community" (Suazo-Flores et al., 2021a, p. 24). Using this definition, we explored practices from interdisciplinary collaborations described in peer-reviewed journal articles and found that MERs engaged in practices such as *working towards research interests*, *cultivating trust and open-mindedness*, and *understanding of institutional support*. Building from this work and using additional interview data, we expanded our definition of practices, and further described *being*, *interacting*, and *operating* as three ways to categorize practices:

> *Being* […] refers to MERs describing their view of themselves and others in the interdisciplinary group including specific roles taken on by group members. *Operating* […] means members' ways of doing in the interdisciplinary group and acknowledging institutional policies and actions in order to complete the work. *Interacting* is […] developing communication standards, negotiating the meaning of ideas that allows the group to collaborate, and explaining work to people outside of the group. (Suazo-Flores et al., 2021b, p. 827, italics added)

Although practices are categorized as *being*, *interacting*, and *operating*, we found that the practices were not mutually exclusive (Suazo-Flores et al., 2021b). For example, MERs' comfort with new vocabulary and frameworks could depend on how they see themselves as MERs, which is an example of how *interacting* and *being* practices are interconnected. The categories of *being*, *interacting*, and *operating* are useful in describing MERs' experiences in interdisciplinary collaborations. We elaborate on each category of practice using existing research.

Being practices

Being practices are associated with the way a member of an interdisciplinary collaboration views their identity and the identity of others (e.g., Suazo-Flores et al., 2021b). We conceptualize identity as non-static, existing in the moment of performance and recognizing characteristics of the self (Darragh, 2016; Gutiérrez, 2013). Identities are central artifacts of disciplinary practices (c.f. Engeström, 2001; Grossman et al., 2009; Williams et al., 2016). For instance, MERs' interactions with others are influenced by MERs' life experiences (Foote & Bartell, 2011; Martin et al., 2010), status, and agency (Lande & Mesa, 2016). MERs' perceptions of self and others influence MERs participation in interdisciplinary collaborations (Wenger, 1998). At the same time, MERs' identities evolve through their work, the groups they join, and the stories they tell about their research and teaching experiences (Darragh, 2016). We have experienced an evolution in our practices after working with practitioners of other disciplines (e.g., Alyami & Bryan, 2022; Gardner et al., 2021; Kastberg & Xin, 2023).

Interacting practices

Interacting practices are ways of communicating and negotiating within and outside an interdisciplinary group (e.g., Suazo-Flores et al., 2021b). Disciplines include different discursive practices to conduct and document work (Williams et al., 2016). For example, Groth (2015) noted that the nature of discourse within the disciplines of mathematics and statistics differs. "Much mathematical discourse is grounded in deductive reasoning and language of definitive proof, whereas the statistical discourse is often characterized by inductive reasoning and qualified conclusions" (Groth, 2015, p. 4). In mathematics education, MERs use learning theories and frameworks to conduct research in mathematics classrooms. Such theories and frameworks could be obstacles when working with practitioners from other disciplines. For instance, mathematicians in Darragh (2022) described how not knowing educational

theories hampered their collaboration with MERs. Moreover, communication is critical in interdisciplinary collaborations. After working with practitioners from other disciplines, Bruce et al. (2017) suggested studying "discipline-specific vocabularies and methodologies" (p. 168) and interacting in ways that allow for listening, valuing, and taking perspectives on what is being said. *Interacting* practices allow members of interdisciplinary collaborations to consider different perspectives and develop common vocabulary and frameworks to communicate in interdisciplinary groups.

Operating practices

Operating practices convey members' common goals and interests, ways of conducting and doing work, and the acknowledgment of institutional policies influencing interdisciplinary work (e.g., Suazo-Flores et al., 2021b). MERs have identified working on problems at the intersection of members' interests as a productive way of operating in interdisciplinary groups (Akkerman & Bakker, 2011; Bruce et al., 2017). For instance, developing mathematics teacher education curricula is a common area of interest for MERs and mathematicians (e.g., Darragh, 2022; Goos & Bennison, 2018). Also, providing better opportunities to learn mathematics and science content and practices has been the focus of interest of MERs and science and special educators (e.g., Gardner et al., 2021; Kastberg & Xin, 2023; Ramful & Narod, 2014; Thompson et al., 2013). Therefore, having a common research topic and working towards a common interest is a way of *operating* in interdisciplinary collaborations. MERs have also used structures and procedures to conduct interdisciplinary work. For example, Bruce et al. (2017) described using a "two-way flow of information" (p. 156) where members of the groups co-present and cross-cite their work as a productive way to gain insight into different discipline-based understandings.

