CONCEPTUAL AND PROCEDURAL INSTRUCTION IN MATHEMATICS: A CONTENT ANALYSIS FROM 1970 TO 2020

A Dissertation

by

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This dissertation meets the standards for scope and quality of
Texas A&M University-Corpus Christi and is hereby approved.

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ABSTRACT

Mathematics scores on the 2019 National Assessment of Education Progress increased by only two points for both fourth and eighth grade students since 2007, indicating that students continue to struggle in mathematics (National Center for Educational Statistics, 2007; U.S. Department of Education, 2019). The development of both conceptual and procedural knowledge is crucial to student success, as such, it is imperative that educators remain informed of past and current research in conceptual and procedural knowledge so they may make important instructional decisions regarding the focus and sequence of instruction.

This research employed a quantitative content analysis to analyze and describe the extent to which mathematics instruction that emphasizes conceptual knowledge, procedural knowledge, both conceptual and procedural knowledge, progression from conceptual to procedural knowledge, and progression from procedural to conceptual knowledge in mathematics have been positioned within the publications of the *Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM) research journals. Results indicate that research pertaining to conceptual and procedural knowledge remained consistent from 1988 to 2020, focusing mostly on conceptual knowledge development, incorporating constructivist practices that require students to make connections, notice patterns and relationships, and explain their thinking. The second most common focus was on developing both conceptual and procedural knowledge, incorporating both behaviorist and constructivist practices.

DEDICATION

This dissertation is dedicated to my amazing husband, Larry, who has been a constant source of encouragement and enthusiasm during this challenging journey. I am beyond grateful for having you in my life. Thank you for being my source of strength when things seemed insurmountable, for your unending patience when I was stressed, and most of all for your everlasting love and support.

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CHAPTER I: INTRODUCTION

For over a century, researchers have endeavored to understand the ways in which students learn. Research has focused on how learning occurs, the factors associated with determining when learning has occurred, instructional strategies that lead to increased learning, and contextual issues that impact the learning process (Brown, 2014). Learning theories such as behaviorism and constructivism seek to describe how prior knowledge impacts new learning, how learners construct knowledge, and how such knowledge is stored. However, how does this knowledge of theory translate into optimal educational practices? Effective instructional design can serve as a bridge between theory and practice (Ertmer & Newby, 2013). According to Ertmer and Newby (2013), "learning theories are a *source* of verified instructional strategies, tactics, and techniques" and "provide a foundation for intelligent and reasoned strategy *selection*" (p. 44). Educators and instructional designers are tasked with interpreting these learning theories and translating them into practical instructional activities. Yet, as Poncy et al. (2010) note, there is great controversy among educators as to the most effective approach to instruction and student learning.

Behaviorism is a theory of learning in which the behaviorist attempts to identify the principles that underly an individual's change in behavior (Watson, 1924). Behaviorist instruction emphasizes direct instruction of skills whereby the teacher explicitly explains concepts and provides learners with solution strategies that promote success (Poncy et al., 2010). "Behaviorism treats the learner as an agent who builds competency through various stages of behavior change" (Booyse & Chetty, 2016, p. 139). This theory of learning suggests that students should be provided with opportunities for repeated skill practice and reinforcement of correct responses (Poncy et al., 2010). However, as Magliaro et al. (2005) note, it is not a lecture-based approach to learning, rather it is an approach that "focuses on the interaction between teachers and students" (p. 41). One such behaviorist instructional practice is known as direct instruction, sometimes referred to as explicit instruction. This teacher-directed, explicit instruction scaffolds support and provides practice that is strategically reduced until independence is attained (Archer & Hughes, 2011). This is accomplished by means of detailed teacher modeling, guided student practice, and teacher monitored independent practice, and is often associated with procedural knowledge development. Procedural knowledge, often referred to as instrumental understanding, is frequently described as a learner's knowledge of rules or solution strategies without necessarily understanding the reasons as to why such rules and strategies work (Skemp, 2006). Early proponents of the behaviorist theory include Pavlov (1960), Watson (1924), and Skinner (1938). Pavlov and Watson pioneered work in what came to be known as conditional reflex. Conditional reflexes are the low-level processes developed out of habit and associated with visual cues that connect new information with the old and help to develop primary memory (Greenspan & Baars, 2005). Later, Skinner studied a second form of conditioning known as operant conditioning. Operant conditioning is a behavior theory which posits that a correct response is required for reinforcement, whether the reinforcement is a reward or the avoidance of punishment (Ewen, 2010).

In contrast to behaviorism, constructivism suggests that meaning and knowledge construction is obtained through active involvement rather than receiving information passively (Xin et al., 2016) and is commonly implemented through inquiry-based practices. According to Gordon (2009), learners "actively create, interpret, and reorganize knowledge in individual ways" (p. 738). Constructivists propose that by building their own knowledge through meaningful experiences, students can build on prior knowledge and apply their new learning to their daily life (Bermejo et al., 2021; Booyse & Chetty, 2016), thus supporting conceptual knowledge development. Conceptual knowledge, or relational knowledge, refers to understanding the underlying concepts and unifying principles, which in turn, allows the learner to apply their learning to new situations (Canobi, 2009; Rittle-Johnson et al., 2015). Early proponents of constructivism include Dewey (1958), Bruner (1960), Piaget (1973), and Vygotsky (1978) (Booyse & Chetty, 2016; Brown, 2014). Dewey and Bruner suggest that the exploration of ideas and connections between concepts play an important role in student learning. Similarly, Piaget posited that the mind processes new information through a logical construction of knowledge that progresses through four stages of development: sensorimotor, preoperational thought, concrete operations, and formal operations (Piaget, 1973; Pulaski, 1980; Wadsworth, 1988). Furthermore, Vygotsky's *zone of proximal development* supports the learning theory of constructivism as students are expected to solve problems that lie just beyond their current level of development with the aid of teacher and peer collaboration (Brown, 2014; Brown et al., 1996).

The work of constructivists, along with growing concern about student performance in mathematics, spurred reform efforts in education (Xin et al., 2016). One reform effort spurred by the lack of student progress was that of the American Recovery and Reinvestment Act of 2009 (ARRA) and provided $4.35 billion for a grant program known as Race to the Top (Popham, 2020; U.S. Department of Education, 2009; U.S. Department of Education, 2015). The Race to the Top program rewarded states for innovative practices and reform efforts aimed at increasing student achievement and teacher effectiveness (U.S. Department of Education, 2009; U.S. Department of Education, 2015). Such practices included: 1) adopting college and career readiness standards, 2) building assessments that measure student growth, 3) tying student achievement to teacher evaluations, and 4) improving the lowest performing schools. An additional reform effort was the creation of the Common Core State Standards (CCSS). The Common Core State Standards, created in 2010, at the behest of state governors and commissioners of education from 48 states, two territories, and the District of Columbia. College and career readiness, as well as kindergarten through twelfth-grade standards were developed by a team of teachers and standards experts from across the country (Common Core State Standards Initiative, 2021). The CCSS placed high importance on the development of conceptual understanding though modeling, sense making, and reasoning (Xin et al., 2016).

Evidence of the lack of student success in mathematics that spurred these reform efforts can be seen by student performance on both the National Assessment of Educational Progress (NAEP) and the Trends in International Mathematics and Science Study (TIMSS). For example, in 2007, the National Center for Educational Statistics (NCES) reported that 61 percent of 4^{th} grade students and 68 percent of 8^{th} grade students scored below proficient in mathematics on the National Assessment of Educational Progress (NAEP). According to the U.S. Department of Education (2019), "NAEP is a congressionally mandated project of the National Center for Education Statistics (NCES)" provides "educators, policymakers, and parents with a common measure of student achievement that allows for direct comparisons among states and participating urban districts" (p. 2) and began assessing in 1973. Students in the United States are assessed in grades 4, 8, and 12 in a variety of subjects, including mathematics (U.S. Department of Education, 2019) and results are published as The Nation's Report Card. According to the U.S. Department of Education (2019), the NAEP assessment measures students' mathematics knowledge as well as their ability to apply their knowledge in problem-solving situations. In mathematics, students are assessed on number properties and operations, measurement, geometry, data analysis, statistics and probability, and algebra (U.S. Department of Education, 2019). Questions are written at three levels of complexity (low, moderate, and high), requiring various amounts of both procedural and conceptual knowledge. Low complexity questions focus on recall or recognition of procedures and concepts. Moderate level questions require students to decide what needs to be done and how to accomplish it. High complexity questions require students to reason, plan, analyze, make judgements, and think creatively (U.S. Department of Education, 2019). The assessment is written with the intent that approximately half of the assessment time is spent on questions of moderate complexity and the remainder of the time spent equally on low and high complexity questions, resulting in approximately 75% of the assessment requiring the use of conceptual knowledge and approximately 25% of the assessment requiring the use of procedural knowledge (see Figure 1). However, NAEP cautions that the balance of complexity types on the assessment are not necessarily the balance teachers should aim for regarding mathematics curriculum and instruction.

Figure 1

NAEP Student Actions and Assessment Time

Student Action	Percentage
Students decide what needs to be done and how	50%
Students reason, plan, analyze, make judgements, and think creatively	25%
Students recall and recognize procedures and concepts	25%

75% Conceptual Knowledge

Additionally, 2003 and 2007 results from the Trends in International Mathematics and Science Study (TIMSS) indicated that students in the United States were failing to compete internationally in mathematics (Grady et al., 2018). The TIMSS assessment began in 1995 and assesses students in fourth and eighth grades in mathematics and science, providing a common measure of student achievement and allowing comparisons between student performance in the United States and other countries. Fourth graders are assessed on number, measurement and geometry, and data. Eighth graders are assessed on number, algebra, geometry, and data and probability (Lindquist et al., 2017). In fourth grade, approximately 40% of the questions focus on recall of facts, concepts, and procedures, 40% focus on applying conceptual understanding to solve problems, and 20% require students to solve problems with unfamiliar contexts and situations and with multi-step solutions, resulting in approximately 60% of the assessment requiring conceptual knowledge (see Figure 2). In eighth grade, 35% focus on recall of facts, concepts, and procedures, 40% focus on applying conceptual understanding, and 25% require students to solve problems with unfamiliar contexts, in unfamiliar situations, and with multi-step solutions, resulting in approximately 65% of the assessment requiring conceptual knowledge (Lindquist et al., 2017) (see Figure 2).

Figure 2

TIMSS Student Actions and Assessment Time

Student Action	4th Grade	8th Grade
Students apply conceptual understanding to solve problems	40%	40%
Students solve problems with unfamiliar contexts and situations	20%	25%
Students recall and recognize procedures and concepts	40%	35%

60% to 65% Conceptual Knowledge

While many educators and theorists alike are divided on their support of constructivism and behaviorism, many endorse an instructional approach that creates a balance between the two. In 2008, the National Mathematics Advisory Panel put forth the claim that inquiry-based instruction, as promoted through constructivism, is not sufficient for student learning and recommended a balance between constructivist and behaviorist approaches to learning. One instructional strategy that seeks to strike a balance between inquiry-based instruction and direct, explicit instruction is that of the concrete-representational-abstract (CRA) instructional sequence (Xin et al., 2016). The CRA instructional sequence assists students in developing conceptual knowledge through guided cognitive activities by means of concrete models, such as manipulatives, as well as representational models, such as pictorials or drawings, alongside abstract notations, such as algorithms. Once students have demonstrated adequate conceptual knowledge, the concrete and representational supports are removed and students utilize only abstract notations, thus demonstrating procedural knowledge. Thus, learners are guided through a progression from conceptual knowledge development to procedural knowledge development.

Statement of the Problem

As mentioned previously, 61 percent of 4th grade students and 68 percent of 8th grade students scored below proficient in mathematics in 2007 on the National Assessment of Academic Progress (NAEP) (National Center for Educational Statistics, 2007). In 2019, the U.S. Department of Education reported that 59 percent of fourth-grade students and 66 percent of eighth-grade students scored below proficient in mathematics on the NAEP, an increase of only two percentage points in each grade level over a 12-year period of students demonstrating proficiency (see Figure 3 and Figure 4).

Figure 3

NAEP 2007 and 2019 Fourth-Grade Results

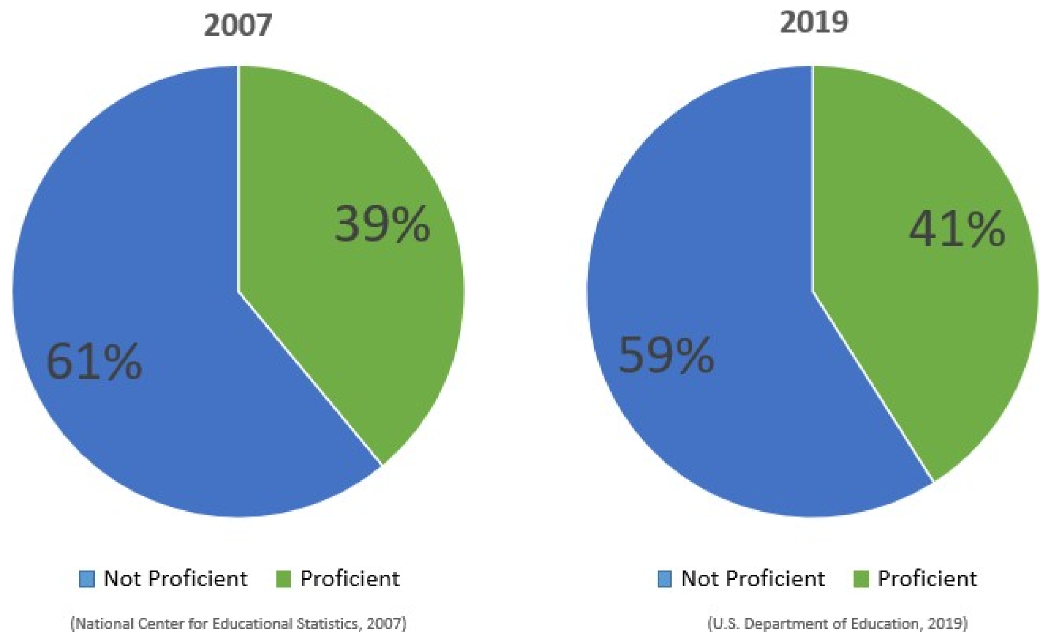
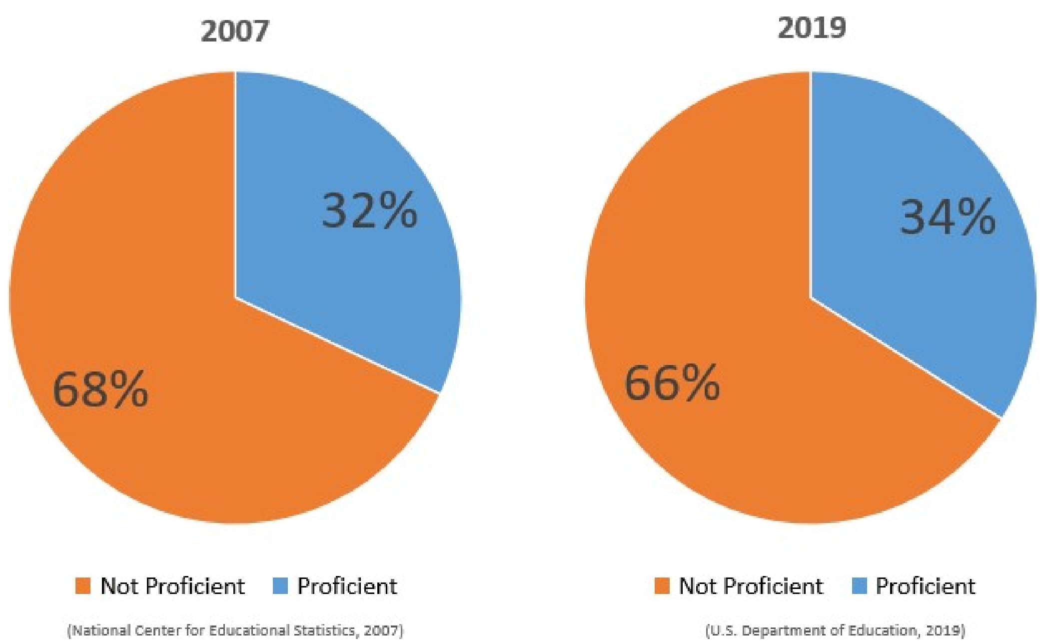


Figure 4

NAEP 2007 and 2019 Eighth-Grade Results



Additionally, 2019 Trends in International Mathematics and Science Study (TIMSS) results indicate that the United States ranks 15th among 64 international education systems for 4th graders and 11th among 46 international education systems for 8th graders, with no significant increase in scores for either grade level from the previous administration in 2015 (TIMSS 2019 U.S. Highlights Web Report). Thus, demonstrating that there has not been a significant increase in the number of students scoring proficient in mathematics since the implementation of the Common Core State Standards (2010) or Race to the Top grant program (2015). According to Gersten et al. (2009), a common hurdle for students' mathematical success is a lack of conceptual understanding of mathematical content, thus limiting their ability to apply their learning to new situations and preventing success with classroom activities and assessments. Conceptual knowledge development aids students in transferring their learning to new mathematical tasks and is important to student success (Skemp, 2006). Procedural knowledge development is also necessary, as it can be easier to understand when viewed within its own context and offers a reliable means to quickly complete mathematical tasks (Skemp, 2006). Furthermore, numerous studies have been conducted on instruction that progresses from conceptual to abstract (procedural) understanding, through concrete-representational-abstract (CRA) instruction, also known as concrete-pictorial-abstract (CPA) instruction or concreteness fading, and virtual-representational-abstract (VRA) instruction, finding that such instructional sequences increase student achievement in mathematics (Bouck, et al., 2017; Flores, 2010; Fyfe, et al., 2015; Hinton & Flores, 2019; Milton et al., 2019; Park, et al., 2020; Root, et al., 2020). As evidenced by the expectations for student actions on both the NAEP and TIMSS assessments, the development of both conceptual and procedural knowledge are imperative to student success. Additionally, student performance on the two assessments demonstrates that students continue to struggle in mathematics, therefore, this present research study seeks to investigate research in mathematics education pertaining to conceptual and procedural understanding dating back to 1970.

Purpose of the Study

The purpose of this research is to describe the extent to which mathematics instruction that emphasizes conceptual knowledge, procedural knowledge, both conceptual and procedural knowledge, progression from conceptual to procedural knowledge, and progression from procedural to conceptual knowledge in mathematics have been positioned within the publications of the *Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM), two prominent mathematics research journals, from 1970 to 2020. The study employs a quantitative content analysis research design.

According to Krippendorff (2019), "content analysis is an empirically grounded method, exploratory in process, and predictive or inferential in intent" (p. 1) that assists researchers in "making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use" (p. 24). Content analysis allows the researcher to examine printed matter, through systematic reading, in order to understand what the material may mean, enable, or convey. Through a content analysis, the researcher can infer trends and learn about the direction specific fields may be taking (Krippendorff, 2019).

Relevance of the Study

Research has been published in the field of mathematics education for half a century, as seen in the two most prominent mathematics research journals, the JRME and ESM. Additionally, there is an abundance of research reflecting the importance of developing both conceptual and procedural knowledge in mathematics (Baroody et al., 2007; Canobi, 2009;

NCTM, 2014; Rittle-Johnson et al., 2016; Rittle-Johnson et al., 2015; Strickland, 2017), yet, as stated previously and evidenced by student performance on the National Assessment of Education Progress (U.S. Department of Education, 2019), students continue to struggle. To ensure future student success, educators must ensure that evidence-based instructional approaches and strategies are implemented and that mathematical concepts are taught in the most effective manner possible. Therefore, it is imperative that educators remain informed of both the historical and current research on instruction and the development of conceptual and procedural knowledge. For educators to remain informed of effective instructional practices related to conceptual and procedural knowledge development, research must continue to be conducted in those areas. Therefore, this study is relevant to researchers in the field of mathematics instruction, as it identifies gaps in the research and pinpoints areas that can be further studied. Additionally, this study is relevant to educators as it can help identify the areas in which classroom instruction should focus.

Significance of the Study

The findings of this study provide a snapshot of the absence or presence of an emphasis on conceptual knowledge, procedural knowledge, both conceptual and procedural knowledge, progression from conceptual to procedural knowledge, and progression from procedural to conceptual knowledge in mathematics abstracted from 50 years of research in two prominent mathematics journals. The continued struggle of students in mathematics proficiency justifies the need for investigating the research on conceptual and procedural knowledge development. These findings are significant to researchers as they contribute to the discourse on the trends and current status of conceptual and procedural knowledge development and instruction that progresses between the two and are particularly important to researchers who wish to begin new research in areas that have limited prior research. Furthermore, the findings in this study are significant to mathematics teachers and instructional coaches. The results provide educators with insight into the direction their instruction may take to improve student success in mathematics. Educators can make informed instructional decisions as to the order in which the two types of knowledge may be developed and if one type of knowledge should receive more emphasis than the other. Finally, this study is significant to school district professional development coordinators as they can use the findings to develop a district or campus professional development plan, focusing on the areas of knowledge development prominent in the research.

Research Questions

The following research questions will guide this study:

1. How has mathematics instruction that emphasizes the following been positioned within prominent research journals from 1970-2020?
    a. only conceptual knowledge
    b. only procedural knowledge
    c. both conceptual and procedural knowledge
    d. the progression from conceptual to procedural knowledge, and
    e. the progression from procedural to conceptual knowledge
2. What textual units (keywords) related to conceptual and procedural knowledge are present within prominent research journals from 1970-2020?
3. How do research trends of the articles published in the journals vary from 1970 to 2020 (i.e., emphasis on procedural instruction only, conceptual instruction only, procedural and conceptual, or a progression from one to the other during certain time periods)?

Definitions

The following terms are used for this study:

- Behaviorism – a guided approach to learning that emphasizes direct instruction of skills whereby the teacher explicitly explains concepts and provides learners with solution strategies that promote success (Poncy et al., 2010).
- Conceptual Knowledge – understanding of underlying concepts and unifying principles, whether abstract or general, of a mathematical domain (Canobi, 2009; Rittle-Johnson et al., 2015). Conceptual knowledge is also commonly referred to as relational understanding (Skemp, 2006).
- Concrete-Representational-Abstract (CRA) Sequenced Instruction – sequence of instruction utilizing manipulatives, two-dimensional representations, and abstract numerals only. It is an iterative process with each phase following the same instructional cycle: the teacher models the concept, students practice the skill with guidance from the teacher, and students independently demonstrate an understanding of the concept (Flores, 2010; Forbringer & Fuchs, 2014). CRA is also referred to as concrete-pictorial-abstract (CPA) instruction and concreteness fading.
- Constructivism – active creation of knowledge through experiences, reflections, and/or social discourse (Clements & Battista, 1990; Yackel et al., 1990).
- Content Analysis – a research method that allows the researcher to examine printed matter, through systematic reading, to understand what the material may mean, enable, or convey (Krippendorff, 2004, 2019).

- Explicit Instruction – a direct and unambiguous method of instructional design and delivery that provides students with scaffolded support and practice until independence is attained (Archer & Hughes, 2011).
- Manipulative – object that an individual can handle in a multisensory way and fosters mathematical thinking, both consciously and subconsciously (Swan & Marshall, 2010).
- Procedural Knowledge – the skills, or knowledge of a series of steps, required to solve mathematical problems (Canobi, 2009; Rittle-Johnson et al., 2015). Procedural knowledge is also commonly referred to as instrumental understanding (Skemp, 2006).
- Virtual-Representational-Abstract (VRA) Sequenced Instruction – instruction that gradually moves students from virtual manipulative use to pictorial representations to abstract numerical representations (Bouck, et al., 2017; Bouck, et al., 2019; Park, et al., 2020).

Summary

Success in mathematics requires that students develop both conceptual and procedural knowledge (LeFevre et al., 2006; Riddle-Johnson et al., 2015). Additionally, the National Council of Teachers of Mathematics (NCTM) (2014) states that "effective mathematics teaching focuses on the development of both conceptual understanding and procedural fluency" (p. 42). However, there are many theories about how such knowledge is developed in mathematics and what the ideal sequence of development should be. Behaviorists suggest that knowledge is developed through direct, explicit instruction of skills, followed by both teacher-guided and independent practice of like problems. In contrast, constructivists posit that learners create knowledge through inquiry-based practices that require students to build upon prior learning and make connections to new concepts. As students continue to struggle in mathematics, it is imperative that educators remain informed of past and current research in conceptual and procedural knowledge so they may make important instructional decisions regarding the focus and sequence of instruction. Moreover, to expand the current research in the areas of conceptual and procedural knowledge, researchers must be knowledgeable of the areas that have a plethora of research and those with limited coverage. Therefore, this study examines how conceptual knowledge, procedural knowledge, and the progression from one to the other have been positioned within prominent research journals utilizing a quantitative content analysis research design.

CHAPTER II: REVIEW OF THE LITERATURE

Teachers are tasked with educating students with a wide range of academic abilities in mathematics. Students may struggle with mathematics in general or with specific concepts. To ensure all students are successful in mathematics and develop a deep understanding of mathematical concepts, it is recommended that teachers tailor their instruction to meet students' individual needs (Frye et al., 2013), provide explicit instruction, allow students the opportunity to work with various representations before expecting mastery at the abstract level (Gersten, Beckmann et al., 2009), and assist students in developing both conceptual and procedural knowledge (NCTM, 2014). This review of the literature first examines the theoretical frameworks of constructivism and behaviorism, the different types of knowledge necessary for concept development, and the role manipulatives and visual representations play in student learning. The review culminates with literature on explicit (direct) instruction, as well as the concrete-representational-abstract (CRA) sequence of instruction, also known as concrete-pictorial-abstract instruction or concreteness fading, and the virtual-representational-abstract (VRA) sequence of instruction.

The review of the literature includes references to mathematics educators, instructional designers, and educational psychologists. The research from educational psychologists, whose research focuses on how knowledge change occurs, specifically in mathematics, links psychological theory and educational practice (Vanderbilt University, 2021). Research topics of these educational psychologists include generating explanations to promote mathematics learning, problem-solving procedures, the effect of early mathematics learning on later math knowledge (Vanderbilt University, 2021), and developmental differences in mathematics, among other topics (Carleton University, 2021). The educational psychologists referenced in the review are highly cited and prominent researchers in their field. Additionally, according to Bransford et al. (2000):

> Research from cognitive psychology has increased understanding of the nature of competent performance and the principles of knowledge organization that underlie people's abilities to solve problems in a wide variety of areas, including mathematics, science, literature, social studies, and history. (p. 4)

Furthermore, collaboration between cognitive psychologists and educators on the design of learning environments, as well as evaluation of such environments, has resulted in new knowledge concerning learning and teaching (Bransford et al., 2000).

Theoretical Framework

The current research is grounded in the theories of constructivism and behaviorism. The following section will provide a review of both constructivism and behaviorism and describe teacher and student behaviors within instructional practices that utilize each theory of learning.

Constructivism

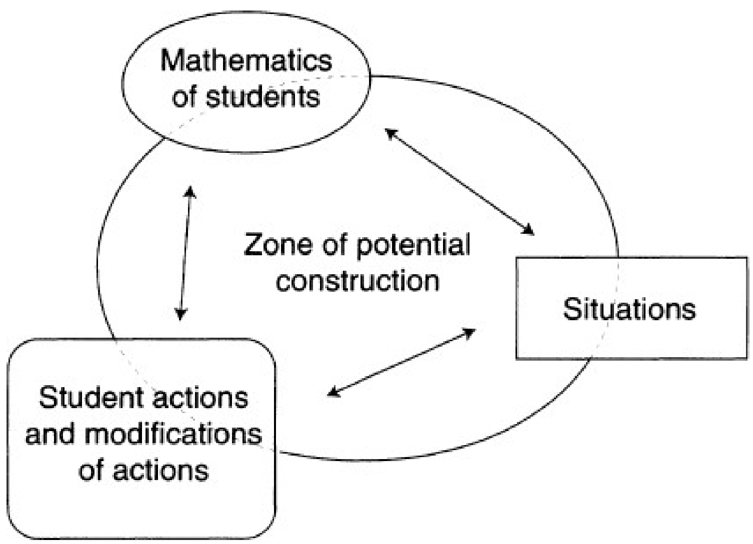
Constructivism can be classified two forms: epistemological and psychological constructivism (Phillips, 2000). In epistemological constructivism, also referred to as social constructivism, meaning is constructed through reading, writing, speaking, listening, and reflecting, all of which are social in nature, requiring the use of language and the exchanging of ideas (Zain et al., 2012). Language, conversation, negotiation, and group acceptance are central to the construction of knowledge in a social constructivist paradigm (Ernest, 1998; Zain et al., 2012). Psychological constructivism, or pedagogical constructivism, refers to the idea that construction of new knowledge occurs when students reconfigure mental connections, ideas, and procedures already learned when presented with information that does not easily correlate to this prior knowledge (Hiebert & Grouws, 2006). Knowledge is everchanging, as it is a "compendium of concepts and actions that one has found to be successful, given the purposes one had in mind" (von Glasersfeld, 1995, p. 5). Von Glasersfeld (1995) posits that knowledge of concepts is relative to particular contexts and must be restructured as new experiences occur. Although social constructivism and pedagogical constructivism differ by epistemology and pedagogy, they also both suggest that knowledge is constructed through experiences. It is important to keep in mind that, as Simon (1995) states, "although constructivism provides a useful framework for thinking about mathematics learning in classrooms and therefore can contribute in important ways to the effort to reform classroom mathematics teaching, it does not tell us how to teach mathematics; that is, it does not stipulate a particular model" (p. 114). Constructivism merely describes the development of knowledge without regard for the presence of a teacher or the act of teaching (Simon, 1995). This research will focus primarily on that of psychological, or pedagogical constructivism.

Mathematical knowledge is created by the student through their experiences (Clements & Battista, 1990; Yackel et al., 1990). Dewey (1958) suggested that *how* things are experienced is just as important as *what* is experienced and that the isolation of objects from the experience reduces the event to a mere procedure rather than an exploration of ideas and connections. Furthermore, one must interact with their environment to form conclusions, make connections between prior experiences and the current topic of study, and to learn (Dewey, 1958). Learners can then construct notations for expressing their constructions and begin to apply them to different materials (Bruner, 1966). As Bruner (1960) proposed, the discovery of mathematics should be thought of as a "process of working" and that intuition, the concretization of unstated ideas through examples and actions, plays an important role in the learning of mathematics (p. 612).

Similarly, Piaget (1973) posited that learners construct their own knowledge through their actions with the environment (Wadsworth, 1988) and that the mind processes new information through four stages of development: sensorimotor, preoperational thought, concrete operations, and formal operations (Pulaski, 1980; Wadsworth, 1988). Prior relevant experiences must be assimilated with students' current actions to make connections and form new knowledge (Wadsworth, 1988). In other words, how does the student interact with objects? In this sense, Piaget postulated that knowledge cannot be transferred from teacher to student through direct spoken or written word (Wadsworth, 1988) and that when students are expected to 'learn' on demand, very little is retained (Piaget, 1973). Likewise, von Glasersfeld (1995) suggested that knowledge of concepts must be conceived rather than transferred from the teacher to the student. Students' curiosity must be piqued so they may form their own ideas and conclusions (Piaget, 1973) and students must experience disequilibrium, the direct conflict between their prior knowledge and current predictions, to restructure their knowledge through concept exploration, assimilation, and accommodation (Wadsworth, 1988). Piaget (1973) suggested that educators promote disequilibrium through cognitive conflict, structured experiences that surprise the learner, well thought out sequencing of curriculum based on the stages of development, and adapting experiences based on individual student differences. Steffe and D'Ambrosio (1995) suggest following the Zone of Potential Construction model, whereby the teacher "interprets the schemes and operations available to the student and anticipates the student's actions when solving different tasks" (p. 154) (see Figure 5). The teacher hypothesizes about what a student might learn, based on the teacher's knowledge of the student's mathematical knowledge and results of interactions between the teacher and student (Steffe & D'Ambrosio, 1995). As students respond to mathematical situations, both verbally and nonverbally, the teacher formulates new hypotheses and adjusts the instructional experiences, thus, engaging in constructivist teaching (Steffe & D'Ambrosio, 1995).

Figure 5

Zone of Potential Construction



Note: Reprinted from *Toward a Working Model of Constructivist Teaching: A Reaction to Simon* (p. 154), by Steffe, L.P, & D'Ambrosio, B.S., 1995, Journal for Research in Mathematics Education, 26(2).

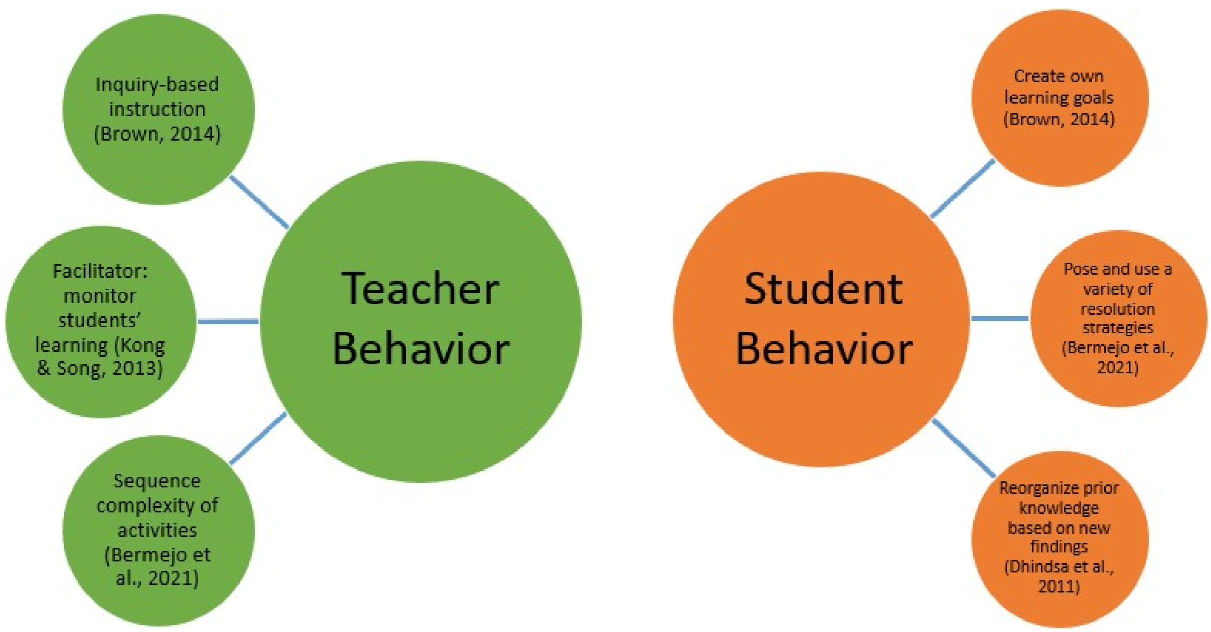
Like Dewey and Piaget, Vygotsky (1978) held that students must make sense of things for themselves through their lived experiences, that development and learning take place simultaneously, and students must be active participants in their learning (Wink & Putney, 2002; Vygotsky, 1978). Vygotsky developed what is known as the *zone of proximal development*. The zone of proximal development is that level of cognition between a student's current development and that of their potential development (Brown et al., 1996; Vygotsky, 1978). This development is determined by the student's ability to problem solve independently versus their ability to problem solve with the assistance of an adult (Brown et al., 1996). Working within a student's zone of proximal development supports the construction of knowledge by ensuring the appropriate amount of teacher support is provided.

Teacher and student behaviors. Instructional models that follow the constructivist theory of learning are often associated with inquiry-based instruction in which students are encouraged to self-discover and reflect on their learning (Brown, 2014). Students often create their own learning goals and work to reach those goals in cooperative groups with other students (Brown, 2014). Teachers take on the role of facilitator, monitoring students' process of learning and intervene to provide instructional support in a timely manner (Kong & Song, 2013). However, this is not to say that teachers do not have input into the learning goals and outcomes. The misconception that teachers should not have input or directly provide information to students "confuses a theory of pedagogy (teaching) with a theory of knowing" (Bransford et al., 2000). Mathematical content "must be sequenced in terms of the complexity and significance for the student as well as contextualized at all times" (Bermejo et al., 2021). This requires the teacher to have fundamental knowledge of the development of mathematical thinking (Bermejo et al., 2021), and the ability to organize and embed student learning in real-life situations, suggest problem-solving tasks, and structure activities so they build upon students' prior knowledge (Tiilikainen et al., 2017). Constructivists place high importance on the role of memory and the process of encoding (Ertmer & Newby, 2013). Teachers are tasked with organizing information in a way that assists learners in the process of encoding and is often accomplished with the use of graphic organizers and hierarchical maps (Ertmer & Newby, 2013).

Meanwhile, the students pose and use a variety of resolution strategies to construct their own knowledge (Bermejo et al., 2021) and reorganize prior knowledge based on their new findings (Dhindsa et al., 2011). A summary of teacher and student behaviors in constructivism can be found in Figure 6.

Figure 6

Constructivism Teacher and Student Behaviors



Behaviorism

Behaviorism is a guided approach to learning that emphasizes direct instruction of skills whereby the teacher explicitly explains concepts and provides learners with solution strategies that promote success (Poncy et al., 2010). According to Bransford et al. (2000), behaviorists conceptualized "learning as a process of forming connections between stimuli and responses" (p.

6). Within mathematics, students are provided repeated practice, aimed at developing automaticity, by means of explicit, targeted math problems (Poncy et al., 2010). Tasks are broken down into small steps and ordered hierarchically, allowing students to master prerequisite skills prior to increasing the complexity of problems (Poncy et al., 2010). Learning is determined by observable changes in student behavior (Ertmer & Newby, 2013). According to Ertmer and Newby (2013), "responses that are followed by reinforcement are more likely to recur in the future" (p. 48) and students react to environmental conditions rather than making discoveries in their environment. The learner can generalize what they have learned and apply it to a situation or problem with like elements (Ertmer & Newby, 2013).

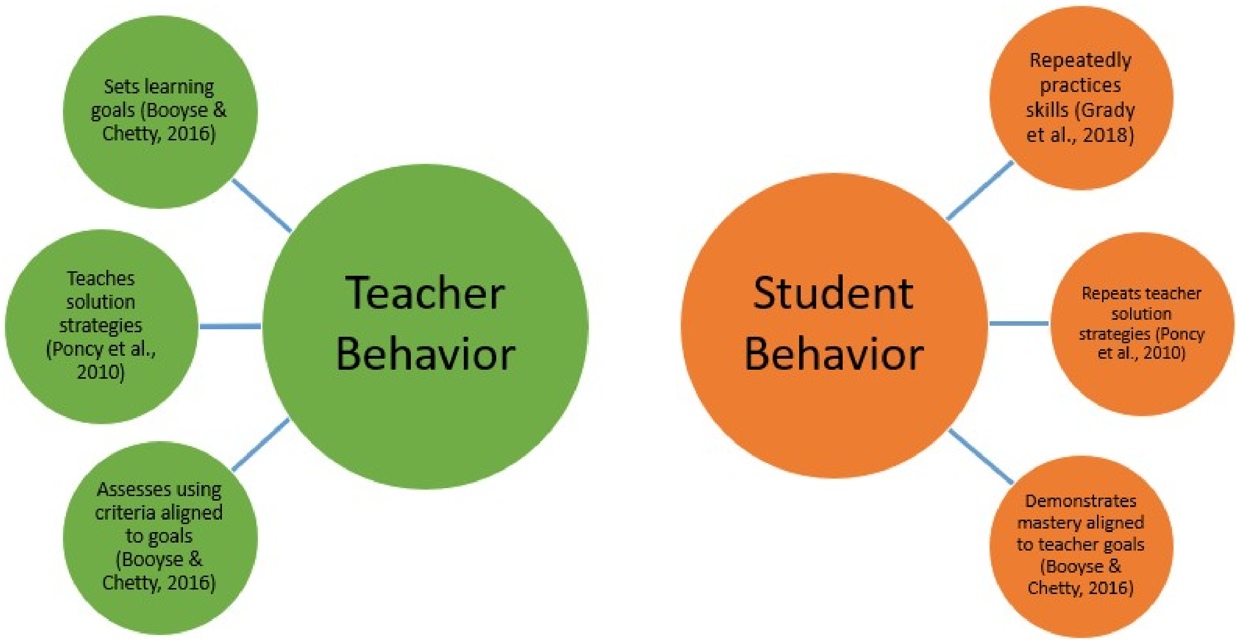
Behaviorism's roots are grounded in the work of Ivan Pavlov (1960), John B. Watson (1924), and B. F. Skinner (1938). Pavlov's study of conditional reflexes with dogs, utilizing food and tones, to initiate the salivation response, pioneered the study of behaviorism. Conditional reflexes are the low-level processes developed out of habit and associated with visual cues that connect new information with the old and help to develop primary memory (Greenspan & Baars, 2005). According to Ewen (2010), an "organism learns that one stimulus will be followed by another stimulus because the two stimuli repeatedly occur closely together in time" (p. 287). Later, John B. Watson supported the work of Pavlov and began experiments with conditioned reflex, widely promoting the theory (Greenspan & Baars, 2005).

B. F. Skinner (1938), another great supporter of Pavlov's work, studied a second form of conditioning, labeled operant conditioning. According to Ewen (2010), Skinner posited that a correct response is required for reinforcement, whether the reinforcement is a reward or the avoidance of punishment. Skinner further noted that a response is more likely to occur if it is reinforced and less likely to reoccur if it is not reinforced (Ewen, 2010). Thus, behavior change is a result of environmental design (Ewen, 2010). Additionally, Skinner suggested that the manner and intensity of the reinforcement strongly impacts learning and behavioral change (Ewen, 2010). Skinner explained that thinking is a behavior and decisions are made based on the strength of a response (Ewen, 2010). Such responses are tied to the environment, therefore, problem solving stems from altering the environment or situation so that an “appropriate response can be made” (Ewen, 2010, p. 301). These elements of operant conditioning are applied to education through explicit, direct instruction.

Teacher and student behaviors. Instructional models that follow the behaviorism theory of learning are often associated with a more traditional approach to instruction (Grady et al., 2018). The process begins with the teacher assessing students to determine an instructional starting point and set learning goals (Booyse & Chetty, 2016). Once the teacher has set the learning goals, he or she determines what information is most important for the student to learn to accomplish the goals (Booyse & Chetty, 2016). Tasks are broken into small steps, taught explicitly, often through the direct instructional model (Poncy et al., 2010), and repeated until students demonstrate mastery (Grady et al., 2018). Mastery is determined by assessing students using criteria aligned to the original instructional goals (Booyse & Chetty, 2016). A summary of teacher and student behaviors in behaviorism can be found in Figure 7.

Figure 7

Behaviorism Teacher and Student Behaviors



Conceptual and Procedural Knowledge

Mathematical understanding can be broken into two facets: instrumental understanding and relational understanding (Skemp, 2006). According to Skemp (2006), instrumental understanding, sometimes called procedural understanding, refers to one's knowledge of rules without any reason as to why or how such rules work. Students accept the rules required to arrive at an answer to a mathematical problem but do not necessarily understand the underlying reasons for the procedures they carried out (Loong, 2014). It provides students with knowledge of the skills or series of steps required to solve mathematical problems (Canobi, 2009; Rittle-Johnson et al., 2015) and execute procedures fluently. Instrumental understanding occurs when students learn fixed steps or exact procedures to solve specific types of tasks (Skemp, 2006). Lithner

(2008) referred to the rote learning of strategies to solve specific mathematical tasks without an understanding of the meaning behind the steps as imitative reasoning and that such steps are learned by repeating the solution steps until they are memorized. Lithner (2008) noted that imitative reasoning is a primary factor in student learning difficulties.

In contrast, relational understanding, sometimes called conceptual understanding, refers to "knowing what to do and why" (Skemp, 2006, p. 89). These are the underlying concepts and unifying principles, whether abstract or general, of a mathematical domain (Canobi, 2009; Rittle-Johnson et al., 2015); the "understanding of relationships and connections within a domain" (Crooks & Alibali, 2014, p. 348). Fletcher, et al. (2019) performed a concept analysis of conceptual learning and found it to have the following attributes: 1) recognition of patterns in information, 2) formation of links with a concept, 3) acquisition of deeper understanding of an individual concept, 4) discovery of relevance and value, and 5) application of concepts to new situations. Relational understanding occurs when students implement a variety of solution plans, beginning at various starting points, to solve a mathematical task and where understanding is the ultimate goal rather than the completion of set steps (Skemp, 2006). Hiebert and Lefevere (1986) described conceptual knowledge as a connected web in which the relationship between concepts is just as important as the individual pieces of information. Lithner (2008) referred to conceptual knowledge as creative reasoning, by which students create novel problem-solving sequences. Students understand "a mathematical operation" and are "able to apply that knowledge in new situations" (Osterman & Brating, 2019, p. 462). While procedural understanding involves the use of multiple rules that are applied to specific mathematical tasks, conceptual understanding involves principles that can be applied more generally and transferred among a variety of mathematical tasks (Skemp, 2006). According to von Glasersfeld (1995), conceptual understanding is necessary to solve problems that differ from those presented during instruction and student success is dependent upon having conceptual understanding of a domain.