Given that our goal is to explore practices MERs report using in interdisciplinary collaborations, we use the conceptualization of practices as ways of *being*, *interacting,* and *operating* to study MERs' experiences in interdisciplinary groups. Our goal is to further unpack *being*, *interacting*, and *operating* practices to provide research-based examples of practices that would allow others to have a starting point when engaging in interdisciplinary collaborations.

Methods and analysis

Participants and context

This research explored practices as part of a larger project focused on describing MERs' lived experiences when working in interdisciplinary collaborations (Suazo-Flores et al., 2021a, b). Participants were recruited based on having published research in peer-reviewed journals as part of an interdisciplinary collaboration that included at least one MER or by self-identifying as a MER who was part of an interdisciplinary collaboration. The recruitment included asking potential MER participants to share their experiences working in an interdisciplinary collaboration during a recorded interview.

Through the recruitment, four MERs from three different projects volunteered to participate in this study: Amelia, Ian and Alexis, and Iris (pseudonyms). Amelia was a university faculty member as well as a primary developer and leader of her interdisciplinary project. Her project involved working with mathematics educators, mathematicians, science educators, and scientists to improve the curriculum and pedagogy for university pre-service mathematics education programs. Ian and Alexis collaborated on a project to create engineering tasks that would allow students to learn new mathematics content. Ian was a graduate student, and his dissertation research focused on developing curricula to learn mathematics through engineering tasks. Alexis was a university faculty member and Ian's advisor, who also worked on the project. They worked with engineers, scientists, and mathematicians to develop the tasks. Iris was a university faculty member and leader of a project with mathematicians and mathematics teacher educators. Her project developed curricular modules to help high school students learn advanced mathematics content.

Data source and analysis

To collect data on practices MERs used in interdisciplinary collaborations, we conducted three semi-structured interviews (Kvale, 1996) with the four MERs (Ian and Alexis were interviewed together). Each interview was audio-recorded and lasted approximately 90 min. Interview questions focused on encouraging MERs to report practices that developed and were used during their interdisciplinary collaborations. The following are examples of questions that we asked each of the MERs: Can you tell us how your interdisciplinary group was formed? Were there any routines that developed as your group worked together? Were there any central concepts or theoretical ideas that were critical elements for your group and how did the group come to agreement on these? Can you describe situations when working in an interdisciplinary collaboration was an asset or a constraint? Data analyzed for this research are the transcriptions of the three recorded interviews.

Developing a codebook: Grounded theory analysis

We used our preliminary definition of practices as existing or emerging ways of *being*, *interacting*, and *operating* to identify the pieces of transcripts representing such a definition. To further conceptualize specific practices within the categories of *being*, *interacting*, and *operating*, we used grounded theory (Charmaz, 2005). Key ideas were identified within each category of practices, which led us to create a codebook. We used an inductive approach to analyze the data and develop descriptions of specific practices in each category based on patterns in the data. Our first completed interview was with Amelia, so the initial analysis was done in three phases using Amelia's transcript (Suazo-Flores et al., 2021b). In the first phase, we used our general definition of practices to note instances where Amelia described specific types of practices her interdisciplinary group used. Each instance of a practice was identified as *being*, *interacting*, or *operating*. For example, the following excerpt was coded as a practice in the *operating* category.

> Amelia: I guess everyone has been in a situation where we realize that, what we want to work on, improving [mathematics] teacher education is a shared problem. We've just been working on it separately previously so that having, being able to identify something that we actually care about the same thing.

This quote was identified as a way of *operating* because it provided evidence of Amelia's view of the members of the group having a common goal or interests, in this case, a shared problem to improve mathematics teacher education.

In the second phase, we used descriptive phrases for practices in the *being*, *interacting*, and *operating* categories. We grouped responses that described specific practices with common themes (e.g., holding regular meetings, and views of other members in the group) and edited the descriptive phrases to summarize the practices in each category in the code book. These edited descriptive phrases became codes used to identify specific practices within *being*, *interacting*, and *operating* categories. For instance, in the transcript shared above, the descriptive phrase *common goal* was used because Amelia referred to working on this problem together and the members "car[ing] about the same thing." Examples of each type of practice were added to aid with ongoing coding.