As Skemp (2006) described, instrumental (procedural) and relational (conceptual) understanding each have advantages. Procedural understanding can be easier to understand when viewed within its own context. It offers a reliable means to quickly complete mathematical tasks using easily remembered rules and steps, providing students with a feeling of success and increasing their confidence. Alternatively, conceptual understanding is easily transferred and adapted to new mathematical tasks. If one understands the why behind a mathematical task, they are better able to relate it to a variety of situations. Although conceptual understanding can be more difficult to learn than procedural understanding, once learned, it is easier to remember. Furthermore, conceptual understanding allows the learner to connect individual concepts to see how they form a whole.

According to Hiebert and Grouws (2006), in general, the correlation of skill efficiency, or procedural understanding, and features of teaching are similar in all subjects. These features are: 1) rapid paced instruction, 2) teacher-directed modeling, and 3) smooth transitions from teacher demonstration to student practice. The teachers' role in procedural instruction is to organize content presentation, establish appropriate pacing, and provide well-defined learning goals. Meanwhile, Hiebert and Grouws (2006) suggested that there are two key features of conceptual understanding from an instructional standpoint: 1) teaching should attend explicitly to concepts and connections between mathematical procedures and ideas, and 2) students must expend effort, or productively struggle, to make sense of mathematical concepts. Teachers can attend explicitly to concepts and connections by incorporating structured discussions of key mathematical ideas. Such discussions may focus on the meaning of the underlying procedural processes related to a concept, inquiring about alternate solutions, exploring how problems build upon previous mathematical concepts, and drawing students' attention to how the current topic of study aligns with previous and upcoming lessons. Importantly, Hiebert and Grouws (2006) noted that "conceptual development of the mathematics can take many pedagogical forms" and that "concepts can be developed through teacher-centered and highly structured formats" or "through student-centered and less structured formats" (p. 387) and instruction that focuses on conceptual development also facilitates skill, or procedural, development. As noted previously, students must also expend effort, or productively struggle, to make sense of mathematical concepts (Hiebert & Grouws, 2006). The process of productive struggle reconfigures the mental connections, ideas, and procedures that students already have when they are presented with information that does not easily correlate to this prior knowledge (Hiebert & Grouws, 2006), thus, constructing new knowledge.

The relation between conceptual and procedural knowledge has historically taken one of three views: 1) concepts-first, 2) procedures-first, and 3) iterative. According to Rittle-Johnson (2017), the concepts-first view proposes that children first develop conceptual knowledge either naturally or by learning from adults. Students then develop procedural knowledge by repeatedly solving problems based on their previously learned conceptual knowledge. The procedure-first view suggests that children learn procedures by first imitating adults and then develop conceptual knowledge by abstracting principles from the problems. The iterative view submits that conceptual and procedural knowledge development is bidirectional; that conceptual knowledge increases procedural knowledge and vice versa. A comparison of the three views can be found in Table 1.

Table 1

Views on Conceptual and Procedural Knowledge Development

Concepts First	Procedures First	Iterative
Conceptual knowledge developed naturally or by learning from adults	Procedural knowledge developed by imitating adults	Conceptual and procedural knowledge development is bidirectional
Procedural knowledge developed by repeatedly solving problems based on previously learned conceptual knowledge	Conceptual knowledge developed by abstracting principles from problems	Conceptual knowledge increases procedural knowledge
		Procedural knowledge increases conceptual knowledge

The National Council of Teachers of Mathematics (NCTM, 2014) emphasizes the development of conceptual knowledge prior to procedural instruction; "Conceptual understanding (i.e., the comprehension and connection of concepts, operations, and relations) establishes the foundation, and is necessary, for developing procedural fluency (i.e., the meaningful and flexible use of procedures to solve problems") (p. 7). Research conducted by Pesek and Kirshner (2000), comparing the effects of instrumental (procedural) instruction and relational (conceptual) instruction, supports this recommendation. Pesek and Kirshner (2000) administered a pretest, provided an intervention to experimental groups with an embeded intermediate test, and concluded with a post-test. Following the intervention and assessments, interviews were conducted with the student participants. The purpose of the research was to investigate if prior instrumental instruction interferes with relational instruction, whether cognitively, metacognitively, or attitudinally. Results suggested that: 1) students' relational understanding was negatively impacted by initial instrumental instruction, 2) cognitive, metacognitive, and attitudinal interference all contributed to student misunderstandings, and 3) students in the relational instruction only experimental group outperformed those students in the instrumental instruction prior to relational instruction experimental group.

Similarly, Rittle-Johnson et al. (2016) conducted a study to evaluate the interaction between the type of instruction and the timing of such instruction within a lesson. Second graders were randomly assigned to one of four instructional groups: 1) conceptual instruction prior to practice, 2) conceptual instruction after practice, 3) conceptual and procedural instruction combined prior to practice, and 4) conceptual and procedural instruction combined after practice. A pretest-intervention-post-test design was utilized. Results indicated that students who received two iterations of instruction focused on conceptual knowledge had better retention of both conceptual and procedural knowledge than students who received a mixture of both conceptual and procedural instruction within one lesson. The order of conceptual and procedural instruction within the single lesson did not impact student outcomes.

In contrast, the iterative and concurrent development of conceptual and procedural knowledge is most widely accepted by researchers (Verschaffel et al., 2006) and many research studies support this iterative view (Kieran, 2013; LeFevre et al., 2006; Pirie, 1988; Rittle-Johnson et al., 2015; Voutsina, 2012). Kieran (2013) proclaims that the dichotomy between conceptual understanding and procedural skills is false and explains that procedures are conceptual in nature "during their period of elaboration" and that "even when they function as automatized skills" they "are regularly being updated, revised, and extended by means of conceptual elements" (p. 154). Thus, the interaction between conceptual understanding and procedural skill is not dichotomous, rather it is an ongoing recursive process (Kieran, 2013). To demonstrate this, Kieran (2013) conducted a study with 10th grade students that intertwined technical work and conceptual reflection in Algebra in which students completed a task aimed at developing new factoring techniques for polynomial expressions. Results of the study indicated that conceptual knowledge was integral to students developing new procedural techniques, through elaboration of conceptual ideas, while simultaneously integrating these new techniques into their conceptual knowledge.

Voutsina (2012) conducted a study to analyze the types of mathematical knowledge students employ when solving multi-step addition tasks. The researcher utilized a combination of clinical interviews and the micro-genetic method with ten 5-6-year-old students to document changes in the students' problem-solving behaviors. Students were provided an arithmetic task and researchers studied their problem-solving approaches and behaviors over five separate sessions. Students were asked to solve each task and describe their problem-solving process and strategy. Researchers recorded and analyzed students' verbalizations, movements, and hesitations and focused on changes in the students' behaviors. Results indicated that there is an interplay between task representations, procedures, and conceptual development.

Likewise, research shows that the development of conceptual and procedural knowledge is bidirectional, meaning that procedural knowledge supports conceptual knowledge and vice versa (Riddle-Johnson et al., 2015). This bidirectional relationship was determined through a review of empirical evidence by examining longitudinal relations between conceptual and procedural knowledge and causal evidence from experimental studies. The researchers examined the research design used in each study as well as the measures utilized to measure both conceptual and procedural knowledge. Although the relationship was found to be bidirectional, no empirical evidence was found as to the optimal ordering of instruction.

According to Pirie (1988), procedural and conceptual understanding do not develop in a linear fashion. Likewise, Bergeron, Herscovics, and Bergeron contend that the construction of mathematical concepts evolves from the simultaneous development of intuition, procedural understanding, abstraction and formalization (as cited by Pirie, 1988, p. 6). Procedural understanding provides students with the skills necessary to work within the conceptual realm and, in turn, the process of developing conceptual understanding provides a context to apply their procedural understanding (Reason, 2003). “It is as if ‘relational’ and ‘instrumental’ form a duality rather than being distinct” (Reason, 2003, p. 7).

Mathematics Instruction

Success in mathematics requires that students develop both conceptual and procedural knowledge (LeFevre et al., 2006). Instructional practices such as concrete and virtual manipulative use as well as the use of visual representations assist learners with developing conceptual knowledge. To aid students in the development of procedural knowledge, explicit instruction can be utilized. Additionally, the concrete-representational-abstract (CRA) and the virtual-representational-abstract (VRA) instructional sequences support the progression from conceptual to procedural knowledge development in mathematics. These instructional strategies provide scaffolds for student understanding and help to bridge the gap between conceptual and procedural knowledge development.

Instructional Strategies that Support Conceptual Knowledge Development

Concrete manipulative use. To assist students in developing conceptual (relational) understanding of mathematical concepts, educators often utilize manipulatives. Mathematics manipulatives are objects that an individual can handle in a multisensory way (sight, touch, and/or sound) to support mathematical thinking (Hurrell, 2018). Pattern blocks, base ten blocks, Unifix cubes, counters, square tiles, and Cuisinaire rods are examples of manipulatives (Swan & Marshall, 2010). Manipulatives such as these help students explore and clarify mathematical concepts to consolidate their relational understanding (Golafshani, 2013; Loong, 2014). They create an external representation of mathematical ideas that are later developed into internal representations (Hurrell, 2018) and assist students in translating these ideas into spoken language (Flores & Hinton, 2019).

It is important to note that manipulative use does not guarantee student learning (Furman, 2017; National Center on Intensive Intervention [NCII], 2016; Puchner et al., 2008, Uribe-Florez & Wilkins, 2010). Hurrell (2018) cautioned that just because the teacher understands the relationship between a specific manipulative and the abstract mathematical concept it represents, it does not necessarily mean that students see the relationship. As such, educators must not only carefully select manipulatives to ensure they are suitable for stimulating student thinking, but the mathematics must also be clearly articulated (Hurrell, 2018). Additionally, Puchner et al. (2008) suggested that connections between the concrete manipulatives and a mathematical concept must be constructed by students and educators must take care to not turn manipulative use into a procedural (instrumental) task. Manipulatives should be used to test out solutions and construct knowledge before procedures are introduced (Puchner et al., 2008). Kablan (2016) conducted a study with seventh-grade students to evaluate the various times spent on concrete manipulative use in combination with traditional abstract mathematics instruction and its effect on student achievement. The researcher evaluated the effects of three instructional models: 50 percent concrete and 50 percent abstract, 30 percent concrete and 70 percent abstract, and 100 percent abstract. Results of the study indicated that the instructional models that incorporated both concrete manipulatives and abstract traditional instruction attended to a variety of learning styles and resulted in increased student achievement (Kablan, 2016). Liggett (2017) investigated the impact of manipulative use on students' mathematical achievement with second graders.

Students were randomly divided into one control group without access to manipulatives and one treatment group with access to manipulatives and given a pre- and posttest to assess any differences in mathematical achievement. The results supported the claim that manipulatives increase student achievement as the treatment group improved more than the control group from pretest to posttest (Liggett, 2017).

Uribe-Florez and Wilkins (2017) conducted a cross-sectional correlation analysis and longitudinal analysis to examine the relationship between elementary (K-5) student learning of mathematics and the use of manipulatives. The researchers utilized The Early Childhood Longitudinal Study (ECLS) 1998/2004 data to examine: 1) grade level differences in manipulative use, 2) the relationship between student achievement and the use of manipulatives, and 3) longitudinal student learning as a result of manipulative use in the elementary grades. Results indicated that as the grade level increases, manipulative use decreases. Manipulative use was highest in kindergarten and progressively decreased in subsequent grade levels, with fifth-grade utilizing them the least. Manipulative use was found to have a limited relationship with student achievement, as assessed with a single snapshot, however, when examined longitudinally across all elementary grades, there was a positive relationship between manipulative use and student learning.

Carbonneau et al. (2013) investigated the effect of manipulative use during mathematics instruction compared to instruction that focused on abstract symbolic representations by means of a meta-analysis. The researchers examined 55 studies across different grade levels (kindergarten to college) that compared the efficacy of manipulative use to that of abstract symbolic instruction. Additionally, the researchers analyzed retention, problem solving, transfer, and justification learning outcomes. Results of the study indicated a small to moderate effect size for the use of manipulatives during math instruction and moderate to large effect size on students' mathematics retention. However, the researchers found that manipulative use had a small effect size on students' problem solving, transfer and justification when compared to abstract symbolic instruction.

Virtual manipulative use. Virtual manipulatives are also commonly used by educators to assist students in developing conceptual (relational) understanding. Virtual manipulatives are "an interactive, technology-enabled visual representation of a dynamic mathematical object" (Moyer-Packenham & Bolyard, 2016, p. 3). They are designed to represent the corresponding concrete manipulative in an Internet or app-based environment (Bouck, et al., 2020) and are often represented as three-dimensional objects that can be rotated (Bouck & Sprick, 2019).

Gecu-Parmaksiz and Delialioglu (2019) conducted a quasi-experimental study with 72 preschool children to compare the effect of concrete and virtual manipulatives. Students were divided into an experimental and control group. The experimental group of students utilized a tablet to work with virtual manipulatives while the control group of students used concrete manipulatives. Students were administered both a pre- and posttest to measure their geometric shape recognition prior to the intervention and after the intervention. Results indicated that students in the experimental group (virtual manipulatives) showed a statistically significant mean difference when compared to the control group (concrete manipulatives).

To research the use of virtual manipulatives to teach the concepts of area and perimeter, Satsangi and Bouck (2015) utilized a multiple baseline design with three high school students with learning disabilities. Baseline data was collected through paper and pencil tasks, students were then given a 40-minute lesson on the concepts of area and perimeter, taught how to use the virtual manipulatives on a computer, asked to solve problems similar to those in the baseline tasks, assessed for maintenance two weeks after the intervention, and assessed on their ability to generalize their learning to abstract word problems. Results of the study showed that all three students' performance improved from their baseline data. Additionally, all three students demonstrated improved performance through the maintenance and generalization stages of the study.

Bouck, Park, and Stenzel (2020) investigated the use of app-based virtual manipulatives as assistive technology with three middle school students in special education classes. The researchers first modeled solving division with remainder problems with virtual Cuisenaire Rods on an iPad while also communicating their problem-solving method aloud, students then solved two problems with guidance, and finally three problems independently. A multiple baseline, multiple probe across participants single case research design was utilized. As a functional relationship was found between the use of app-based virtual manipulatives and student achievement, the findings support the use of virtual manipulatives.

To compare how students used concrete base-ten blocks and virtual base-ten blocks to learn about place value, Burris (2013) conducted a study with four third-grade classes. Students in two classes utilized concrete manipulatives while students in the other two classes utilized virtual manipulatives. All four classroom teachers used the same lesson plans and students worked in pairs in all classes during the study. Students manipulated their respective manipulatives to build numbers, identify quantities, and then write the corresponding numerals. The interactions between paired students were video-taped and the recordings analyzed. The researchers found that students used both the virtual and concrete manipulatives in similar ways and that both groups of students were able to construct numbers, identify quantities, and write numerals that corresponded to the numbers built.

Representations (Visual). The National Council of Teachers of Mathematics (NCTM) suggest teaching practices for effective mathematics instruction. One suggestion is for educators to "use and connect multiple representations" (NCTM, 2014, p. 24). Additionally, NCTM suggests that educators "allocate substantial instructional time for students to use, discuss, and make connections among representations" (NCTM, 2014, p. 29). These suggestions support the development of conceptual, or relational, understanding in students. Visual representations assist students in bridging the abstract mathematical ideas presented in a mathematics problem with problem context through mathematical problem-solving practices (Murata & Stewart, 2017). Additionally, visual representations assist students and teachers by creating a platform with which to analyze and refine their thinking (Murata & Steward, 2017).

Further support for the use of visual representations can be found in research conducted by Heinze et al. (2009). The researchers conducted a research of literature related to the flexible and adaptive use of strategies and representations in mathematics education. The authors explain that flexibility refers to one's capacity to flexibly select among a variety of strategies, although the authors note that the strategy or representation chosen by the individual may not be the most appropriate strategy to utilize. Furthermore, the authors define adaptive use of strategies as one's capacity to select the most appropriate strategy for the task at hand. The researchers concluded that both flexibility and adaptive strategy and representation selection impact students' success in mathematics.

Jitendra et al. (2016) reviewed research on the use of both concrete and visual representations to solve problems with students who have mathematics difficulties. The researchers analyzed the literature for studies focused on the effects of representations on student performance, applied Gersten et al. (2005) guidelines of evidence standards, and found that 13 out of 25 studies met the guidelines set forth by Gersten et al. (2005). Therefore, the researchers concluded that the use of representations is an evidence-based strategy for solving mathematical problems and increasing student performance.

As Pirie (1988) noted, teachers present visual representations to help students make connections between the conceptual underpinnings of mathematical concepts and the abstract symbolic notations and representations. However, as with manipulatives, the use of visual representations does not ensure student learning. “Teachers present pictures and cut up paper in an attempt to relate mathematical theory to reality for their pupils” (Pirie, 1988, p. 5). However, it is not guaranteed that students view the pictures as *real*, rather they may view them simply as an abstract diagram, lacking the bridge between conceptual understanding and the abstract symbolization that the teacher was seeking (Pirie, 1988). Therefore, as with the use of manipulatives, appropriate representations must be utilized, and the mathematics must be clearly articulated.

Physical and visual representations support the transition from problem presentation to abstract solutions. The concrete-representational-abstract (CRA) sequence of instruction is a model of instruction that combines the use of manipulatives and visual representations to further guide students towards construction of knowledge, conceptual understanding of mathematical concepts, and scaffolds student learning to promote eventual independence.

Instructional Strategy that Supports Procedural Knowledge Development

Explicit instruction. Explicit instruction, also known as direct instruction, is a direct and unambiguous method of instructional design and delivery attributed to Siegfried Engelmann and his colleagues at the University of Illinois at Champaign-Urbana in the 1960s (Magliaro et al., 2005). This type of instruction provides students with scaffolded support and practice that is strategically reduced until independence is attained (Archer & Hughes, 2011) by means of detailed teacher modeling, guided student practice, and teacher monitored independent practice. Explicit instruction supports instructional interactions between teachers and students (Doabler & Fien, 2013) and both The National Mathematics Advisory Panel (NMAP, 2008) and The Institute of Education Sciences (Gersten, Beckman et al., 2009) recommend its use. A meta-analysis conducted to synthesize the results of 42 interventions designed to test the effects of mathematics instructional strategies for students with learning disabilities found that explicit instruction was the most important instructional strategy of those analyzed and resulted in significant effect sizes (Gersten, Chard et al., 2009). Although various alternative explicit (direct) instructional models have been developed over the years in the realm of behaviorism, according to Magliaro et al. (2005), they all have the following similar components:

1) Materials and curriculum are broken down into small steps and arrayed in what is assumed to be the prerequisite order.
2) Objectives must be stated clearly and in terms of learner outcomes or performance.
3) Learners are provided with opportunities to connect their new knowledge with what they already know.
4) Learners are given practice with each step or combination of steps.
5) Learners experience additional opportunities to practice that promote increasing responsibility and independence (guided and/or independent; in groups and/or alone).
6) Feedback is provided after each practice opportunity or set of practice opportunities (p. 44).

When utilizing explicit instruction, teachers provide highly structured instruction, clear models and specific strategies for solving problems using a variety of examples, teachers sequence instructional materials and problems to highlight critical features of the content, students are provided extensive opportunities to practice newly learned strategies and skills, students are provided numerous opportunities to ask questions and think aloud, and teachers provide students with immediate feedback (NMAP, 2008).

Teacher modeling and precise questioning are critical to explicit instruction. Doabler and Fien (2013) explained that teachers should use precise and consistent language during the modeling phase and involve students in the demonstration. Teachers should also model how to think aloud about both the steps they are carrying out and the reasons for doing so (Gersten, Beckmann et al., 2009). Problems should be varied, sequenced to gradually increase student proficiency, and scaffolded to allow transfer of the work from the teacher to the students as they are able to complete the problems more independently (Gersten, Beckman et al., 2009). Teachers should design their questioning to guide mathematical discourse in the classroom and encourage students to explain their thinking and strategies, as well as their solutions to problems (Doabler & Fien, 2013; Gersten, Beckman et al., 2009). Finally, academic feedback should be provided to students in a timely manner and help to clarify what students are doing correctly as well as what needs to be corrected (Doabler & Fien, 2013; Gersten, Beckman et al., 2009). Teacher modeling, precise questioning, and immediate feedback allow educators to scaffold their instruction to meet the needs of all students.

As mentioned previously, Siegfried Engelmann and his colleagues at the University of Illinois at Champaign-Urbana developed the original direct instruction, or explicit instruction, model. According to Bereiter and Engelmann (1966), the direct instruction model has the following characteristics:

1. Fast pace,

2. Reduced task-irrelevant behavior,
3. Strong emphasis on verbal responses,
4. Carefully planned small-step instructional units with continual feedback, and
5. Heavy work demands (pp. 55-56).

Additionally, the model contains three stages. Magliaro et al. (2005) explains that students are first introduced to the new concept or skill to be learned, followed by a teacher led instructional presentation. Next, practice problems are posed and solved as a whole class, directed by the teacher, by means of rapid, purposefully sequenced questions, designed to steer students to the teacher's interpretation of the information. Once students have demonstrated that they are ready to apply their learning, students independently practice solving similar problems as the teacher provides immediate corrective feedback and reinforcement. According to Bereiter and Engelmann (1966), "through direct instruction … children are fully aware of what they have learned and aware that they have learned it through their own effort, concentration, and intelligence" (p. 62). In contrast, Bereiter and Engelmann (1966) argue that with indirect teaching methods lead to a vague awareness that learning has occurred, therefore, students don't realize the significance of their accomplishments. A summary of the model can be found in Table 2.

One variation of Engelmann's direct instruction model is the Rosenshine explicit teaching model. Like Engelmann's model, Rosenshine's model consists of the following elements: 1) review of previous learning, 2) presentation of new concept or skill, 3) guided practice of new concept or skill, 4) correction and feedback with respect to the new concept or skill, 5) independent practice, and 6) weekly and monthly reviews (Magliaro et al., 2005; Rosenshine, 2012). The main tenants of this model are that information is presented in small steps and that students are provided the opportunity for guided practice after each step is presented (Magliaro et al., 2005; Rosenshine, 2012). A summary of the model can be found in Table 2.

Another variant of Engelmann's direct instruction model is the expository teaching model. Although the expository teaching model is a teacher-centered approach and closely parallels Engelmann's model, it also includes cognitive elements (Magliaro et al., 2005). To ensure the lesson objectives are met, convergent questions are utilized by the teacher and practice examples are scaffolded to support the gradual knowledge development of the concept or skill (Magliaro et al., 2005). Immediate corrective feedback, as well as reinforcement, are provided to ensure students do not develop misconceptions (Magliaro et al., 2005). A summary of the model can be found in Table 2.

In line with the direct instruction model, Madeline Hunter developed a direct instruction framework to assist teachers in designing effective classroom instruction. The Madeline Hunter Model contains the following components: anticipatory set, objectives/standards, teaching/input, modeling, guided practice, checking for understanding, independent practice, and closure (Han et al., 2013; Hunter, 1982). This model is summarized in Table 3.

Table 2

Comparison of Direct Instruction Models

Stages of Direct Instruction	Engelmann's Direct Instruction Model	Rosenshine's Explicit Teaching Model	Expository Teaching Model
Introduction	Introduction of new concept based on previously mastered skills and knowledge	Review: Homework, relevant previous learning, prerequisite skills, and knowledge for lesson	Visual presentation of targeted concept, abstraction, or generalization Inform learner of intended learning outcome
Main Presentation of the Lesson	Fast paced, scripted explanation or demonstration designed to elicit only one interpretation of concepts. The targeted concept must be reinforced with appropriate examples and nonexamples	State lesson goals and/or provide outline, teach in small steps, model procedures, provide concrete positive and negative examples Use clear language, check for student understanding, avoid digressions	Define concepts, abstractions, or generalizations Link to prior knowledge Provide positive and negative examples
Practice	Students are provided with opportunities to verbally respond, either through a set of question or tasks, in order to indicate their learning of the concept Teacher either confirms correct student response or provides corrections and repetition of the missed items After group work, students engage in self-directed practice in workbooks. Teacher monitors progress and provides guidance when needed	High frequency of questions or guided practice, all students respond and receive feedback, high success rate, continue practice until students are fluid Give process feedback when answers are correct but hesitant, give sustaining feedback, clues, or reteaching when answers are incorrect, reteach when necessary Students receive help during initial steps or overview, practice continues until students are automatic, teacher provides active supervision, routines are used to give help to slower students Weekly and monthly reviews	Classify or explain teacher examples, provide additional examples

Note. Adapted from *Direct instruction revisited: A key model for instructional technology* (pp. 46, 50), by S. G. Magliaro, B. B. Lockee, & J. K. Burton, 2005, Educational Technology, Research, and Development, 53(4).

Table 3

Madeline Hunter Model

Stages of Direct Instruction	Teacher Action
Anticipatory Set	Focus students' attention by providing a "hook" that relates the lesson's learning to previous student experiences
Objectives/Standards	Communicate the purpose of the day's lesson and what students should be able to achieve by the end of the lesson
Teaching/Input	Impart the vocabulary, skills, concepts, and facts the students need to know to be successful
Modeling	Provide a model/example of desired response or product that meets the lesson's objectives
Guided Practice	Provide direct guidance on practice exercises – lead students through required steps to achieve the lesson's objectives
Check for Understanding	Utilize a variety of questioning techniques to determine student understanding
Independent Practice	Release students to complete practice exercises on their own
Closure	Provide cues to students about what they have learned

As with the previous models, students are first introduced to the new concept or skill to be learned, followed by a teacher led instructional presentation that includes purposefully sequenced questions to check for understanding and is designed to guide students to the teacher's interpretation of the information. Once students have demonstrated that they are ready to apply their learning, similar problems are presented to students to solve independently as the teacher provides assistance and corrective feedback.

Research conducted by Al-Makahleh (2011) investigated the effect of direct mathematics instruction on fourth and fifth-grade students' achievement. The students selected for the study were identified as having learning difficulties and received instruction in a resource setting. An experimental pre- and post-test design was utilized, and data was collected through both an achievement test and mathematics attitudes scale. Results suggested that student achievement and attitudes improved as a result of the direct instruction.

Shin and Bryant (2015) conducted a review of literature to determine the effectiveness of evidence-based mathematics interventions on student achievement of fraction concepts. Seventeen studies were analyzed for the interventions' features, instructional components, and effectiveness on student achievement. The studies reviewed consisted of randomized controlled trials, quasi-experimental designs, and single-subject designs. Additionally, the studies were categorized based on the instructional component(s) utilized: 1) concrete and visual representations, 2) explicit instruction, 3) concrete to abstract sequenced instruction, 4) heuristic strategies, and 5) real-world problem use. Results of the analysis indicated that the use of these instructional components had a positive effect on student achievement and that explicit instruction improved student outcomes when utilized in conjunction with concrete and visual representations.

Instructional Strategies that Support the Progression from Conceptual to Procedural Knowledge Development

Concrete-Representational-Abstract (CRA) sequenced instruction. Bruner (1966) posits that students progress through three phases when developing new concepts. Bruner labeled the three phases as enactive, iconic, and symbolic. In the enactive stage, students utilize objects, or manipulatives, without internal representations (Hurrell, 2018; Milton et al., 2019). During the iconic phase, students begin to develop pictorial representations (Hurrell, 2018) that aid in the creation of mental images (Milton et al., 2019). The final phase, known as the symbolic phase, refers to the process of converting and storing the mental images into symbols which convey meaning to the student (Hurrell, 2018; Milton et al., 2019). The concrete-representational-abstract (CRA) instructional sequence, sometimes referred to as the concrete-pictorial-abstract (CPA) instructional sequence or concreteness fading, follows the developmental sequence posed by Bruner and consists of three instructional phases: 1) use of manipulatives, 2) use of two-dimensional drawings or visual representations, and 3) use of words and symbols only and various researchers (Agrawal & Morin, 2016; Doabler et al., 2012; Flores, 2010; Flores et al., 2014; Flores et al., 2016; Mudaly & Naidoo, 2015; National Center on Intensive Intervention [NCII], 2016; Strickland, 2017) support its use. The concrete-representational-abstract sequence of instruction is an iterative process with each phase following the same instructional cycle: the teacher models the concept, students practice the skill with guidance from the teacher, and students independently demonstrate an understanding of the concept (Flores, 2010; Forbringer & Fuchs, 2014). Although described as a sequential process or progression, CRA is in fact a combination of both sequential and non-sequential processes. Students are presented and work with the abstract symbolic representations simultaneous to the concrete or visual representations. This assists students with understanding what the abstract symbolic representations mean (Miller et al., 2011).

During the concrete phase, students work with concrete objects, or manipulatives, alongside the corresponding numerals and symbols, to gain a deeper conceptual understanding of the mathematical concepts (Agrawal & Morin, 2016; Flores, 2010; Flores et al., 2014; Flores et al., 2016; Forbringer & Fuchs, 2014; Hurrell, 2018; Milton et al., 2019; NCII, 2016; Strickland, 2017). The use of manipulatives allows students to make meaning of the mathematics (Flores & Hinton, 2019) by creating an external representation of the concept and their thinking (Hurrell, 2018). The teacher assists students in making connections between the mathematical concept and how the manipulatives represent that concept, as well as how those concepts are then abstractly represented with numerals and symbols by using consistent language throughout the stages and explicitly modeling the multiple representations simultaneously. In the representational phase, students use two dimensional pictures and/or drawings of the manipulatives, alongside the corresponding numerals and symbols, to solve problems similar to those completed in the concrete phase (Agrawal & Morin, 2016; Flores, 2010; Flores et al., 2014; Flores et al., 2016; Forbringer & Fuchs, 2014; Hurrell, 2018; Milton et al., 2019; Strickland, 2017). Once again, the teacher assists students in making connections between the various forms of representations (concrete manipulatives, pictures/drawings, and numerals/symbols) through explicit modeling and consistent language. Finally, during the abstract phase, students solve problems similar to those in the concrete and representational phases using numerals only (Flores, 2010; Flores et al., 2014; Forbringer & Fuchs, 2014; Hurrell, 2018; Milton et al., 2019; NCII, 2016; Strickland, 2017).

Research carried out by Fyfe et al. (2015) examined the effectiveness fading students from the use of concrete objects, or manipulatives, to visual representations on worksheets, to abstract representations only. The study utilized a between-subjects design with 64 third-grade students. Students were randomly placed into one of the following instructional groups: 1) concrete only, 2) abstract only, 3) concrete fading to abstract, and 4) abstract fading to concrete. A transfer assessment was administered to examine which instructional condition yielded the most transfer. Students that participated in the concrete fading to abstract yielded higher transfer than the other experimental conditions.

Flores (2010) evaluated the effectiveness of CRA on students' subtraction with regrouping computation fluency and maintenance of regrouping utilizing a multiple probe across students design with embedded changing behaviors. Six third-grade students with demonstrated mathematics difficulties participated in the study. All six students demonstrated improved performance with subtraction with regrouping and four out of six demonstrated maintenance after six weeks, suggesting that CRA is an effective instructional strategy.

In addition to the above effectiveness of CRA with student performance with subtraction, a functional relationship was established between CRA instruction and multiplication and division. Milton et al. (2019) researched the effects of CRA while alternating between multiplication and division instruction on students' conceptual understanding and unknown fact mastery. The study utilized a multiple probe across student design with fourth through sixth-grade students to determine fact mastery. Additionally, interviews were conducted to obtain data on students' conceptual understanding. In addition to improved fact mastery, students' ability to communicate conceptual understanding also increased.

Research conducted by Hinton and Flores (2019) examined the effects of CRA instruction on two third-grade students' rounding, subtraction with regrouping, and fraction comparison achievement. Students were provided small group intervention for 25 minutes per session, four days per week, over a 12-week period, and multiple probes were administered throughout to evaluate changes in student achievement. Both students' achievement on rounding, subtraction with regrouping, and fraction comparisons increased, and error patterns observed at the beginning of the study were no longer present at completion.

The three stages of CRA allow educators to scaffold student learning, help students develop a conceptual understanding of concepts, and aid in their ability to eventually work independently at the abstract level (Agrawal & Morin, 2016; Forbringer & Fuchs, 2014). To assist students in developing deeper conceptual knowledge, educators should explicitly link the concrete, visual representations, and abstract notations by utilizing consistent language across all three stages of instruction (Forbringer & Fuchs, 2014). The iterative process of CRA assists students in developing both conceptual and procedural knowledge through scaffolded instructional delivery and research suggests that it is an effective instructional method (Bouck et al., 2018; Flores, 2010; Fyfe et al., 2015; Hinton & Flores, 2019).

Virtual-Representational-Abstract (VRA) sequenced instruction. The virtual-representational-abstract (VRA) instructional sequence is a variation of the concrete-representational-abstract (CRA) instructional sequence (Bouck et al., 2017; Bouck & Sprick 2019; Park, et al., 2020; Root et al., 2020). Like CRA, VRA gradually moves students from manipulative use to abstract numerical representations, however, the concrete manipulatives are replaced with virtual manipulatives (Bouck, et al., 2017; Bouck, et al., 2019; Park, et al., 2020). The representational and abstract phases of the instructional sequence remain the same as with CRA (Bouck & Sprick, 2019).

Similar to the findings of Milton et al. (2019) regarding the concrete-representational-abstract instructional sequence with multiplication and division, Bouck et al. (2017) found a functional relation between the VRA instructional sequence and the students' ability to solve fraction equivalent problems. Bouck et al. (2017) researched the effectiveness of the VRA instructional sequence for equivalent fractions through a multiple probe across-students single-case design. The researchers worked with three middle school students with learning disabilities.

The researchers investigated student improvement in finding equivalent fractions and performance maintenance after an intervention of using virtual manipulatives to solve equivalent fraction problems while in the manipulative stage of the VRA instructional sequence. Student achievement was determined by means of a preassessment, baseline probes, intervention, and maintenance tasks.

Park et al. (2020) conducted a study to evaluate the effectiveness of the VRA instructional sequence on students' multiplication acquisition and skill maintenance after an intervention. The researchers utilized a single-case multiple probe across-participants design with three middle school students with disabilities. Baseline data was collected, an intervention was provided using virtual manipulatives during the manipulative phase of the VRA instructional sequence, and maintenance tasks were administered beginning one week after the intervention concluded. Results of the study indicated that student performance increased after the intervention and a functional relation was found between student performance and the VRA instructional sequence.

Research conducted by Root et al. (2020) investigated the effects of the VRA instructional sequence on student's multiplicative problem-solving skills. The researchers utilized a single-case multiple probe across participants research design and collected baseline, intervention, and maintenance data on three middle school students with developmental disabilities. Seven baseline sessions were conducted using virtual materials, representation materials, abstract materials, and maintenance materials. Students were then introduced to the intervention following the VRA process of virtual manipulative instruction, representation instruction, and abstract instruction. After the intervention was complete, students completed two maintenance probes in which visual supports were removed. All three students' multiplicative problem-solving accuracy increased and had positive maintenance performance when visual supports were removed after the intervention.

Summary

As students present with a wide range of academic abilities, it is imperative that educators present math content in a manner that supports students' construction of both conceptual and procedural knowledge. This begs the question of what is the most effective way of accomplishing this task? Two learning theories at the forefront of education are that of constructivism and behaviorism. Behaviorists suggest that knowledge is developed through direct, explicit instruction of skills, followed by both teacher-guided and independent practice of like problems. In contrast, constructivists posit that learners create knowledge through inquiry-based practices that require students to build upon prior learning and make connections to new concepts. Still yet, many researchers and educators alike posit that the most effective approach to successful student learning is a combination of the two learning theories with strategies that assist students in progressing from conceptual knowledge development to procedural knowledge development through a continuum of scaffolded instruction that sequences instructional materials and highlights critical features of the mathematics content (Flores, 2010; Fyfe et al., 2015; Milton et al., 2019; NMAP, 2008). Concrete manipulatives, virtual manipulatives, visual representations, explicit instruction, the concrete-representational-abstract (CRA) instructional sequence, and virtual-representational-abstract (VRA) instructional sequence provide scaffolds for student understanding and help to bridge the gap between conceptual and procedural knowledge development.

CHAPTER III: METHODOLOGY

To investigate how conceptual and procedural knowledge instruction in mathematics has been positioned within prominent research journals, this study utilized a quantitative content analysis design. This chapter provides an overview of the research methodology, outlining the purpose of the study, research questions, journal selection, procedures, and data analysis. The chapter culminates with a discussion on validity, reliability, and ethical considerations.

Purpose of the Study

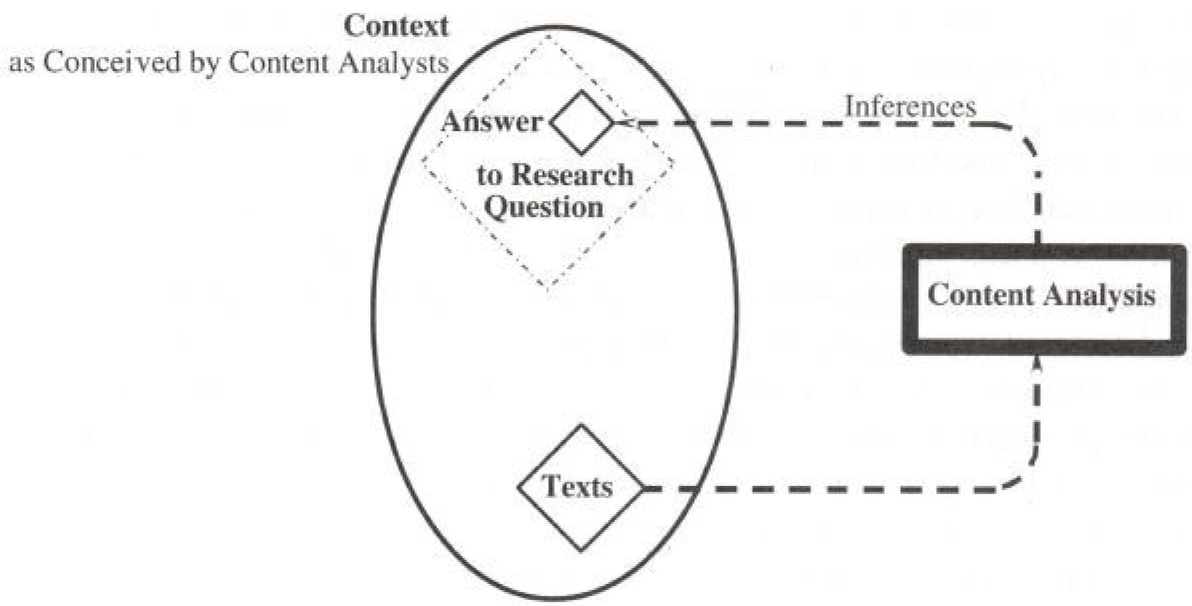
The purpose of this research is to describe the extent to which mathematics instruction that emphasizes conceptual knowledge, procedural knowledge, both conceptual and procedural knowledge, the progression from conceptual to procedural knowledge, and the progression from procedural to conceptual knowledge have been positioned within the publications of the *Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM) from 1970 to 2020. The study employed a quantitative content analysis research design.

Content Analysis

According to Krippendorff (2019), "content analysis is an empirically grounded method, exploratory in process, and predictive or inferential in intent" (p. 1) that assists researchers in "making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use" (p. 24). Content analysis allows the researcher to examine printed matter, through systematic reading, to understand what the material may mean, enable, or convey (Krippendorff, 2004, 2019). Through a content analysis, the researcher can infer trends and learn about the direction specific fields may be taking (Krippendorff, 2004, 2019). Krippendorff (2019) sets forth the following framework for content analysis: 1) an available body of text, 2) a research question, 3) a context with which the researcher uses to make sense of the text, 4) an analytical construct, 5) inferences aimed at answering the research question, and 6) evidence to validate the content analysis (see Figure 8). As noted in the framework, research questions for content analysis are answered by means of inferences drawn from the selected texts, therefore, the content analyst reads the selected texts with a specific purpose in mind (Krippendorff, 2004, 2019). According to Krippendorff (2019), the content analyst selects texts, conducts the content analysis, and makes inferences based on the findings to answer the research questions. The texts and research findings are situated in the context conceived by the content analyst.

Figure 8

Content Analysis Framework



Note. Reprinted from *Content Analysis: An Introduction to Its Methodology* (4th ed.) (p. 87), by K. Krippendorff, 2019, Sage Publications. Copyright 2019 by Sage Publications, Inc.

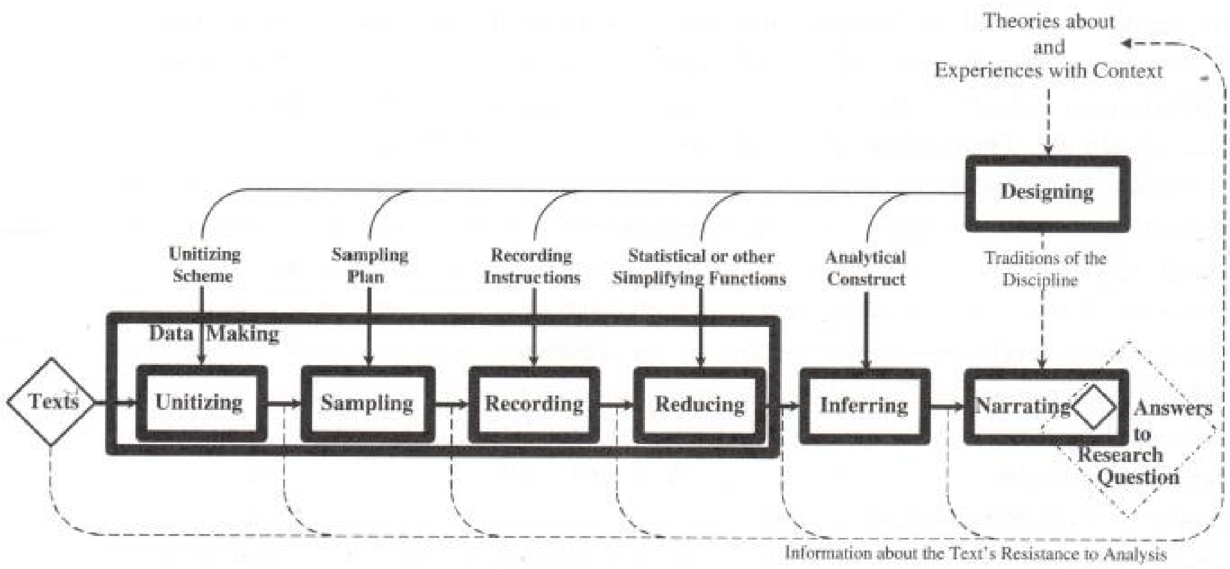
This research utilized both a designations analysis and a sign-vehicle analysis, both of which are specific types of the semantical content analysis. A semantical content analysis enables the researcher to categorize textual units according to their meanings (Krippendorff, 2004, 2019). A designations analysis focuses on the specific people, groups, things, or ideas discussed in a text (Krippendorff, 2004, 2019). A sign-vehicle analysis provides the frequency with which specific textual units appear within the text, for example, counting the number of times a specific word appears in the text (Krippendorff, 2004, 2019). "The frequency with which a symbol, idea, reference, or topic occurs in a stream of messages is taken to indicate the importance of, attention to, or emphasis on that symbol, idea, reference, or topic in the messages" (Krippendorff, 2019, p. 66).

According to Krippendorff (2019), a content analysis research design is comprised of six components (see Figure 9):

- Unitizing: systematically distinguishing segments of text that are of interest and pertain to the topic of study
- Sampling: limiting the units to a manageable and representative amount
- Recording/coding: transforming texts to analyzable representations
- Reducing: listing of types and frequencies for efficient representation
- Inferring: drawing inferences about phenomena and justifying such claims
- Narrating: explaining the results of the content analysis in a comprehensible way

Figure 9

Content Analysis Research Design



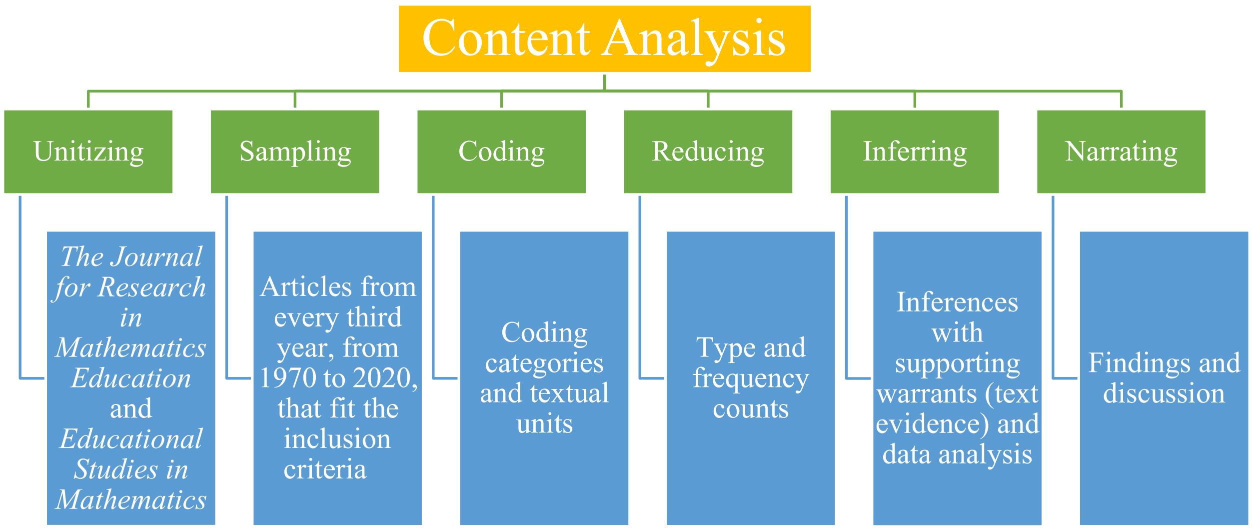
Note. Reprinted from *Content Analysis: An Introduction to Its Methodology* (4th ed.) (p. 90), by K. Krippendorff, 2019, Sage Publications. Copyright 2019 by Sage Publications, Inc.