Once the analysis reached a point where each practice from the transcript aligned with a code, the third and final analysis phase began. In this phase, we used the category definitions, codes for practices, and examples of practices to re-code Amelia's transcript, identifying all practices that she shared during her interview. The three iterations of analysis resulted in a codebook with definitions of the practice categories improved with evidence from data, codes, and descriptions for practices within each of the three categories, and examples of each of the different practices.

Using codebook: Content analysis

Once the codebook was developed, we used a deductive approach to code all three interviews and analyze the transcript data consistent with qualitative content analysis (Mayring, 2015). For reliability, each transcript was coded by a member of the research team using the codebook and then checked by a second member of the research team. When coding differences between researchers were found, the coded practices were discussed by the entire research team to share reasoning and clarify codes. Once we reached an agreement regarding the use of a code or its definition, the codebook was updated. Then, the coded items in each of the transcripts were reviewed for alignment with the updated codebook (c.f., Mayring, 2015).

Following the final coding of the three transcripts, frequency tables were created to identify how often each practice occurred in the data (see Table 1). The frequencies of the coded practices were disaggregated by projects. For example, there were 22 instances where the *being* practice *view of others* was coded across the three interview transcripts. Disaggregation showed six codes from Amelia's transcript, nine from Ian's and Alexis' transcript, and seven from Iris' transcript. The frequency table allowed for the identification of practices that were common among all of the participants. This research presents the five practices that were reported in each of the three transcripts at least three times.

Table 1 Frequency table for common practices

Category	Practice	Amelia	Ian and Alexis	Iris	Total
Being	*View of self*	4	6	20	30
	View of others	6	9	7	22
Interacting	*Developing ideas*	6	7	3	16
Operating	*Common goals*	9	5	5	19
	Working together	4	3	4	10

Findings

Our analysis identified five practices MERs reported using in interdisciplinary collaborations: *view of self*, *view of others*, *developing ideas*, *common goals*, and *working together*. The first two practices are in the *being* category, the third practice is in the *interacting* category, and the last two practices are in the *operating* category. Below, we describe each practice and use examples from the data to illustrate the category.

Being: View of self

View of self was coded as a type of *being* practice. This practice involved MERs describing their individual identity, including their dispositions, interests, sense of efficacy, and roles to engage with others in the group's work. Participants provided evidence of recognizing their roles and dispositions in their interdisciplinary collaborations. For example, Ian indicated that he "was the central person" who, at the beginning of the project, "was trying to meet the needs of so many people," which we interpreted as an acknowledgment of his role in the group. In Iris's research group, she described how she was "not a mathematician" but brought her vast teacher education background to the project: "I have lots of experience in working with teachers." Amelia reflected on her interdisciplinary experiences: "It's been a wonderful growing experience. So, I don't feel like I've lost anything because I still have my life in my discipline, and I have a much enriched and expanded life as well by having these experiences that I didn't realize I was going to get." Amelia's quote provides evidence of how she saw herself still being part of the mathematics education discipline and having learned and grown as an MER after being part of interdisciplinary collaborations.

Being: View of others

View of others was coded as a *being* practice. This practice involved MERs describing how they perceived others as part of the interdisciplinary collaboration. The code *view of others* was used when participants described others in the group as members of a discipline, as taking on special roles in the group, or as influential due to their added diversity of knowledge or experience. For instance, Iris described a

member of her interdisciplinary collaboration as having a "very strong mathematics background" with experience in the "high-tech industry" and "a high school teacher." Iris acknowledged the member's expertise and experiences and appreciated that she and this member were both committed to "narrowing the gap between school mathematics and contemporary mathematics," which was the main goal of the interdisciplinary project. As a member of Ian's interdisciplinary collaboration, Alexis described roles and expertise of other members of the collaboration. Alexis stated, "The primary role that the team members played was to bring their expertise to the table," and she noted that "Ian really was the lead in designing the material or coming up with the context and coming up with the activities".