The six components for the current study include the following (see Figure 10):

- Unitizing: *The Journal for Research in Mathematics Education and Educational Studies in Mathematics*
- Sampling: Articles from every third year, from 1970 to 2020, that fit the inclusion criteria
- Recording/coding: coding categories and textual units
- Reducing: Type and frequency counts
- Inferring: Inferences with supporting warrants (text evidence) and data analysis
- Narrating: Findings and discussion

Figure 10

Content Analysis Research Design for Current Study



The following research questions will guide this study:

1. How has mathematics instruction that emphasizes the following been positioned within prominent research journals from 1970-2020?
    a. only conceptual knowledge
    b. only procedural knowledge
    c. both conceptual and procedural knowledge
    d. the progression from conceptual to procedural knowledge, and
    e. the progression from procedural to conceptual knowledge
2. What textual units (keywords) related to conceptual and procedural knowledge are present within prominent research journals from 1970-2020?
3. How do research trends of the articles published in the journals vary from 1970 to 2020 (i.e., emphasis on procedural instruction only, conceptual instruction only, procedural and conceptual, or a progression from one to the other during certain time periods)?

Researcher Credentials

The researcher holds a bachelor's degree in Interdisciplinary Studies, with a specialization in mathematics, and master's degree in Curriculum and Instruction. Additionally, the researcher has 20 years of experience as a classroom teacher in the elementary grades, serving as a mathematics curriculum developer for 10 years, two years of experience as a mathematics instructional coach and interventionist, and two years of experience as a, gifted and talented, Response to Intervention (RtI), and mathematics education specialist. The researcher has 18 hours mathematics education, 9 hours at master's level and 9 hours at doctoral level.

Second Coder Credentials

A second rater served to establish reliability of the coding categories and keywords, as well as demonstrate replicability and validity. The second rater is an Assistant Professor in the Curriculum, Instruction, and Learning Sciences department at Texas A&M University – Corpus Christi and holds a Ph.D. in Mathematics Education from the University of Texas – Austin. The rater has 10 years of experience as a Coordinator for Mathematics Initiatives for the Texas Regional Collaboratives at the state level housed at the University of Texas in Austin and is a national level professional development instructional leader for Cognitively Guided Instruction (K-5th grades) and Extending Children's Mathematics (3rd-6th grades).

Data Corpus

The units, or distinguishing segments of text that are of interest and pertain to the topic, for the current study were all research articles published in *The Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM) from 1970 to 2020 (fifty years), enabling the researcher to examine the research trends in mathematics education pertaining to conceptual and procedural knowledge. These journals were selected based on their prestige in the mathematics community and the quality of their publications.

Journal quality is most commonly determined by one of two methods: citation-based and opinion-based (Williams & Leatham, 2017). Citation-based methods rely on measuring the citations within a given journal, while opinion-based methods rely on the opinions of scholars in the field (Williams & Leatham, 2017). Williams and Leatham (2017) conducted their own citation-based and opinion-based studies by investigating the relative quality of 20 mathematics journals. The researchers found that ESM ranked number one and JRME ranked number two using the citations-based method. The researchers also found the JRME ranked number one and ESM ranked number two using the opinion-based method. This indicates that JRME and ESM are the two most highly ranked mathematics journals.

As described on the journal's website, *The Journal for Research in Mathematics Education* (JRME) is a peer-reviewed publication of the National Council of Teachers of Mathematics (NCTM) and focuses on research pertaining to the teaching and learning of mathematics from preschool through college (National Council of Teachers of Mathematics, 2020). The journal is published five times per year online, began in 1970, has an SJR of 2.916, which places it in quartile 1, and an H index of 74. As mentioned above, JRME is one of the two most highly ranked mathematics journals and is of very high quality (Williams & Leatham, 2017).

The *Educational Studies in Mathematics* (ESM) journal aims to present mathematics education developments and ideas by means of peer-reviewed high-level content that focuses on methodological and pedagogical subjects (Springer, 2020). According to Williams and Leatham (2017), ESM is one of the two most highly ranked mathematics journals, and like JRME, is considered very high quality. ESM is published an average of nine times per year, began in 1969, has an SJR of 1.57, placing it in quartile 1, and an H Index of 60.

This research study will focus on articles published beginning in 1970, as that is the earliest common date of publication for both journals. The ending year for data collection was set as 2020, as that is the last year available for both journals at the time this research was being conducted.

Procedures

Pilot Study

A pilot study was conducted to ensure the data collection methods were viable and assist in the development of the main study's recording/coding and textual units (keywords). A preliminary list of textual units (keywords) was developed based on the researcher's knowledge of the topic to employ in the pilot study. One year, 2019, from the *Journal of Mathematical Behavior* (JMB) was utilized. The JMB was chosen because of its similarities to the JRME and the ESM. JMB is an international, peer reviewed journal, is of very high quality (Williams & Leatham, 2017), and focuses on the learning and teaching of mathematics. All research articles form 2019 were downloaded and the study's inclusion criteria applied. The articles that remained were analyzed to determine the appropriate coding category (conceptual only, procedural only, both, progression between the two) and the frequency of specific textual units (keywords). During the analysis stage, the researcher noted additional textual units (keywords) that were present in the articles and would be important to the final study, thus developing a comprehensive codebook (see Appendix A).

Descriptive statistics were run to determine the frequency of coding categories represented in the pilot study data. According to Urdan (2017), descriptive statistics are used to describe the characteristics of a specific sample. Forty percent of the articles analyzed in the pilot study were found to focus on conceptual development only, 20% focused on both conceptual and procedural, and 40% focused on a progression from conceptual to procedural development. The results are summarized in Table 4 and Figure 11.

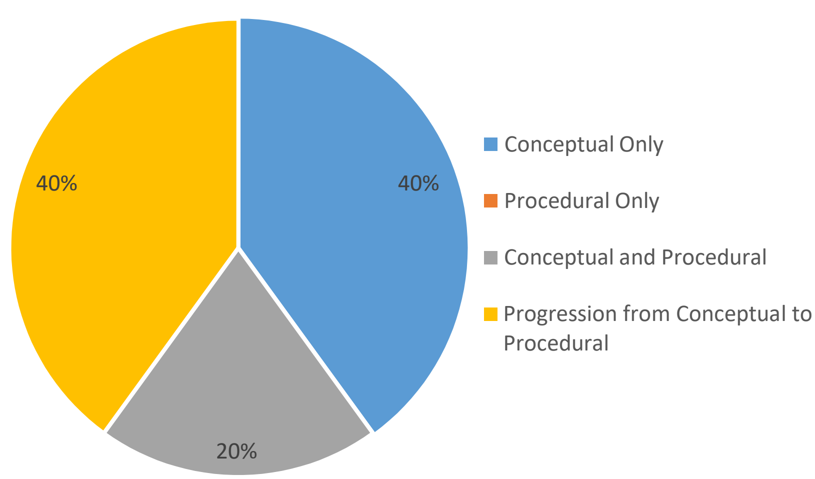
Table 4

Frequency of Coding Categories (JMB)

Coding Category	Frequency	Percent
Conceptual Only	2	40.0%
Conceptual and Procedural	1	20.0%
Progression from Conceptual to Procedural	2	40.0%
Total	5	100.0%

Figure 11

Coding Category Percentages (JMB)



Final Study Protocol

This research systematically reviewed literature from the JRME and ESM utilizing an *a priori* defined protocol (see Table 5). This *a priori* defined protocol was used to identify and screen articles, apply inclusion criteria, and delineate the final study sample. An *a priori* defined protocol is a protocol that has been formed or conceived beforehand to aid in a researcher's systematic review. The resulting final studies were then coded using quantitative content analysis.

Table 5

A Priori Defined Protocol - Identification, Screening, and Inclusion Procedures

Identification	Screening and Eligibility	Inclusion
Articles identified through JSTOR, Springer, and NCTM journal archives (units)	Articles screened based on title Articles screened based on abstract Articles screened based on full text	Articles included in the content analysis (sample set)

Eligibility Criteria

Eligible studies consisted of studies that were:

- Non-interventional
- Focused on instructional strategies and/or instructional practices
- Conducted in K-12 educational settings
- Written in English
- Published between 1970 and 2020.

Interventional studies were excluded from the data set as they often contain multiple experimental and control groups, which would result in one study being coded as fitting several coding categories simultaneously in the current study. Additionally, the data analysis methods required to analyze the various combinations of research methodologies in interventional studies was beyond the means of this study. To be included in the current study, studies must also focus on instructional strategies or practices. Exploratory studies that documented students' current knowledge or achievement on a topic, tested the validity of assessments, or assessed students' attitudes or perceptions were excluded from the data set.

Identification and Screening of Studies

Articles from ESM were obtained from JSTOR (1970-2015) and Springer (2018 and 2020), while articles from JRME were obtained from the National Council of Teachers of Mathematics (NCTM) website (see Table 5). Each entry within a volume of the ESM (based on the table of contents listings) was downloaded as individual PDF files, resulting in a total of 979 files. Each volume of the JRME from 1970 to 1994 was only available as one large PDF containing the entire volume. Individual entries (based on the table of contents listings) of each volume were extracted and saved as individual PDF files. From 1997 through 2020, each entry within a volume was provided by JSTOR as its own PDF. In total, the JRME produced 672 individual PDF files. A first level screening was conducted based on article titles to remove articles not written in English, those not focused on instructional strategies and/or instructional practices, and those that were interventional (see Table 5).

A second level screening was conducted to review the remaining study sample article abstracts to apply the eligibility criteria (see Table 5). If an abstract was not present in the article, the screening was based on the paragraph within the text that stated the study's purpose. Studies not focused on instructional strategies and/or instructional practices and those that were interventional were removed from the study sample. Third level screening consisted of reading the remaining study sample articles in full to apply the eligibility criteria (see Table 5). The articles that remained, 40 ESM and 12 JRME, were deemed the sample set for the study.

Study Review and Coding

The remaining sample set articles, 40 ESM and 12 JRME, were read in full a second time to analyze the content and code the findings (see Table 5). A comprehensive codebook was utilized to standardize the coding process and recording/coding units (see Appendix A). Coding categories and textual units (keywords) were developed based on the researcher's knowledge of the topic as well as a pilot review of articles published in the *Journal of Mathematical Behavior* (JMB) in 2019. Employing the codebook, the researcher coded study characteristics, conceptual and procedural concept category information, and textual unit characteristics.

Study characteristics coded included the title of the study, grade band focus of the study (e.g. elementary, secondary, elementary and secondary, not specified/determined), name of the journal, and year of publication (see Appendix B). Text reference to conceptual knowledge development, procedural knowledge development, the progression from conceptual to procedural knowledge development, and the progression from procedural to conceptual knowledge development, whether specifically stated or implied, was coded as conceptual and procedural concept category information along with supporting warrants. As noted by Krippendorff (2019), "the research questions of content analysis must be answered through inferences drawn from texts" (p. 39) while supported by appropriate warrants. Textual unit characteristics included the frequency with which specific textual units (keywords) appear in the text. Each article was coded against a pre-defined list of keywords (developed *a priori* after the pilot study) (see Appendix

B). As content analysts rely on text already in print, in a multitude of formats, the researcher cannot anticipate all the terms their data sources may utilize (Krippendorff, 2004, 2019). Therefore, additional textual units were added to the initial list as new terms are discovered during the final study. Abstracts were not included in the keyword count, as many of the abstracts repeated sentences from the main text. Additionally, tables, figures, and diagrams were excluded from the keyword count. The meaning of each keyword was considered before coding the word to ensure it was used in a way that was consistent with the intent of this study.

Data Analysis

Data was quantitative analyzed using SPSS. Descriptive statistics, including frequency counts, mean, standard deviation, and range were employed. Crosstabulations, including Cohen's Kappa and Chi-Square, were also utilized to determine associations, or relationships, between variables.

Establishing Reliability and Validity

According to Krippendorff (2019), "a research procedure is reliable when it responds to the same phenomena in the same way regardless of the circumstances of its implementation" (p. 277). To establish reliability, the researcher selected a random sample of five research articles included in the final study to reanalyze 14 days after the initial analysis of each article to collect reliability data and Cohen's Kappa was used to test for stability. Krippendorff (2019) describes stability as such:

> Stability is the degree to which a process is unchanging over time. It is measured as the extent to which a measuring or coding procedure yields the same results on repeated trials. The data for such assessments are created under *test-retest* conditions; that is, one observer rereads, recodes, or reanalyzes the same text, usually after some time has

> elapsed, or the same measuring device is repeatedly applied to one set of objects. (pp. 280-281)

Additionally, an external mathematics researcher and educator served as a second rater. The external mathematics educator analyzed a sample of five research articles to establish reliability and demonstrate replicability. According to Krippendorff (2019), "Demonstrating replicability requires reliability data that are obtained under *test-test* conditions: for example, two or more individuals, working independently of each other, apply the same recording instructions to the same units of analysis" (p. 281). Following Krippendorff's (2019) recommendation, assessment of the reliability of the data will meet the following criteria: 1) communicable instructions, 2) communicable criteria for the selection of individual coders, and 3) independently generated data work. The results by both the researcher and the external mathematics educator will be analyzed for inter-coder reliability using Cohen's Kappa.

Semantic validity was established by means of the external mathematics researcher and educator. Semantic validity is "The extent to which the categories of an analysis of texts corresponds to the meanings these texts have within the chosen context" (Krippendorff, 2019, p. 366). The external mathematics educator served as a source of validating evidence for the categories the researcher uses in the study. Face validity was established through plausible inferences. Face validity "is fundamentally an individual's judgment with the assumption that everyone else would agree with it" (Krippendorff, 2004, p. 361).

Delimitations

One inclusion criterion for this research study was the years analyzed. The years selected to be analyzed were every third year from 1970 to 2020. The study utilized the following years: 1970, 1973, 1976, 1979, 1982, 1985, 1988, 1991, 1994, 1997, 2000, 2003, 2006, 2009, 2012, 2015, 2018, and 2020. Furthermore, this study excluded interventional studies and only focused on non-interventional studies, as the complexities associated with the data collection and analysis of interventional studies was beyond the means of this research. Finally, the study was limited to the *Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM) research journals.

Ethical Considerations

Human participants were not involved in this research. Therefore, informed consent, voluntary participation, confidentiality, and anonymity do not apply.

Summary

This study utilized a quantitative content analysis design to investigate how conceptual and procedural knowledge are positioned within the *Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM) from 1970 to 2020. The study employed a quantitative content analysis research design to infer trends and learn about the direction the field of mathematics understanding and learning has taken regarding conceptual and procedural knowledge development. The researcher first conducted a pilot study to ensure viability of the selected research method and to assist in developing a comprehensive codebook to be utilized in the final study. For the final study, the researcher selected an available body of text (JRME and ESM), applied an *a priori* defined protocol to identify and screen articles, applied the study's inclusion criteria, and coded each remaining article (see Appendix C). Studies eligible for coding were non-interventional, focused on instructional strategies and/or instructional practices in K-12 educational settings, were written in English, and published between 1970 and 2020. To ensure reliability, a random sample was selected from the final study sample and reanalyzed 14 days after the initial coding. An additional test for reliability was conducted by means of an external mathematics educator and researcher, whose findings for five randomly chosen articles were compared to that of the researcher, and found to have moderate agreement.

CHAPTER IV: FINDINGS

This chapter describes the results of the data analyses conducted for the main study to test for reliability, identify the frequency of coding categories and textual units, and determine trends that may exist in the data. A crosstabulation was conducted, utilizing Cohen's Kappa, to establish reliability in the category coding procedure from both the original researcher's coding as well as the second rater's coding. Descriptive statistics, including frequency counts, mean, standard deviation, and range, were employed to answer research questions one and two. Descriptive statistics were also employed to describe the findings from the first 15 years of the two journal's publications. Additionally, a crosstabulation was conducted, using Chi-Square, to determine associations, or relationships, between variables to answer research question three.

Results

Reliability

Five articles were randomly selected from the main study's initial data set for reanalysis by the researcher 14 days after the initial analysis to establish reliability. The purpose of this reanalysis and establishment of reliability was to demonstrate consistency in the researchers coding results. Cohen's Kappa (κ) was used to determine if there was agreement between the two analysis ratings on whether the coding category of each article was conceptual only, procedural only, both conceptual and procedural, or a progression between the two. Four of the five articles were coded as conceptual only and one was coded as both conceptual and procedural in both the original analysis and the reanalysis. Cohen's Kappa reflects the degree of agreement between two ratings classifying data into mutually exclusive categories and ranges from -1 (perfect disagreement) to +1 (perfect agreement) (Vogt et al., 2014). There was perfect agreement between the two analysis ratings, $\kappa = 1.000$, $p < .05$. This perfect agreement demonstrates that the codes assigned to each article during the researcher's first analysis matched the codes assigned to each article during the researcher's second analysis perfectly. These results are summarized in Table 6 and Table 7.

Table 6

Category Rating 1 and Rating 2 Crosstabulation

Category Rating 1	Category Rating 2					
	Conceptual Only		Conceptual and Procedural		Total	
	N	%	N	%	N	%
Conceptual Only	4	100.0%	0	0.0%	4	80.0%
Conceptual and Procedural	0	0.0%	1	100.0%	1	20.0%
Total	4	100.0%	1	100.0%	5	100.0%

Table 7

Cohen's Kappa of Category Rating 1 and Category Rating 2 Crosstabulation

Measure of Agreement	Value	Asymptotic Standard Error[a]	Approximate T[b]	Approximate Significance
Kappa	1.000	.000	2.236	.025

In addition, an external mathematics researcher and educator served as a second rater to further test for reliability. The purpose of this reliability test was to demonstrate agreement with the original researcher's findings. The same five articles that the original researcher analyzed and then reanalyzed after 14 days were also analyzed by the external researcher and mathematics educator and coded as conceptual only, procedural only, both conceptual and procedural, or a progression between the two. The study's researcher coded four of the five articles as conceptual only and one as both conceptual and procedural. The second rater coded three articles as conceptual, one article as both conceptual and procedural, and one article as a progression from conceptual to procedural. The article coded differently between the researcher and the second rater was The Nature of Student Predictions and Learning Opportunities in Middle School Algebra in *Educational Studies in Mathematics* (2011). Cohen's Kappa was run to demonstrate interrater reliability and was found to be moderate at $\kappa = .583$, $p = .05$. These results are summarized in Table 8 and Table 9.

Table 8

Cohen's Kappa of 2nd Rater Category Rating 1 and Category Rating 2 Crosstabulation

Measure of Agreement	Value	Asymptotic Standard Error[a]	Approximate T[b]	Approximate Significance
Kappa	.583	.324	1.957	.050

Table 9

2nd Rater Category Rating 1 and Rating 2 Crosstabulation

Category Rating 1		Category Rating 2: Conceptual Only	Conceptual and Procedural	Progression from Conceptual to Procedural	Total
Conceptual Only	Count	3	0	1	4
	% within CodingCat 1	75.0%	0.0%	25.0%	100.0%
	% within CodingCat 2	100.0%	0.0%	100.0%	80.0%
	% of Total	60.0%	0.0%	20.0%	80.0%
Conceptual and Procedural	Count	0	1	0	1
	% within CodingCat 1	0.0%	100%	0.0%	100.0%
	% within CodingCat 2	0.0%	100%	0.0%	20.0%
	% of Total	0.0%	20.0%	0.0%	20.0%
Total	Count	3	1	1	5
	% within CodingCat 1	60.0%	20.0%	20.0%	100.0%
	% within CodingCat 2	100%	100%	100.0%	100.0%
	% of Total	60%	20%	20.0%	100.0%

Articles from 1970 to 1985

No articles from 1970 to 1985 within every third year, in either journal, met the inclusion criteria of the study and were therefore excluded from the data set. There was a total of 389 files excluded because they were either not written in English, an interventional study, not situated in K-12 education, not focused on an instructional strategy or practice, or not a research study. Based on a review of the file titles, 128 files were excluded. 96 were not studies, and 32 were not written in English. Based on the abstract or a full read of the text, 261 files were excluded. 99 were not a study, 39 were interventional, 31 were not conducted in a K-12 educational setting, and 50 were not an instructional strategy or practice. Some files violated multiple inclusion criteria. Nine were a combination of not a study, not in a K-12 educational setting, and not an instructional strategy or practice. Five were a combination of not in a K-12 educational setting and not an instructional strategy. Three were a combination of not a study and not in a K-12 educational setting. Seven were a combination of not a study and not an instructional strategy or practice. Seven were a combination of interventional and not an instructional strategy. Four were a combination of interventional and not in a K-12 educational setting. Finally, seven were a combination of interventional and not a study. A table of the excluded files can be found in Appendix D and Appendix E. The results of the exclusion findings are summarized in Table 10.

Table 10

Articles Excluded Based on Title, Abstract, or Full Read from 1970 to 1985

Exclusion Criteria (s)	Based on Title		Based on Abstract or Full Read	
	Frequency	%	Frequency	%
Not a Study	96	75.0%	99	37.9%
Not Written in English	32	25.0%	0	0%
Interventional	0	0.0%	39	14.9%
Not K-12	0	0.0%	31	11.9%
Not Instructional Strategy	0	0.0%	50	19.2%
Not a Study, Not K-12, Not an Instructional Strategy	0	0.0%	9	3.5%
Not K-12, Not an Instructional Strategy	0	0.0%	5	1.9%
Not a Study, Not K-12	0	0.0%	3	1.2%
Not a Study, Not an Instructional Strategy	0	0.0%	7	2.7%
Interventional, Not an Instructional Strategy	0	0.0%	7	2.7%
Interventional, Not K-12	0	0.0%	4	1.5%
Interventional, Not a Study	0	0.0%	7	2.7%
Total	128	100%	261	100%

Research Question 1 Results

Research question one sought to determine how mathematics instruction that emphasizes the following have been positioned within prominent research journals from 1970-2020?

a. only conceptual knowledge

b. only procedural knowledge

c. both conceptual and procedural knowledge

d. the progression from conceptual to procedural knowledge, and
e. the progression from procedural to conceptual knowledge

Each article in the study was read and coded with one of the above category codes.

To answer research question one, descriptive statistics were run to determine the frequency of coding categories represented in the data. Seventy-five percent of the *Educational Studies in Mathematics* (ESM) articles analyzed in the study were found to focus on conceptual development only, 17.5% focused on both conceptual and procedural, and 7.5% focused on a progression from conceptual to procedural development. Seventy-five percent of the *Journal for Research in Mathematics* (JRME) articles analyzed in the study were found to focus on conceptual development only, 8.3% focused on procedural development only, 8.3% focused on both conceptual and procedural, and 8.3% focused on a progression from conceptual to procedural development. These results are summarized in Table 11.

Table 11

Frequency of Coding Categories in ESM and JRME

Coding Category	ESM		JRME	
	Frequency	%	Frequency	%
Conceptual Only	30	75%	9	75%
Procedural Only	0	0%	1	8.3%
Conceptual and Procedural	7	17.5%	1	8.3%
Progression from Conceptual to Procedural	3	7.5%	1	8.3%
Total	40	100%	12	100%

Additionally, the total frequency of coding categories represented in the data from both ESM and JRME were analyzed. Seventy-five percent of the articles were found to focus on conceptual development only, followed by 15.4% that focused on both conceptual and procedural development, 7.7 that focused on a progression from conceptual to procedural, and 1.9% that focused on procedural development only. These results are summarized in Table 12 and Figure 12.

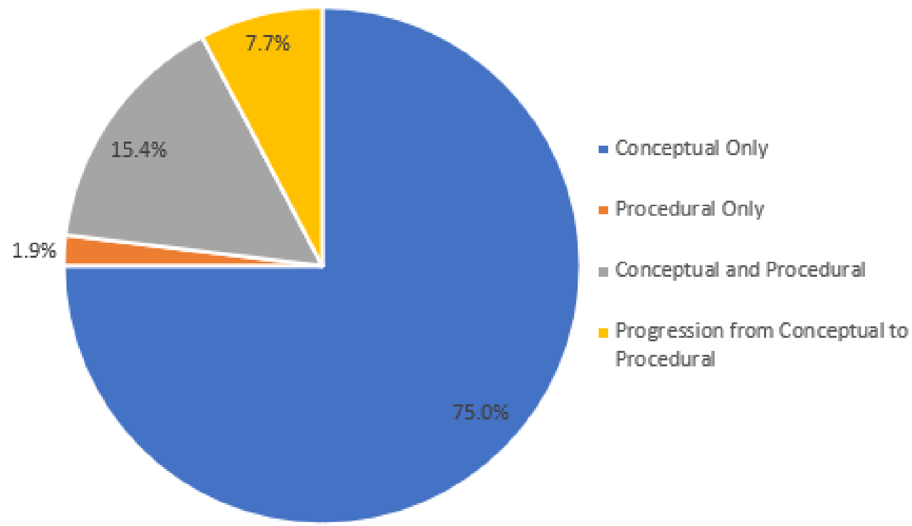
Table 12

Total Frequency of Coding Categories (ESM and JRME)

Coding Category	Frequency	Percent
Conceptual Only	39	75.0%
Procedural Only	1	1.9%
Conceptual and Procedural	8	15.4%
Progression from Conceptual to Procedural	4	7.7%
Total	52	100.0%

Figure 12

Coding Category Percentages (ESM and JRME)



For the two journals, both individually and collectively, 75% of the articles included in the study focused on conceptual knowledge only. These articles included content that support instructional practices that promote sense-making and interpretation. For example, in *Educational Studies in Mathematics*, English (2012) explained that data modeling "engages children in extended and integrative experiences in which they generate, test, revise, and apply their own models in solving meaningful problems" (p. 27) and highlighted research that emphasizes the understanding and interpretation of data in place of following procedures. Likewise, in *Educational Studies in Mathematics*, Elbers (2003) discusses research in which a teacher stimulated the students "to construct mathematical knowledge, for which he gave them space, rather than asking them to keep to the guidelines set out by a textbook" and that "he wanted his students to engage in mathematical thinking, and this would not be consistent with the performance of mechanical calculations or mindless acceptance of teachers' instructions" (p. 79). A final example from the study sample of articles can be found in the *Journal for Research in Mathematics Education*, whereas Yackel et al., (1991) explain that "it was never the teacher's intent to show the students a procedure for completing the activities or to explain how to do them" (p. 393) and that "the children explained and justified their interpretations of activities and solution attempts" (p. 394).

The second most common category collectively coded in the study, at 15.4%, was a focus on both conceptual and procedural knowledge. These articles included content that support instructional practices that promote student learning of a prescribed series of steps, procedures, and skills as well as sense-making and interpretation. For example, in *Educational Studies in Mathematics*, Pirie and Schwarzenberger (1988) posited that understanding encompasses "the comprehension of concepts, the relationships between these concepts and ordinary language or physical objects" (p. 461). The authors continued by stating that "such comprehension must also include the procedural and process skills which depend upon familiarity with these relationships" (p. 461). Another example can be found in *Educational Studies in Mathematics*, whereas Jaworski and Potari (2009) described a teacher's instructional goal during mathematics instruction. The authors stated that the teacher's "goal was that his students should understand and be involved in doing mathematics and also develop mathematical skills" (p. 224). These research studies support the notion that conceptual and procedural knowledge are iterative, and as previously noted in the review of literature, procedures are conceptual in nature "during their period of elaboration" and that "even when they function as automatized skills" they "are regularly being updated, revised, and extended by means of conceptual elements" (Kieran, 2013, p. 154).

The third most common category collectively coded in the study, at 7.7%, was a focus on the progression from conceptual knowledge to procedural knowledge. These articles included content that support instructional practices that guide students through developing conceptual knowledge before progressing to procedural knowledge. For example, in *Educational Studies in Mathematics*, Parzysz (1988) stated that only after students pass through a phase of using 3D representational models can they learn to do without them to solve problems in space geometry. Likewise, in *Educational Studies in Mathematics*, Van Den Heuvel-Panhuizen (2003), conducted research on a Dutch approach to mathematics instruction in which models progress from representational drawings to strip diagrams used for estimation and culminate with the use of abstract tools. A final example from the study sample of articles can be found in the *Journal for Research in Mathematics Education*, whereas Murata and Fuson (2006) described a teacher's actions as assisting constructive paths by shifting instructional emphasis from conceptual to a 'short-cut' way of solving problems involving addition. These support the various researchers discussed in the review of literature that posit that learners progress through the enactive, iconic, and symbolic phases when developing new concepts (Bruner, 1966) as well as research conducted by various other researchers on utilizing the concrete-representation-abstract (CRA) instructional sequence (Agrawal & Morin, 2016; Doabler et al., 2012; Flores, 2010; Flores et al., 2014; Flores et al., 2016; Mudaly & Naidoo, 2015; National Center on Intensive Intervention [NCII], 2016; Strickland, 2017).

Furthermore, the results of research question one help support the theoretical framework of constructivism which grounds this current research. As noted previously in the review of literature, constructivism refers to the idea that construction of new knowledge occurs when students reconfigure mental connections, ideas, and procedures already learned when presented with information that does not easily correlate to this prior knowledge (Hiebert & Grouws, 2006) and as von Glasersfeld (1995) posited, knowledge of concepts must be conceived rather than transferred from the teacher to the student.

Research Question 2 Results

As previously discussed in the review of literature, conceptual learning has the following attributes: 1) recognition of patterns in information, 2) formation of links with a concept, 3) acquisition of deeper understanding of an individual concept, 4) discovery of relevance and value, and 5) application of concepts to new situations (Fletcher, et al., 2019). Additionally, conceptual understanding occurs when students implement a variety of solution plans, beginning at various starting points, to solve a mathematical task and where understanding is the ultimate goal rather than the completion of set steps (Skemp, 2006). Common instructional practices that are attributed to assisting students in developing conceptual knowledge are the use of concrete manipulatives and visual representations. As such, the following textual units (and their variations) were chosen as linked to conceptual knowledge: conceptual, conceptual knowledge (or relational knowledge), relationship, connection, manipulative, concrete, concrete manipulative, representation, concrete representation, visual representation, graphical representation, pictorial representation, pictorial, graphical, drawing (a pictorial representation either drawn by the student or presented to the student to aid in concept visualization), and scaffold (see Table 13).

Students learn fixed steps or exact procedures to solve specific types of tasks (Skemp, 2006). Behaviorist approaches to learning are often associated with procedural knowledge development as tasks are broken into small steps, taught explicitly, often through the direct instructional model (Poncy et al., 2010), and repeated until students demonstrate mastery (Grady et al., 2018). Procedural instruction provides students with knowledge of the skills or series of steps required to solve mathematical problems (Canobi, 2009; Rittle-Johnson et al., 2015) and execute procedures fluently. Therefore, the following textual units (and their variations) were chosen as linked to procedural knowledge: procedural, procedural knowledge (or instrumental knowledge), procedure, abstract representation, symbolic representation, explicit instruction, and direct instruction (see Table 13).

Table 13

Textual Units

Conceptual Knowledge Textual Units	Procedural Knowledge Textual Units
Conceptual	Procedural
Conceptual knowledge (relational knowledge)	Procedural knowledge (instrumental knowledge)
Relationship	Procedure
Connection	Abstract representation
Manipulative	Symbolic representation
Concrete	Explicit instruction
Concrete manipulative	Direct instruction
Representation	
Concrete representation	
Visual representation	
Graphical representation	
Pictorial	
Graphical	
Drawing	
Scaffold	

The textual units were ordered based on type of knowledge and words that describe student actions associated with such knowledge, followed by instructional tools and strategies that support the two types of knowledge. For example, for conceptual knowledge, the units *conceptual*, *conceptual knowledge*, *connection* and *relationship* were grouped together. For procedural knowledge, *procedural*, *procedural knowledge*, and *procedure* were grouped together. Finally, *manipulative, concrete, concrete manipulative, representation, concrete representation, visual representation, pictorial, drawing, abstract representation, explicit instruction,* and *scaffold* were grouped together as instructional tools and strategies. The results are summarized in Figure 13 and Table 14. Frequency denotes how many times each textual unit was counted, mean denotes the average number of occurrences, standard deviation denotes the amount of variation or dispersion within the set of values, and range denotes the difference between the lowest and highest frequency value for each textual unit.

To answer research question two, descriptive statistics were run to determine the frequency of textual units (keywords) represented in the data that relate to conceptual and procedural knowledge. The five most prevalent keywords, including variations of each, were found to be *relationship* [f=378, *M*=7.27, *SD*=9.021], *representation* [f=278, *M*=5.35, *SD*=10.546], *procedure* [f=182, *M*=3.50, *SD*=9.668], *conceptual* [f=164, *M*=3.15, *SD*=8.819], and *connection* [f=138, *M*=2.65, *SD*=5.009]. These results support the findings from research question one as 75% of the collective research study articles were coded as conceptual knowledge and the terms relationship, conceptual, and connection are related to conceptual knowledge. For example, in the article The Nature of Student Predictions and Learning Opportunities in Middle School Algebra in *Educational Studies in Mathematics*, coded as focusing on conceptual knowledge, the textual unit *relationship* was found 28 times and *connection* was found 25 times. Additionally, in the article Supporting Latino First Graders' Ten-Structured Thinking in Urban Classrooms in the *Journal for Research in Mathematics Education*, coded as focusing on conceptual knowledge, the textual unit *relationship* was found 19 times, *conceptual* was found 60 times, and *connection* was found one time. The textual unit analysis results also support the finding that the second highest percentage of study articles were coded as both conceptual and procedural as the terms relationship, conceptual, and connection are related to conceptual knowledge and the term procedure is related to procedural knowledge. For example, in the article An Analysis of Diverging Approaches to Simple Arithmetic: Preference and its Consequences in *Educational Studies in Mathematics*, coded as focusing on both conceptual and procedural knowledge, the textual unit *relationship* was found 8 times, *conceptual* was found 5 times, and *procedure* was found 21 times. Likewise, in the article Epistemological Investigation of Classroom Interaction in Elementary Mathematics Teaching in *Educational Studies in Mathematics*, coded as focusing on both conceptual and procedural knowledge, the textual unit *relationship* was found 51 times, *conceptual* was found 10 times, *connection* was found 9 times, and *procedure* was found 6 times.

Figure 13

Frequency of Textual Units (Keywords)

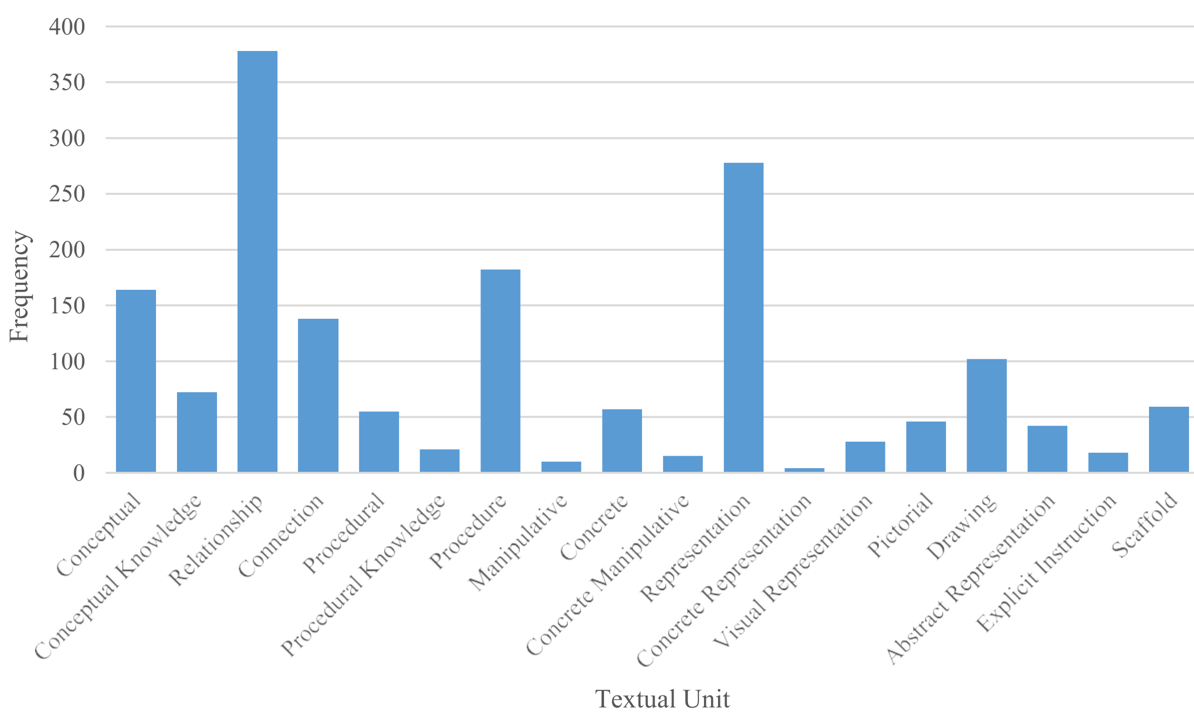


Table 14

Textual Units (Keywords) Descriptive Statistics

Textual Unit	Frequency	Mean	Std. Deviation	Range
Conceptual	164	3.15	8.819	60
Conceptual Knowledge	72	1.38	3.769	26
Relationship	378	7.27	9.021	51
Connection	138	2.65	5.009	25
Procedural	55	1.06	2.993	15
Procedural Knowledge	21	.40	1.445	7
Procedure	182	3.50	9.668	62
Manipulative	10	.19	.715	4
Concrete	57	1.10	2.507	15
Concrete Manipulative	15	.29	1.035	5
Representation	278	5.35	10.546	49
Concrete Representation	4	.08	.436	3
Visual Representation	28	.54	1.614	9
Pictorial	46	.88	2.111	10
Drawing	102	1.96	5.974	33
Abstract Representation	42	.81	1.482	7
Explicit Instruction	18	.35	.883	4
Scaffold	59	1.13	3.332	20

Research Question 3 Results

Research question three sought to determine how research trends of the articles published in the journals vary from 1970 to 2020 (i.e., emphasis on procedural instruction only, conceptual instruction only, procedural and conceptual, or a progression from one to the other during certain time periods)? No articles from 1970 to 1985, in either journal, met the inclusion criteria of the study and were therefore excluded from the data set. The articles that remained after the inclusion criteria was applied spanned 1988 to 2020 and were divided into three time periods, 1988 - 1997, 1998 - 2006, and 2007 - 2020. These time periods were chosen as they most equally divided the number of articles among the time periods. Thus, the following hypotheses were developed.

Ho: There will be no statistically significant difference between the proportion of the 5 categories between the periods 1988 – 1997, 1998 – 2006, and 2007 – 2020.

H1: There will be a statistically significant difference between the proportion of the 5 categories between the periods 1988 – 1997, 1998 – 2006, and 2007 – 2020.

A Chi-Square test was conducted. According to Urdan (2017), a chi-square test "allows you to determine whether cases in a sample fall into categories in proportions equal to what one would expect by chance" (p. 205). In other words, it compares categorically coded data with expected frequencies. The chi-square test showed that there were no statistically significant differences in the proportion of articles in each category between the three time periods in YearCat3 ($\chi2(6) = 7.755$, $p=.257$). For the variable YearCat3 (categories coded for each article separated out among the three time periods), with degrees of freedom of 6, the Pearson chi-square test ($\chi2$) resulted in a value of 7.755 with a p value of .257. The researcher, therefore, accepted the Ho, and concluded that there is no evidence that there are significant differences in the proportions.

The Chi-Square test showed that regarding articles coded as focusing on conceptual only, 11 were from 1988 to 1997, 15 were from 1998 to 2006, and 13 were from 2007 to 2020. Regarding articles coded as focusing on procedural only, zero were from 1988 to 1997, zero were from 1998 to 2006, and one was from 2007 to 2020. Articles coded as focusing on both conceptual and procedural knowledge were found to have five from 1988 to 1997, one from 1998 to 2006, and two from 2007 to 2020. Finally, regarding articles coded as focusing on the progression from conceptual to procedural knowledge, two were from 1988 to 1997, two were from 1998 to 2006, and zero were from 2007 to 2020. This suggests that the trends have remained consistent from 1988 to 2020 regarding the direction researchers have taken with reference to conceptual and procedural knowledge with non-interventional research. These results are summarized in Table 15 and Table 16.

Table 15

Chi-Square Test of Category and Time Period

Measure	Value	Df	Asymptotic Significance (2-sided)
Pearson Chi-Square	7.755	6	.257

Table 16

Time Period and Category Crosstabulation

	Time Period			
Coding Category	1988-1997	1998-2006	2007-2020	Total
Conceptual Only				
Count	11	15	13	39
% within Time Period	61.1%	83.3%	81.3%	75%
Procedural Only				
Count	0	0	1	1
% within Time Period	0.0%	0.0%	6.3%	1.9%
Conceptual and Procedural				
Count	5	1	2	8
% within Time Period	27.8%	5.6%	12.5%	15.4%
Progression from				
Conceptual to Procedural				
Count	2	2	0	4
% within Time Period	11.1%	11.1%	0.0%	7.7%
Total				
Count	18	18	16	52
% within Time Period	100.0%	100.0%	100.0%	100.0%

Summary

This chapter described the results of content analyses of the main study. Data was quantitative and analyzed using SPSS, utilizing a variety of statistical methods. Reliability of the coding category coding was tested with a crosstabulation, utilizing Cohen's Kappa, to determine if there was consistency between two trials of coding and showed perfect agreement. To answer research questions one and two, descriptive statistics were analyzed, including frequency counts, mean, standard deviation, and range. Results indicated that, in total, 75% of the research articles focused on conceptual knowledge only, 1.9% focused on procedural knowledge only, 15.4% focused on both conceptual and procedural knowledge, and 7.7% focused on the progression from conceptual to procedural knowledge. Additionally, the textual unit analysis found that the five most frequent textual units were r*elationship*, *representation*, *procedure*, *conceptual*, and *connection*. Finally, to answer research question three, a crosstabulations using Chi-Square was conducted to determine relationships between the coding categories and time periods and indicated that there was no statistically significant difference between the proportion of the 5 categories between the periods 1988 – 1997, 1998 – 2006, and 2007 – 2020.

CHAPTER V: DISCUSSION AND CONCLUSION

As indicated by results on the National Assessment of Academic Progress (NAEP) in 2019 and the Trends in International Mathematics and Science Study (TIMSS) assessment in 2019, and despite reform efforts, such as the creation and implementation of the Common Core State Standards in 2010, students continue to struggle in mathematics. This study sought to describe the extent to which two prominent mathematics research journals, *Educational Studies in Mathematics* (ESM) and the *Journal for Research in Mathematics Education* (JRME), emphasize mathematics instruction that focuses on conceptual knowledge, procedural knowledge, both conceptual and procedural knowledge, progression from conceptual to procedural knowledge, and progression from procedural to conceptual knowledge from 1970 to 2020. Results indicate that most of the research published in these two journals from 1988 to 2020 (every third year) focus on conceptual knowledge development followed by a focus on both conceptual and procedural knowledge development. Additionally, the data suggests that this focus has remained consistent from the 1988 to 2020 publications. This chapter provides further interpretations and insights into the findings and their significance and connects the results to the theoretical frameworks of constructivism and behaviorism, as well as the research pertaining to conceptual and procedural knowledge development and instructional strategies that support both types of knowledge. Additionally, this chapter discusses implications for researchers and educators, the impact such findings have on educational policy, research limitations, and recommendations for future research.

Although one journal is national and the other international, the results of this study indicate that the two journals, *Educational Studies in Mathematics* (ESM) and the *Journal for Research in Mathematics Education* (JRME), are similar in nature with respect to research articles meeting the inclusion criteria (written in English, non-interventional, and focus on instructional strategies) that focus on conceptual knowledge, procedural knowledge, both types of knowledge, and a progression between the two. Results indicate that most of the studies focused on conceptual knowledge development in which students made connections between mathematical concepts to form new knowledge, utilized concrete manipulatives and visual representations, and were encouraged to make sense of the mathematics. Research noted in this study's review of the literature suggests that such actions are associated with constructivist learning and teaching practices. The learning theory of constructivism suggests that students construct knowledge by reconfiguring mental connections, ideas, and procedures already learned once presented with information that does not easily correlate with their prior knowledge (Hiebert & Grouws, 2006). The second most common focus in research published in the two journals focused on the development of both conceptual and procedural knowledge in which, in addition to those actions associated with constructivism and making sense of the mathematics, students identified mathematical rules and learned fixed steps, procedures, or solution strategies to solve mathematical problems. As indicated by the research in this study's review of the literature, learning and following fixed solution steps and identifying mathematical rules are actions associated with behaviorist learning and teaching practices. These findings are significant, as they support the need for both constructivist and behaviorist instructional practice that include teaching students specific rules and solution strategies through practices such as direct instruction, as well as providing numerous opportunities to make connections between mathematical concepts, explore and identify relationships among concepts, utilize multiple forms of representation, and make sense of the mathematics, supporting the development of both procedural and conceptual knowledge development.