Interacting: Developing ideas

Developing ideas was coded as an *interacting* practice. This practice involved MERs' descriptions of how members of the interdisciplinary collaborations negotiated the meaning of ideas, representations, or frameworks. When MERs described developing understandings and common definitions that allowed members of the group to collaborate and communicate with external audiences, we coded those instances as *developing ideas*. For example, Amelia indicated the importance of group members developing ways of communicating. Amelia described that given the different areas of disciplinary expertise, group members needed "to be good listeners and very respectful and making it possible for questions to be asked and to be able to respond to those questions in a way that is serious and takes things seriously." In another example, Iris indicated that she and a mathematician in the group had "arguments on the buildup, on the [mathematics content] that [the mathematician] thought would be very accessible." Iris and the mathematician had discussions to unpack the mathematics content and make it accessible to students. She explained that such discussions allowed them to develop new ways to represent mathematics to external audiences. "I don't think I would be able to do it on my own, and I don't think [the mathematician] would be able to do it on his own".

Operating: Common goals

Common goals was categorized as an *operating* practice. This practice involved the use of common goals in "ways of doing" for the interdisciplinary collaborations. Participants described this practice when they shared purposes or common goals used by the group to focus on the work being done. *Common goals* was used as a code when (1) the group goals were described by a participant, (2) the goals were established, and (3) the goals were used to focus the work being done. For instance, Amelia's project goals were to foster interdisciplinary collaborations and "design, implement, and evaluate new approaches to teacher education." Those two goals kept the group together. Similarly, Ian described how his group's purpose was to create a curriculum for the "learning of new mathematics and new engineering at the same time."

Operating: Working together

Working together was categorized as an *operating* practice. This practice involved representing "ways of doing" for a group. *Working together* was how an interdisciplinary group conducted work as a team, in other words, a process the group's members used to get work done. Iris' interdisciplinary collaboration included a process where the mathematician would "set up a storyboard" to tell the story around the targeted mathematics concept. Later, Iris and the mathematician worked together to add the "know-how" pieces, which were procedures or ways of thinking involved with the targeted mathematics concept. This process resulted in a new story around the mathematics concept that began "with a question instead of starting with declarations." In Amelia's interdisciplinary collaboration, she described how different groups had tried different approaches, or systems, to teacher education. The interdisciplinary collaboration included sharing different systems already being used, which resulted in a menu of items for all members to consider. According to Amelia, "Then we each picked something from that menu that we decided to try this approach next year, and then we swapped and learned from each other." The group developed a way of *working together* to learn from each other by picking different approaches from the menu.

Discussion and implications

In this research, we addressed Bakker et al.'s (2021) and Cai et al.'s (2020) calls for additional insights into sustainable interdisciplinary collaborations by identifying five practices MERs reported using. Two practices were from the *being* category, one from the *interacting* category, and two from the *operating* category, respectively: *view of self*, *view of others*, *developing ideas*, *common goals*, and *working together*. Such practices add to the literature on how MERs experience interdisciplinary collaborations (e.g., Bruce et al., 2017; Goos & Bennison, 2018).

Disciplines have domains of knowledge and practices that differentiate them from other disciplines (Pimm, 2009; Roth, 2014; Williams et al., 2016). In the specific case of mathematics education, MERs received preparation to use mathematics education research practices such as using frameworks (e.g., Sherin, 2014). MERs in our study provided evidence that their work in interdisciplinary projects involved more than their expertise in mathematics education research practices. Our findings demonstrate that engaging in interdisciplinary collaborations can also involve adapting existing disciplinary practices and creating new practices in collaboration with members of interdisciplinary groups. For example, the data include examples where MERs used *being* practices to recognize themselves and others in the project, *interacting* practices to communicate within the interdisciplinary collaboration, and *operating* practices to conduct the interdisciplinary work. The *being* practices reflect how MERs saw themselves in relation to others and how they perceived others in their groups. The practices of *view of self* and *view of others* allowed MERs to navigate work with others by situating themselves and others in the project, acknowledging differences, and seeing differences as assets. The *interacting* practice, *developing*

ideas, involved the development and use of discourse norms within interdisciplinary groups to listen to each other's perspectives and build common understandings. The *operating* practices of *common goals* and *working together* described ways of doing in interdisciplinary collaborations. Members of interdisciplinary collaborations worked towards goals that met their interests and aligned with the interdisciplinary project. They also developed strategies for getting work done, like creating a storyboard or picking from a menu, that built on the disciplinary expertise and backgrounds of the group members.