The similarity between the findings of this research on the two journals' focus on conceptual and procedural knowledge is also mirrored in the focus on conceptual and procedural knowledge in assessments administered to students nationally and internationally that compare student achievement. As noted previously, the United States administers the National Assessment of Education Progress (NAEP) which provides a common measure of student achievement and focuses on both conceptual and procedural knowledge (U.S. Department of Education, 2019). The Trends in International Mathematics and Science Study (TIMSS) assesses both conceptual and procedural knowledge and allows for the comparison of student achievement in different countries, including the United States. Approximately three-fourths of the testing time on the NAEP is intended to be spent answering questions of moderate to high complexity, requiring students apply conceptual knowledge to make decisions about what needs to be done, make decisions about how to accomplish a task, reason, plan, and analyze to solve problems (U.S. Department of Education, 2019). Similarly, approximately sixty percent of the TIMSS assessment focuses on applying conceptual knowledge to solve problems with unfamiliar context and multi-step solutions (Lindquist et al., 2017). These actions correlate with constructivist teaching and learning. As discussed in the review of literature, conceptual knowledge is utilized when students reason, analyze, and make decisions about how to solve a problem. One-fourth of the testing time on the NAEP is intended to be spent on low complexity problems, requiring students to recall facts, concepts, and procedures (U.S. Department of Education, 2019), while approximately forty percent of the TIMSS assessment focuses on recalling facts, concepts, and procedures (Lindquist et al., 2017), which correlate with behaviorist teaching and learning.

Although a greater percentage of the questions in both assessments focus on conceptual knowledge, procedural knowledge is also important. The percentages of focus on conceptual and procedural knowledge within both assessments, in conjunction with the research noted in this study's literature review, as well as the results of this study, provide insight into what the greater educational community considers important in learning and instruction. This is significant to educators, curriculum planners, and professional development coordinators as it suggests that although a greater amount of conceptual knowledge development may be beneficial to student success, procedural knowledge is also important and should not be overlooked, as research indicates that conceptual and procedural knowledge development is bidirectional (Rittle-Johnson et al., 2015). Furthermore, the types of questions on both the NAEP and TIMSS that focus on conceptual and procedural knowledge suggest that educators should employ instructional strategies that combine elements of both constructivism and behaviorism.

The findings of this study also suggest that the direction researchers have taken with reference to conceptual and procedural knowledge with non-interventional research has remained consistent from 1988 to 2020. One possible reason for this may be the apparent disconnect between what research has shown to be successful and the academic success of students in the mathematics classroom. As discussed in this study's review of the literature, instructional strategies that develop conceptual understanding only, both conceptual and procedural understanding, or a progression from conceptual to procedural understanding can improve student success in mathematics, it stands to reason that research in those areas would continue to be conducted to remediate the disconnect. Another possible reason for the consistent research focus is that national organizations, such as The National Council of Teachers of Mathematics (NCTM, 2014), emphasize the development of conceptual knowledge prior to procedural instruction. For example, as discussed in the review of literature, NCTM states that "Conceptual understanding (i.e., the comprehension and connection of concepts, operations, and relations) establishes the foundation, and is necessary, for developing procedural fluency (i.e., the meaningful and flexible use of procedures to solve problems") (p. 7). A final possible reason for the consistency in the focus of research from 1988 to 2020 is that, as previously mentioned and indicated by both the NAEP and TIMSS assessments, the greater educational community appears to place significant importance on the development of both conceptual and procedural knowledge development, with conceptual development receiving greater focus, thus the continued similar research among the two journals.

Implications for Researchers

Seventy-five percent of the articles analyzed in this research study focused on instructional practices that aim to improve students' conceptual knowledge and placed importance on noticing relationships and making connections between mathematical concepts as evidenced by the frequency of keywords, yet 59 percent of fourth-grade students and 66 percent of eighth-grade students scored below proficient in mathematics on the National Assessment of Education Progress in 2019 (U.S. Department of Education, 2019). According to Gersten et al. (2009) one reason for students' difficulty in mathematics is a lack of conceptual understanding of mathematical content which in turn limits their ability to apply their learning to novel situations and problems. The larger research community may benefit from additional research into investigating why there appears to be a disconnect between what research has shown to be effective instructional practices in the reviewed studies from 1988 to 2020 and student success in current classrooms. Why do students continue to struggle in mathematics when research shows that there are numerous instructional strategies that have been shown to be effective in improving student success in the research setting? Additionally, as noted in the literature review, instructional strategies that progress students from conceptual to procedural knowledge development, such as the concrete-representational-abstract instructional sequence, have shown positive results on student achievement with students identified as having mathematics difficulties within the respective studies, yet only 7.7% of the articles analyzed in this study were coded as focusing on the progression. Is instruction that progresses from conceptual to procedural also effective with students not identified as having mathematics difficulties? This indicates a possible gap in the research and warrants further study.

Implications for Teachers and Professional Development Coordinators

As the results of this study indicate, instructional practices that focus on conceptual development make up a large percentage of the research pertaining to conceptual and procedural knowledge, followed by instructional practice that focus on developing both types of knowledge. Additionally, this research showed that these areas of focus have remained relatively unchanged from 1988 to 2020, indicating that instructional practices that focus on both conceptual and procedural knowledge, with conceptual knowledge receiving more instructional time, is a worthwhile practice to implement in the classroom. These findings, combined with the composition of the NAEP and TIMSS assessments, demonstrate the importance of developing both conceptual and procedural knowledge to ensure student success and achievement. As NAEP is congressionally mandated and allows for direct comparisons among states and the TIMSS provides a common measure of student achievement and comparisons between countries, these assessments provide educators with a framework for instruction. Teachers can use this information to adjust their current instruction to include more instructional practices that focus on conceptual knowledge development, such as providing opportunities for students to describe relationships and connections they have discovered, providing opportunities for students to represent content and ideas in a variety of ways, while also assisting students in making connections between the conceptual knowledge and the associated procedural processes. This information also impacts the development of the curriculum and can influence the selection of curricular materials.

Furthermore, with students continuing to struggle to gain proficiency in mathematics, it may be beneficial for teachers to seek out professional development on ways to infuse classroom instruction with instructional practices that provide students the opportunity to develop deep conceptual understanding by noticing and describing relationships and making connections between mathematical concepts, utilizing multiple forms of representation, and connecting that learning to procedures. Finally, this study is significant to school district professional development coordinators as they can use the findings to develop a district or campus professional development plan, focusing on the areas of knowledge development prominent in the research.

Impact on Education Policy

Reform efforts such as the Race to the Top grant program were spurred in part from concerns over student performance on the NAEP and TIMSS assessments. As noted previously, the Race to the Top grant program, authorized by the American Recovery and Reinvestment Act of 2009 (ARRA), rewarded states for innovative practices and reform efforts aimed at increasing student achievement and teacher effectiveness (Popham, 2020; U.S. Department of Education, 2009; U.S. Department of Education, 2015). Such practices included: 1) adopting college and career readiness standards and improve instructional practices, 2) building assessments that measure student growth, 3) tying student achievement to teacher evaluations, and 4) improving the lowest performing schools. Race to the Top funding provided a means for schools to redesign their teacher evaluation systems, tying student achievement on assessments to teacher evaluations (Popham, 2020; U.S. Department of Education, 2009; U.S. Department of Education, 2015) and, in many school districts, to teacher compensation (U.S. Department of Education, 2015). The revised evaluation systems also tied teacher evaluations to teachers' participation in professional development and school improvement efforts (U.S. Department of Education, 2015), driving the professional development plans that school districts develop and provide to teachers. The NAEP and TIMSS assessments provide a blueprint for classroom instructional focus as well as state standardized tests. This in turn, influences the focus of professional development provided to teachers and the amount of conceptual and procedural instruction provided to students. As educational research, assessments such as the NAEP and TIMSS, instructional practices, student achievement, reform efforts, and subsequent funding associated with such reform efforts, are inexorably linked, they therefore have a direct impact on the content of educational materials, professional development opportunities offered to teachers, and the curricular decisions and policies made in each state and school district. For example, both the 2003 and 2007 TIMSS results indicated that students in the United States were failing to compete internationally in mathematics (Grady et al., 2018), while the 2007 NAEP results reported that 61 percent of 4^{th} grade students and 68 percent of 8^{th} grade students scored below proficient in mathematics (National Center for Educational Statistics, 2007). As mentioned previously, these two assessments promote both conceptual and procedural knowledge, with a greater focus placed on conceptual knowledge. Soon after, the Race to the Top reform program was announced in 2009, rewarding states for innovative practices and reform efforts aimed at increasing student achievement and teacher effectiveness (U.S. Department of Education, 2009;

U.S. Department of Education, 2015), and the Common Core State Standards were introduced in 2010, placing high importance on the development of conceptual understanding through modeling, sense making, and reasoning (Xin et al., 2016). These Common Core State Standards were then implemented in 48 states, two U.S. territories, and the District of Columbia (U.S. Department of Education, 2009). Furthermore, these interconnected factors demonstrate the need for effective mathematics instruction and improved student success.

Limitations

One limitation of the study is that the inclusion criteria may have excluded articles that may have added to the study. This study only looked at articles from every third year of the *Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM) research journals from 1970 to 2020 and only articles that were written in English, non-interventional, conducted in K-12 educational settings, and focused on instructional strategies and/or instructional practices were analyzed.

Future Research

Recommendations for future research include the following:

- It is recommended that future research explores the possible disconnects between instructional practices shown to increase student achievement and their application in classrooms.
- It is also recommended that future research considers conducting more non-interventional studies on instructional practices that focus on developing both conceptual and procedural knowledge.
- It is recommended that future research on how conceptual and procedural knowledge have been positioned in prominent research journals include all published years of the

Journal for Research in Mathematics Education (JRME) and *Educational Studies in Mathematics* (ESM) research journals.

- It is recommended that future research on how conceptual and procedural knowledge have been positioned in prominent research journals include additional mathematics journals.
- It is further recommended that future research explores instructional practices that support the progression from conceptual to procedural knowledge with the general student population.

Conclusion

This research study provides a glimpse into mathematics research in two prominent research journals, the *Educational Studies in Mathematics* (ESM) and the *Journal for Research in Mathematics Education* (JRME), over the last 50 years. The extent to which these two journals emphasize mathematics instruction that focuses on conceptual knowledge, procedural knowledge, both conceptual and procedural knowledge, progression from conceptual to procedural knowledge, and progression from procedural to conceptual knowledge from 1970 to 2020 was examined. Results indicate that much of the research pertaining to conceptual and procedural knowledge development instructional practices in mathematics focus on conceptual knowledge. Furthermore, the results suggest that the instructional practices utilized in the research that focused on conceptual knowledge incorporate aspects of constructivism as they included an emphasis on students noticing relationships and making connections between mathematical content, thus conceiving knowledge rather than receiving it from the teacher. The second most common focus was on developing both conceptual and procedural knowledge. These research studies included features of both constructivism and behaviorism by utilizing a combination of explicit, direct instruction to develop foundational skills as well student-centered, inquiry-based instruction that required students to utilize those foundational skills in tasks that required noticing relationships, making connections between mathematical concepts, and explaining their thinking. Finally, this research provides evidence that the amount of research in each of the research categories (conceptual only, procedural only, both conceptual and procedural, and a progression between the two) has remained relatively unchanged over the last half century. These findings are important to educators and researchers alike. The results demonstrate that developing both conceptual and procedural knowledge are worthwhile practices and therefore should be included in both classroom instruction and future research.

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Appendix A

Codebook

Study Characteristics

- Name of Journal
- Year of Publication
- Title of Study
- Grade band of focus:
  - Elementary
  - Secondary
  - Elementary and Secondary
  - Not specified

Coding Categories

- Conceptual Knowledge Only
- Procedural Knowledge Only
- Conceptual and Procedural
- Progression from Conceptual to Procedural
- Progression from Procedural to Conceptual

Textual Units (Keywords)

- Conceptual
- Conceptual/Relational Knowledge
- Relationship
- Connection
- Procedural
- Procedural/instrumental Knowledge
- Procedure
- Manipulative
- Concrete
- Concrete manipulative
- Representation
- Concrete Representation
- Visual/graphical/pictorial representation
- Pictorial/graphical
- Drawing
- Abstract/symbolic Representation
- Explicit/Direct Instruction
- Scaffold

Appendix B

Example of Data Collection

Scaffold/scaffolded/ scaffolding	Textual Units	0	0	0	0
Explicit instruction/teaching direct instruction/teaching		0	1	0	0
Abstract/symbolic/numerical/ number/quantitative/ algebraic/arithmetical		0	0	0	4
Drawing/drawings		0	0	1	0
Pictorial/graphic/graphical/ graphically		0	0	0	10
Visual/graphical/pictorial representation/representations/ sketch/sketches		0	0	0	9
Concrete representation/ representations		0	0	0	0
Representation/representations/ representational		3	0	0	49
Concrete manipulative/ manipulatives/material/ materials/model/models/thing		0	0	0	0
Concrete/concretely		7	1	0	0
Manipulative/manipulatives		0	0	0	1
Procedure/procedures		0	21	62	1
Procedural/instrumental knowledge/understanding/ fluency		0	7	0	0
Procedural/procedurally/ proceduralize/proceduralized		0	15	0	0
Connection/connections/ interconnection/ interconnections		4	0	4	16
Relationship/relationships/ relation/relations/relational/ interrelation/interrelations		21	8	5	4

Example of Data Collection (continued)

Study Characteristics				Coding Category						Textual Units	
Journal	Year of Publication	Article Title	Grade Band	Conceptual/Relational Knowledge	Procedural/Instrumental Knowledge	Conceptual/Relational and Procedural/Instrumental Knowledge	Progression from Conceptual to Procedural Knowledge	Progression from Procedural to Conceptual Knowledge	Warrants for Coding Category Selected	Conceptual/conceptually/ conceptualize/conceptualizing/ conceptualization	Conceptual/relational knowledge/ understanding/ development/thinking/level
ESM	1988	“Knowing” vs “Seeing. Problems of the Plane Representation of Space Geometry Figures	Secondary				X		“It is compulsory to pass through a phase of using a 3D representation (model), even at high school level. We believe it necessary - for various reasons - for the pupils to learn to do without that kind of representation, but that can be done only after some time, when the mental images are truly set up.”	4	2
ESM	1991	The Concept of Chance in Everyday Teaching: Aspects of a Social Epistemology of Mathematical Knowledge	Elementary	X					“Our specific interest will be to understand better how in everyday teaching, processes of concept development are organized and how the meaning of mathematical concepts is constituted through social interaction. This concrete context of their experiences with the game is a fundamental source for the students, one which has to be maintained throughout the whole process of developing the concept of chance.”	5	3
JRME	2015	How Students’ Everyday Situations Modify Classroom Mathematical Activity: The Case of Water Consumption	Secondary		X				“We discuss the tensions and contradictions that evolve when a generic school procedure emphasized by the teacher meets the specific procedures applicable to everyday situations proposed by the students.”	0	0
JRME	1997	Designing Representations: Reasoning About Functions of Two Variables	Secondary			X			Algebra is viewed as the study of relationships among quantities. Symbolic representations are introduced at a later stage of algebraic reasoning. In part, this is done to allow the symbolization to be carried out meaningfully, after students have formed a base of conceptual understanding of functions and variables and after they’ve developed other representations for feedback and explanations (Yerushalmy & Schwartz, 1993).”	0	1

Appendix C

Articles Analyzed in Final Study

Journal	Year of Publication	Title	Grade Band: Elementary (prek-5), Secondary (6-12)	Conceptual/Relational Knowledge	Procedural/Instrumental Knowledge	Conceptual/Relational *and* Procedural/Instrumental Knowledge	Progression Conceptual to Procedural	Progression Procedural to Conceptual
Study Characteristics				Coding Categories				
ESM	1988	"Knowing" vs "Seeing". Problems of the Plane Representation of Space Geometry Figures	Secondary				X	
ESM	1988	Mathematical Discussion and Mathematical Understanding	Secondary			X		
ESM	1991	The Concept of Chance in Everyday Teaching: Aspects of a Social Epistemology of Mathematical Knowledge	Elementary	X				
ESM	1991	An Analysis of Diverging Approaches to Simple Arithmetic: Preference and its Consequences	Elementary and Secondary			X		
ESM	1994	The Role of Schemes in Two-Step Problems: Analysis and Research Findings	Elementary and Secondary	X				
ESM	1994	Negotiation of Mathematical Meaning and Learning Mathematics	Elementary	X				
ESM	1994	A Model for Nurturing and Assessing Multidigit Number Sense Among First Grade Children	Elementary	X				
ESM	1994	Interactive Development of Subject Matter in the Mathematics Classroom	Secondary			X		
ESM	1997	When Negotiation of Meaning is Also Negotiation of Task: Analysis of the Communication in an Applied Mathematics High School Course	Secondary	X				
ESM	1997	Concept Formation of Triangles and Quadrilaterals in the Second Grade	Elementary				X	
ESM	1997	A Participatory-Inquiry Approach and the Mediation of Mathematical Knowledge in a Multilingual Classroom	Secondary	X				

Journal	Year of Publication	Title	Grade Band: Elementary (prek-5), Secondary (6-12)	Conceptual/Relational Knowledge	Procedural/Instrumental Knowledge	Conceptual/Relational *and* Procedural/Instrumental Knowledge	Progression Conceptual to Procedural	Progression Procedural to Conceptual
ESM	1997	The Equation, The Whole Equation and Nothing But the Equation! One Approach to the Teaching of Linear Equations	Secondary	X				
ESM	1997	Epistemological Investigation of Classroom Interaction in Elementary Mathematics Teaching	Elementary			X		
ESM	2000	Learning to Prove by Investigations: A Promising Approach in Dutch Secondary Education	Secondary	X				
ESM	2000	Creating Meaning For and With the Graphing Calculator	Secondary	X				
ESM	2000	A Multiple-Perspective Analysis of Learning in the Presence of Technology	Secondary			X		
ESM	2000	Teaching Maths through Theme-Based Resources: Pedagogic Style, 'Theme' and 'Maths' in Lessons	Secondary	X				
ESM	2003	The Didactical Use of Models in Realistic Mathematics Education: An Example From a Longitudinal Trajectory on Percentage	Secondary				X	
ESM	2003	Focusing on Informal Strategies When Linking Arithmetic to Early Algebra	Elementary and Secondary	X				
ESM	2003	Some Reflections on Mathematics Classroom Notebooks and Their Relationship to the Public and Private Nature of Student Practices	Secondary	X				
ESM	2003	An Investigation of Communicative Competence in an Upper-Secondary Class Where Using Graphics Calculators was Routine	Secondary	X				
ESM	2003	Developing Mathematical Thinking and Self-Regulated Learning: A Teaching Experiment in a Seventh-Grade Mathematics Classroom	Secondary	X				
ESM	2003	Classroom Interaction as Reflection: Learning and Teaching Mathematics in a Community of Inquiry	Elementary	X				
ESM	2006	Examining the Tasks of Teaching When Using Students' Mathematical Thinking	Secondary	X				

Journal	Year of Publication	Title	Grade Band: Elementary (prek-5), Secondary (6-12)	Conceptual/Relational Knowledge	Procedural/Instrumental Knowledge	Conceptual/Relational *and* Procedural/Instrumental Knowledge	Progression Conceptual to Procedural	Progression Procedural to Conceptual
ESM	2006	Constructing and Consolidating of Algebraic Knowledge Within Dyadic Processes: A Case Study	Secondary	X				
ESM	2006	Mathematical Modeling in the Primary School: Children's Construction of a Consumer Guide	Secondary	X				
ESM	2006	Classroom Practices for Context of Mathematics Word Problems	Elementary and Secondary	X				
ESM	2009	Constructing Mathematics in an Interactive Classroom Context	Secondary	X				
ESM	2009	Working with Artefacts: Gestures, Drawings and Speech in the Construction of the Mathematical Meaning of the Visual Pyramid	Elementary	X				
ESM	2009	Using Graphing Software to Teach About Algebraic Forms: A Study of Technology-Supported Practice in Secondary-School Mathematics	Secondary			X		
ESM	2009	Didactical Designs for Students' Proportional Reasoning: An "Open Approach" Lesson and a "Fundamental Situation"	Secondary	X				
ESM	2009	Bridging the Macro- and micro-divide: Using an Activity Theory Model to Capture Sociocultural Complexity in Mathematics Teaching and its Development	Secondary			X		
ESM	2011	The Nature of Student Predictions and Learning Opportunities in Middle School Algebra	Secondary	X				
ESM	2011	The Role of Visual Representations for Structuring Classroom Mathematical Activity	Elementary	X				
ESM	2012	Data Modelling with First-Grade Students	Elementary	X				
ESM	2012	When Does an Opportunity ecome and Opportunity? Unpacking Classroom Practice Through the Lens of Ecological Psychology	Secondary	X				

Journal	Year of Publication	Title	Grade Band: Elementary (prek-5), Secondary (6-12)	Conceptual/Relational Knowledge	Procedural/Instrumental Knowledge	Conceptual/Relational *and* Procedural/Instrumental Knowledge	Progression Conceptual to Procedural	Progression Procedural to Conceptual
ESM	2012	Developing Fluency in the Mathematical Register Through Conversation in a Tenth-Grade Classroom	Secondary	X				
ESM	2015	The Rise and Run of a Computational Understanding of Slope in a Conceptually Focused Bilingual Algebra Class	Secondary	X				
ESM	2015	Student Participation in Elementary Mathematics Classrooms: The Missing Link Between Teacher Practices and Student Achievement?	Secondary	X				
ESM	2018	Reconfiguring Mathematical Settings and Activity Through Multi-Party, Whole-Body Collaboration	Secondary	X				
JRME	1991	Small-Group Interactions as a Source of Learning Opportunities in Second-Grade Mathematics	Elementary	X				
JRME	1994	Capitalizing on Errors as "Springboards for Inquiry": A Teaching Experiment	Secondary	X				
JRME	1997	Designing Representations: Reasoning About Functions of Two Variables	Secondary			X		
JRME	1997	Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning	Secondary	X				
JRME	1997	Supporting Latino First Graders' Ten-Structured Thinking in Urban Classrooms	Elementary	X				
JRME	2000	Learning of Geometry Through Design	Elementary	X				
JRME	2003	Low-Performing Students and Teaching Fractions for Understanding: An Interactional Analysis	Elementary	X				
JRME	2006	Teaching Geometry With Problems: Negotiating Instructional Situations and Mathematical Tasks	Secondary	X				

Journal	Year of Publication	Title	Grade Band: Elementary (prek-5), Secondary (6-12)	Conceptual/Relational Knowledge	Procedural/Instrumental Knowledge	Conceptual/Relational *and* Procedural/Instrumental Knowledge	Progression Conceptual to Procedural	Progression Procedural to Conceptual
JRME	2006	Teaching as Assisting Individual Constructive Paths Within an Interdependent Class Learning Zone: Japanese First Graders Learning to Add Using 10	Elementary				X	
JRME	2015	How Students' Everyday Situations Modify Classroom Mathematical Activity: The Case of Water Consumption	Secondary		X			
JRME	2020	Beyond Rise Over Run: A Learning Trajectory for Slope	Secondary	X				
JRME	2020	Dimensions of Learning Probability Vocabulary	Elementary	X				