Our findings also expand existing research on the use of practices in interdisciplinary collaborations (Bruce et al., 2017; Goos & Bennison, 2018) and our previous work (e.g., Suazo-Flores et al., 2021b) by providing research-identified examples of practices. Acknowledging that mathematics education as a discipline evolves due to the work of its practitioners (e.g., González et al., 2001; Gutiérrez, 2013; Lerman, 1996, 2000; Steffe & Thompson, 2000), our findings offer five practices MERs can use when engaging in interdisciplinary collaborations. For our participants, developing and using these five practices allowed them to navigate interdisciplinary collaborations successfully. Below, we discuss implications in the context of existing research which is organized using the three categories for practices.

Being practices

Research has communicated how MERs' backgrounds and ways of being impact their work (e.g., de Freitas & Sinclair, 2013; Foote & Bartell, 2011; Lande & Mesa, 2016; Martin et al., 2010). MERs' backgrounds could restrict their interactions with practitioners from other disciplines when working in interdisciplinary collaborations. Our study shows that it is important for MERs to understand and characterize their role and interdisciplinary identity with respect to their existing identity, which includes perceptions of self as MERs (Darragh, 2016). For instance, Amelia reported, "I still have my life in my discipline," which shows how Amelia recognized herself as a MER after working in interdisciplinary groups. Moreover, Amelia's work in interdisciplinary groups shifted her view of self as an MER, "I have a much enriched and expanded life." Amelia provided evidence of how she viewed her interdisciplinary experiences as enhancing her life as a MER.

On the other hand, MERs should consider the roles of others in interdisciplinary collaborations. Addressing significant research problems through interdisciplinary work takes the skills of multiple experts; as Iris recognized, "I don't think I would be able to do it on my own, and I don't think he [the mathematician] would be able to do it on his own." Alexis described how Ian acknowledged each team member's disciplinary expertise with respect to his role. Recognition of team members' roles and expertise contributed to the success of his collaboration. Recognizing and leveraging the diversity of expertise among members is an asset that can support the successful completion of interdisciplinary projects. Thus, we see *being* practices as exemplifying Darragh's (2016) definition of identity in the context of interdisciplinary collaborations, which involves how we position ourselves and others.

Interacting practices

Interdisciplinary work involves communication and collaboration (Bruce et al., 2017); therefore, there is a need to develop ways of communicating to negotiate and develop a common language to be used internally and externally (e.g., Suazo-Flores et al., 2021b). During the interviews, Iris described the need to discuss ideas with colleagues from different disciplinary backgrounds to construct new knowledge. Amelia's description of team members' approaches to interactions as "good listeners and very respectful" aligns with descriptions from Bruce et al. (2017) of team members' dispositions to gain perspective on each other's work.

Practitioners of disciplines have specialized discourse (Williams et al., 2016) that can hamper communication in interdisciplinary collaborations (e.g., Bruce et al., 2017; Pimm, 2009). Members of interdisciplinary collaborations should recognize communication challenges created by disciplinary discourse and expertise and work to develop common meanings for frameworks, keywords, and phrases related to the group's tasks. We found that the common language developed by the members of the interdisciplinary collaborations was unique to the group and the project's final product. For instance, Iris reflected that the group had discussions to develop representations of their work product to others. Iris' interactions with colleagues in mathematics and their considerations of how external audiences (in this case, students in secondary classrooms) would use the project was instrumental in developing a way of communicating the work. *Developing ideas* is an *interacting* practice that can help develop discourse around key terms that will support MERs in interdisciplinary collaborations.

Operating practices

Our findings are consistent with others who wrote about MER practices (Akkerman & Bakker, 2011; Bruce et al., 2017; Goos & Bennison, 2018) in that working toward shared problems or common goals is a productive practice for MERs in interdisciplinary collaborations. Common goals should be developed during the early stages of interdisciplinary work to help ensure that each group member from different disciplinary backgrounds can contribute to the interdisciplinary collaboration and receive recognition for their work from their respective disciplines. The development of the goals could be considered an *interacting* practice, since group members are negotiating the goals to conduct and document work considering the different disciplines and related practices. Once the common goals are established, they become an *operating* practice guiding the individual and collective work of the group.

Our study extends existing research that has explored interdisciplinary groups (Bruce et al., 2017; Goos & Bennison, 2018) as we further unpacked the practice of *working together*, which includes specific ways of operating in interdisciplinary groups. The practice of *working together* that emerged from this study adds to what Bruce et al. (2017) described as the "two-way flow of information" (p. 156). For instance, Iris and her group used the storyboarding system to align mathematics

content with know-how prompts that would allow the audience to have entry points to engage. Amelia and her group developed menus with existing curricular examples, or systems in place, instead of a mandatory system, to share and learn from each other's expertise. In these examples, *working together* involved systems that organize work to be done and likely help the group to be more efficient. *Working together* can include other systems like taking notes and assigning tasks, also designed to keep interdisciplinary collaborations organized and efficient.

Future directions

To address significant questions in mathematics education, MERs are joining interdisciplinary collaborations, and with this, they contribute to evolving forms of discourse and work (Williams et al., 2016). Yet, MERs could be dissuaded from working in interdisciplinary collaborations because of communication challenges or if the interdisciplinary work is not considered for promotion. MERs who learn how to work beyond the discipline productively can expand their expertise and extend the borders of mathematics education as others have. This research offers five practices MERs can use to initiate and increase the likelihood of success in interdisciplinary collaborations.

The five practices show evidence that interdisciplinary collaborations are more than every member contributing their expertise. The *being* and *interacting* practices are evidence of personal involvement and the emergence of new discourse that facilitates the sense of belonging and communication in interdisciplinary groups. We call for the recognition and enactment of practices as a social phenomenon (Williams et al., 2016), which implies that disciplines are made by people who, through communication and personal effort, can contribute to evolving and expanding practices. Because practices are learned over time while conducting and publishing mathematics education research (e.g., Sherin, 2014), we advocate for professional development spaces where MERs can reflect on practices that are ways of *being*, *interacting*, and *operating* in interdisciplinary collaborations. Building from our findings, we share the following questions which can be used to guide such reflections:

- How are the proposed project's goals aligned with MERs' professional interests?
- What is my contribution to the interdisciplinary collaboration, and how am I considering others' expertise?
- What key terminology (e.g., modeling, learning, and data and data analysis) am I hearing or using, and how can I create a space to learn how those terms are understood in different disciplines?
- In what ways am I acting and recognizing that new ideas in interdisciplinary collaborations will emerge dialogically? Am I allowing myself to gain new perspectives on mathematics education?
- How can I find ways to align my interdisciplinary work with my institution's organizational structures and reward systems?

This research focused on identifying practices MERs reported using in interdisciplinary collaborations. Further study is needed on how these practices can be used in disciplinary-specific collaborations (e.g., a group consisting only of mathematics educators). We acknowledge that the practices shared in this research are based on interview data with leaders of productive interdisciplinary collaborations. MERs, as members of interdisciplinary collaborations led by others, may experience practices not captured in this study. Additional research is also needed to understand practices from the perspective of people who are not leaders or in less productive interdisciplinary collaborations that include MERs. An additional limitation is due to the data for this research consisting of interview transcripts from one interview with four MERs. A longitudinal observational study with more MERs could provide different evidence for the breadth of MERs' experiences in interdisciplinary collaborations. We also did not study the role of MERs' race and ethnicity and how these influence MERs' interpersonal relationships and practices in interdisciplinary collaborations, another area that needs study.

Acknowledgements The authors are grateful for the time participants invested during the data collection of this study.

Funding The authors declare the research study that underpins this publication was conducted without funding.

Declarations

Ethics approval The research study that underpins this publication was provided by Purdue University, registration number 1904022085. The participants were recruited, consented, and interviewed online.

Competing interests The authors declare no competing interests.

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Authors and Affiliations

Elizabeth Suazo-Flores[1] · William S. Walker III[2] · Signe E. Kastberg[2] · Mahtob Aqazade[3] · Hanan Alyami[4]

✉ Elizabeth Suazo-Flores
Elizabeth.suazo@und.edu

William S. Walker III
wswalker@purdue.edu

Signe E. Kastberg
skastber@purdue.edu

Mahtob Aqazade
mahtob.aqazade@tamucc.edu

Hanan Alyami
alyamih@pfw.edu

[1] University of North Dakota, Grand Forks, ND, USA

[2] Purdue University, West Lafayette, USA

[3] Texas A&M University-Corpus Christi, Corpus Christi, USA

[4] Purdue University Fort Wayne, Fort Wayne, USA