Appendix D

Files Excluded Based on Title 1970 to 1985

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice
ESM	1970	Volume Information	x				
ESM	1970	Les six étapes de l'apprentissage des structures		x			
ESM	1970	Une expérience dans un lycée classique: Élèves de 14-15 ans		x			
ESM	1973	Sur l'assimilation des programmes de 6ème-5ème		x			
ESM	1973	Volume Information	x				
ESM	1973	Logique formelle et raisonnement naturel des élèves dans l'enseignement de la mathématique: Un fragment d'expérience		x			
ESM	1973	Quelques remarques sur le comportement des élèves concernant les problèmesmathématiques		x			
ESM	1976	Volume Information	x				
ESM	1976	Difficultés Liées à la Présentation des Questions Mathématiques		x			
ESM	1976	Une Expérience à Montrouge		x			
ESM	1976	Ou le premier n'est pas toujours premier...: Piéce probabiliste en trois actes pour desenfants de 10 ans		x			
ESM	1976	Stratégies Pour une Approche de z		x			
ESM	1976	Problématique Dans L'apprentissage de la Mathématique		x			
ESM	1976	Wie Kann man im Mathematikunterricht den Denkstufen Rechnung Tragen?		x			
ESM	1976	Les Probabilités à L'école Élémentaire		x			
ESM	1976	Quelques Reflexions sur L'enseignement de la Numération aux Enfants de 7, 8 ou 9 ans		x			
ESM	1976	Évaluation-Sondage Dans le Premier Cycle (12-16 ans)		x			

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice
ESM	1976	Die Didaktischen Systeme von V. V. Davidov/ D. B. Elkonin Einerseits und L. V. ZankovAndererseits		x			
ESM	1976	La Géométrie Projective à L'école		x			
ESM	1976	Balancierte Graphen - Ein Thema Für die Didaktische Diskussion zum Geometrieunterricht		x			
ESM	1976	Approche des Statistiques en Classe de Sixième: Analyse de Résultats Sportifs		x			
ESM	1979	Locutions inductrices et distractrices: 'de plus que', 'de moins que'		x			
ESM	1979	Loto-questionnaires (pour l'evaluation et l'auto-controle en mathématiques)		x			
ESM	1979	Back Matter	x				
ESM	1979	Coniques et Gravitation Universelle		x			
ESM	1979	Back Matter	x				
ESM	1979	Volume Information	x				
ESM	1979	Editorial Statement	x				
ESM	1979	Back Matter	x				
ESM	1979	Le Lancement des projectiles		x			
ESM	1979	La perception de quelques difficultés en mathématiques par les professeurs en classe de troisième dans l'enseignement secondaire au Sénégal		x			
ESM	1979	Langage, jeu et activité mathématique: Un essai à l'école primaire		x			
ESM	1982	Les premieres acquisitions de la notion de fonction lineaire chez l'enfant de 7 à 11 ans		x			
ESM	1982	Review	x				
ESM	1982	Fiabilité, validité et pertinence: critères de la recherche sur l'enseignement de la mathématique		x			
ESM	1982	L'influence du 'décor' et du langage dans des épreuves de type 'logique' portantapparemment sur l'implication		x			
ESM	1982	Back Matter	x				

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice
ESM	1982	Résolution de problèmes de division au cycle élémentaire dans deux types de situations didactiques		x			
ESM	1982	Reviewed Work(s): Stochastik im Schulunterricht by W. Dörfler and R. Fischer	x				
ESM	1982	Reviewed Work(s): IDM-Unterlagen zum Mathematik-Unterricht by Institut für Didaktik der Mathematik Bielefeld	x				
ESM	1982	Volume Information	x				
ESM	1982	L'Observation de Classes et le Paradoxe de l'Observateur		x			
ESM	1982	Appropriation des Propriétés Ordinales du Nombre par l'Eleve du Cours Preparatoire		x			
ESM	1982	Back Matter	x				
ESM	1985	Front Matter	x				
ESM	1985	Signification et fonctionnement du concept de variable informatique chez des élèvesdebutants		x			
ESM	1985	Back Matter	x				
ESM	1985	Volume Information	x				
ESM	1985	Front Matter	x				
ESM	1985	Editorial Statement	x				
ESM	1985	Back Matter	x				
ESM	1985	Volume Information	x				
ESM	1985	Front Matter	x				
ESM	1985	Front Matter	x				
ESM	1985	Influence de la question dans une épreuve relative à la notion d'indépendance		x			
ESM	1985	Introduction	x				
ESM	1985	Reviewed Work(s): Didaktik der Analysis by W. Blum and G. Törner	x				

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice
ESM	1985	Reviewed Work(s): Didaktik der Analysis by W. Blum and G. Törner; Teaching Calculus by Hugh Neill and Hilary Shuard	x				
JRME	1970	Editorial Comment	x				
JRME	1970	Call for Research Manuscripts, Including General Guidelines	x				
JRME	1970	Abstracts of Future Articles	x				
JRME	1970	Editorial Comment	x				
JRME	1970	Editorial Comment	x				
JRME	1970	Information for Contributors to Journal for Research in Mathematics Education, JRME Editorial Board	x				
JRME	1970	Abstracts of Future Articles	x				
JRME	1970	Editorial Comment	x				
JRME	1973	Editorial Comment	x				
JRME	1973	Abstracts of Future Articles	x				
JRME	1973	Editorial Comment	x				
JRME	1973	Critiques of Articles	x				
JRME	1973	Editorial Comment	x				
JRME	1973	Critiques of Articles	x				
JRME	1973	Editorial Comment	x				
JRME	1973	Classified Index, 1972-1973	x				
JRME	1973	Abstracts of Future Articles	x				
JRME	1976	Revised Information for Contributors to the Journal for Research in Mathematics Education	x				
JRME	1976	Introduction	x				
JRME	1976	Research Summaries	x				
JRME	1976	Journal-Published Reports	x				
JRME	1976	Dissertation Abstracts	x				
JRME	1979	Information for Contributors to Journal for Research in Mathematics Education	x				
JRME	1979	Review	x				
JRME	1979	Letter to the Editor	x				
JRME	1979	Telegraphic Reviews	x				

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice
JRME	1979	Abstracts of Future Articles	x				
JRME	1979	Letters to the Editor	x				
JRME	1979	Telegraphic Reviews	x				
JRME	1979	A Forum for Researchers	x				
JRME	1979	Critiques	x				
JRME	1979	Letters to the Editor	x				
JRME	1979	Telegraphic Reviews	x				
JRME	1979	Announcements	x				
JRME	1979	A Forum for Researchers	x				
JRME	1979	Letters to the Editor	x				
JRME	1979	Reviews	x				
JRME	1979	Telegraphic Reviews	x				
JRME	1979	Acknowledgment	x				
JRME	1979	Classified Index, 1978-79	x				
JRME	1982	Reviews	x				
JRME	1982	Telegraphic Reviews	x				
JRME	1982	Abstracts of Future Articles	x				
JRME	1982	Reviews	x				
JRME	1982	Abstracts of Future Articles	x				
JRME	1982	Reviews	x				
JRME	1982	Telegraphic Reviews	x				
JRME	1982	Letters to the Editor	x				
JRME	1982	Reviews	x				
JRME	1982	Telegraphic Reviews	x				
JRME	1985	Editorial	x				
JRME	1985	Critiques	x				
JRME	1985	Reviews	x				
JRME	1985	Telegraphic Reviews	x				
JRME	1985	Letter to the Editor	x				
JRME	1985	Editorial	x				
JRME	1985	Critique	x				
JRME	1985	Review	x				
JRME	1985	Telegraphic Reviews	x				
JRME	1985	Letter to the Editor	x				
JRME	1985	Editorial	x				

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice
JRME	1985	Critique	x				
JRME	1985	Telegraphic Reviews	x				
JRME	1985	Editorial	x				
JRME	1985	Editorial	x				
JRME	1985	Review	x				
JRME	1985	Telegraphic Reviews	x				
JRME	1985	Letters to the Editor	x				
JRME	1985	Acknowledgment	x				
JRME	1985	Index	x				

Appendix E

Files Excluded Based on Abstract or Full Text Reading 1970 to 1985

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice	Justification for Exclusion
ESM	1970	Some Notes on Multiplication of Whole Numbers	x			x	x	Concept development
ESM	1970	A Geometry on the Cube	x			x	x	Concept development
ESM	1970	Combinatorial Analysis and School Mathematics	x			x	x	Concept development
ESM	1970	XIth International Olympiad Bucharest, 5-20 July 1969	x			x	x	Description of problems presented at the 1969 International Olympiad
ESM	1970	The System and the Organization of Further Training for the Mathematics Teachers of the Secondary Schools in Budapest	x			x	x	Description of training programs for teachers
ESM	1970	Research in Mathematics Education	x			x	x	Description of possible research topics for graduate students presented at a conference
ESM	1970	Communication on Primary Education in Mathematics: Practical Work: For What Purpose?	x			x	x	Description of teacher work/planning sessions and examples of student activities
ESM	1970	Relative Effectiveness of Two Different Television Techniques and One Large Lecture Technique for Teaching Large Enrollment College Mathematics Courses				x		Study participants were univeristy students enrolled in Mathematics 417
ESM	1970	Some Ideas in Geometry That Can Be Taught from K-6	x					Concept development
ESM	1970	Sweep Away All Cows, Ghosts, Dragons and Devils: A Report of the Effects of the Great Cultural Revolution on Mathematics Education in Communist China	x					Description of the education system in China
ESM	1970	A Calculus-with-Computer Experiment				x		Study participants students at a university

Journal	Year	Title	Not a Study	Not in English	Interventional	Not K-12	Not Instructional Strategy or Practice	Justification for Exclusion
ESM	1970	Study of Geometry in the Fourth Grade	x					Description of geometry topics taught in 4th grade in the Soviet Union
ESM	1970	An Investigation of Structure in Elementary School Mathematics: Isomorphism			x			Participants were divided into three experimental groups and intervention provided by researcher
ESM	1970	The Importance of Appropriate Problems in the Teaching of Mathematics	x					Concept development
ESM	1970	A Counting Model for Simple Addition					x	Administration of a test and classification/analysis of student responses/achievement
ESM	1973	Problem Length as a Structural Variable in Verbal Arithmetic Problems					x	Administration of a test and classification/analysis of student responses/achievement
ESM	1973	The Concept of Grouping in Jean Piaget's Psychology - Formalization and Applications	x					Concept development
ESM	1973	Charles Godfrey (1873-1924) and the Reform of Mathematical Education	x					Concept development
ESM	1973	Quasi-Child Logics				x	x	Participants were elementary, secondary, and university students and were administered two tests
ESM	1973	Logical Thinking in College Students				x		Participants were college students
ESM	1973	Development and Signification of a Geometry Test	x					Description of the creation of a geometry test
ESM	1973	Teaching College Mathematics by Question-and-Answer	x			x		Description of instruction at the college level
ESM	1973	Using Models of Operations and Equations	x					Concept development

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ESM	1973	The March of the Discoverer	x					Concept development - benefits of describing how one arrived at a result
ESM	1973	The CF/PMM Approach to Learning Mathematics	x					Concept development
ESM	1973	Relations and Probability	x					Concept development
ESM	1973	Around a Game	x					Description of a figure game
ESM	1973	Experiments in Teaching Intuitive Topology in the 5th and 6th Grades	x					Descriptions of instructional units and activities
ESM	1976	A Study of Pupils' Proof-Explanations in Mathematical Situations					x	Administration of a test and classification/analysis of student responses/achievement
ESM	1976	Why Does the Probabilistic Abacus Work?	x					Concept development
ESM	1976	Nomograms and the Foundations of Geometry	x					Concept development
ESM	1976	What Industry Wants a Mathematician to Know and How We Want Them to Know It	x					Concept development
ESM	1976	A Note on the Role of Parameters in Mathematics Teaching	x					Concept development
ESM	1976	Finite Geometries and Non-Measurable Voting Bodies	x					Concept development
ESM	1976	On a Class-Room Incident	x					Description of a conversation among teachers about a classroom incident
ESM	1976	Three Informal Essays	x					Concept development
ESM	1976	Mathematizing around Convexity	x					Concept development
ESM	1976	Set-Theory and Logic in School	x					Concept development
ESM	1976	Computer-Assisted Instruction in Elementary Logic at the University Level				x		Participants were enrolled in Philosophy 57A, introduction to Symbolic Logic at Stanford University

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ESM	1976	XVIth International Mathematical Olympiad: 8-9 July 1974, Erfurt, Weimar, Berlin	x					Description of problems presented at the 1974 International Olympiad
ESM	1976	XVIIth International Mathematical Olympiad Burgas-Sofia, 3-16 July 1975	x					Description of problems presented at the 1975 International Olympiad
ESM	1976	Five Years IOWO	x					Reprint of a book
ESM	1976	Three Determinants of Difficulty in Verbal Arithmetic Problems					x	Administration of a questionnaire
ESM	1976	Enquiry, Discovery and Research: Terminology and Meaning	x					Concept development
ESM	1976	The Naive Concept of Definition in Mathematics				x	x	Participants were high school and college students, administration of a questionnaire
ESM	1976	The Place of Geometry in Mathematics Teaching: An Analysis of Recent Developments	x					Concept development
ESM	1976	A Commentary from IEA on Dr. Freudenthal's Article in: Educational Studies in Mathematics	x					Review of an article
ESM	1976	[A Commentary from IEA on Dr. Freudenthal's Article in: Educational Studies in Mathematics, Vol. 6, No. 2]: Rejoinder	x					Follow up to a review of an article
ESM	1976	Erratum: On Primary School Teachers' Mathematics	x					Correction to an article
ESM	1976	Decision-Making, the Intervening Variable	x					Concept development
ESM	1976	On Primary School Teachers' Mathematics	x					Description of mathematics teachers' training
ESM	1976	Teaching Problem Solving as Viewed through a Theory of Models	x				x	Concept development, interview with a student on the concept
ESM	1976	Mathematical Induction in the Classroom	x					Concept development

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ESM	1979	Women and Girls in Mathematics: Equity in Mathematics Education	x					Concept development
ESM	1979	Young Children (6-8): Ratio and Proportion	x				x	Observation of a boy outside of a classroom, discussion of classroom activities
ESM	1979	Mathematical Olympiads in the People's Republic of China	x					Description of problems presented at the 1978 Chinese Olympiads
ESM	1979	Ways to Report on Empirical Research in Education	x					Concept development
ESM	1979	Sex Differences in Mathematical Performance: An Historical Perspective	x					Description of sex differences in mathematical performance
ESM	1979	The Intuition of Infinity					x	Administration of a questionnaire
ESM	1979	A New Approach to the Assessment of Children's Mathematical Competence					x	Administration of a test and classification/analysis of student responses/achievement
ESM	1979	The Mathematical Education of Engineers	x					Concept development
ESM	1979	Language and Mathematical Education	x					Description of linguistics in education
ESM	1979	Objective Testing in Elementary Analysis				x	x	Participants were first year university mathematics students, investigation into assessment methods
ESM	1979	The Learning of Process Aspects of Mathematics	x					Concept development
ESM	1979	Rings and String	x					Critique of Piaget's Rings and Strings chapter
ESM	1979	Strategies for Teaching Geometry to Younger Children	x					Concept development
ESM	1979	Visualizing and Mathematics in a Pre-Technological Culture				x		Participants were first year university students

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ESM	1979	The Acquisition of Arithmetical Concepts	x					Concept development
ESM	1982	Subtracting Fractions with Different Denominators			x			The researchers provided an intervention to two groups of students, the intervention was altered midway through the experiment for one group
ESM	1982	Curriculum Variables, Theory and Goals: A Comment on Begle's Critical Variables in Mathematics Education	x					Discussion of a book chapter
ESM	1982	Piagetian Tasks as Readiness Measures in Mathematics Instruction: A Critical Review	x					Synthesis of literature
ESM	1982	The Pupil's View of Mathematics Learning					x	Interviews about students' feelings
ESM	1982	A New Look at the Professional Training of Secondary School Mathematics Teachers				x		Discussion on remedy's for teacher training
ESM	1982	Reviewed Work(s): Rechenstein, Experiment, Sprache by P. Damerow and W. Lefèvre	x					Book review
ESM	1982	Reviewed Work(s): Examining in Second Level Education by John Heywood	x					Book review
ESM	1982	Reviewed Work(s): Mathematics, The Loss of Certainty by Morris Kline	x					Book review
ESM	1982	Reviewed Work(s): Cultural Contexts of Science and Mathematics Education: A Bibliographic Guide by Bryan Wilson	x					Book review
ESM	1982	Bilinguals' Understanding of Logical Connectives in English and Sesotho					x	Administration of a test and classification/analysis of student responses/achievement

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ESM	1982	Reviewed Work(s): Mathematical Discovery. On Understanding, Learning and Teaching Problem Solving by G. Polya	x					Book review
ESM	1982	On Some Psychological Aspects of Mathematics Achievement Assessment and Classroom Interaction				x	x	Participants were teachers and students, administration of questionnaires about views and feelings
ESM	1982	The Mathematical Vitality of Secondary Mathematics Graduates and Prospective Teachers: A Comparative Study				x		Participants were university students
ESM	1982	Reviewed Work(s): Children's Understanding of Mathematics: 11-16 by K. M. Hart	x					Book review
ESM	1982	Learning Mathematics in a Second Language: A Problem with More and Less					x	Administration of a test and classification/analysis of student responses/achievement
ESM	1982	The Development of Semantic Categories for Addition and Subtraction	x					Concept development
ESM	1982	The Understanding of Numeration in Primary School					x	Structured interviews to complete assessment tasks and classification/analysis of student responses/achievement
ESM	1985	Two Children's Anticipations, Beliefs, and Motivations					x	Case studies on students' anticipations and beliefs which therefore impact their motivation
ESM	1985	A Longitudinal Study of Children's School Mobility and Attainment in Mathematics					x	Research on attainment levels of mobile students versus nonmobile students
ESM	1985	Proportional Reasoning: A Review of the Literature	x					Synthesis of literature
ESM	1985	MISP: The Mathematics in Society Project	x					Description of MISP

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ESM	1985	Reviewed Work(s): Didactical Phenomenology of Mathematical Structures by Hans Freudenthal	x					Book review
ESM	1985	Mathematical Education versus Critical Education	x					Concept development
ESM	1985	Unravelling the Mysteries of Expert Mental Calculation	x					Concept development
ESM	1985	Reviewed Work(s): The Concept of Activity in Soviet Psychology by J. Wertsch	x					Book review
ESM	1985	Reflection and Recursion	x					Concept development
ESM	1985	Creativity	x					Concept development
ESM	1985	Between Behaviour and Neurology	x					Concept development
ESM	1985	Reviewed Work(s): Acquisition of Mathematics Concepts and Processes by Richard Lesh and Marsha Landau	x					Book review
ESM	1985	Sex-Related Differences in Mathematics: An Overview	x					Concept development
ESM	1985	Autonomous Learning Behavior: A Possible Explanation of Sex-Related Differences in Mathematics	x					Concept development
ESM	1985	Model of Students' Mathematics Enrollment Decisions	x					Concept development
ESM	1985	Preliminary Notes on a Theory of Informal Barriers for Women in Mathematics	x					Concept development
ESM	1985	Reviewed Work(s): Teaching Calculus by Hugh Neill and Hilary Shuard	x					Book review
ESM	1985	The Hand-Held Calculator at the Intermediate Level			x			Experimental and control groups
ESM	1985	What Is the Point of Group Discussion in Mathematics?	x					Concept development
ESM	1985	Second Language Teaching through Maths: Learning Maths through a Second Language	x					Description of multilingual education in London

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ESM	1985	The Acquisition of Basic Multiplication Skills					x	Interview and classification/analysis of student responses/achievement
ESM	1985	Visualizing Rectangular Solids Made of Small Cubes: Analyzing and Effecting Students' Performance			x			Intervention provided for three weeks, pre and posttests administered
ESM	1985	The Number Line as a Teaching Aid	x					Concept development
ESM	1985	Convexity and Shortest Road			x		x	Two experimental groups, test administered and classification/analysis of student responses/achievement
ESM	1985	Qualitative Evaluation of Mathematical Activity and Its Relation to Effective Guidance					x	Description of the clinical application of a mathematical model
ESM	1985	A Mathematical Camp for Bright Pupils	x					Description of activities that can be used for a mathematical camp
ESM	1985	Hemispheric Basis for Schools in Mathematics				x		Participants were college students enrolled in Philosophy of Mathematics
ESM	1985	Search for the Roots of Ratio: Some Thoughts on the Long Term Learning Process (Towards... A Theory): Part II: The Outline of the Long Term Learning Process	x					Description of a learning process for ratio
ESM	1985	The Impact of Secondary Schooling and Secondary Mathematics on Student Mathematical Behaviour					x	Interviews about student perceptions, tests on skills administered, and questionnaires administered to determine attitudes and beliefs
ESM	1985	Memory in Mathematical Understanding	x					Concept development
ESM	1985	A Direct Approach to Indirect Proofs	x					Concept development

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JRME	1970	Message from Julius H. Hlavaty, President, NCTM	x					Commentary about the journal
JRME	1970	Research in Mathematics Education: The Role of Theory and of Aptitude-Treatment-Interaction	x					Concept development
JRME	1970	Attitude Changes in a Mathematics Laboratory Utilizing a Mathematics-Through-Science Approach			x		x	Intervention provided for 4 weeks, multiple sample groups, pre and posttests administered, attituded scale administered
JRME	1970	The Effects of Two Semesters of Secondary School Calculus on Students' Fist and Second Quarter Calculus Grades at the University of Utah				x		Participants were university students enrolled in calculus
JRME	1970	Teacher Expectancy and Mathematics Achievement				x		Participants were teachers
JRME	1970	The Relationship Between a Seventh-Grade Pupil's Academic Self-Concept and Achievement in Mathematics					x	Assessment of self-concept correlated to achievement
JRME	1970	The Comprehensive Mathematics Inventory: An Experimental Instrument for Assessing the Mathematical Competencies of Children Entering School					x	Research on the reliability of a newly created assessment
JRME	1970	A Two-Stage Sequential Strategy in the Placement of Students in an Undergraduate Mathematics Curriculum				x	x	College students' assessment and achievement data analyzed
JRME	1970	Verbal and Nonverbal Assessment of the Conservation of Illusion-Distorted Length					x	Interview/verbal test and classification/analysis of student responses/achievement
JRME	1970	Interactions Between "Structure-of-Intellect" Factors and Two Methods of Presenting Concepts of Modular Arithmetic - A Summary Paper				x		Participants were college students enrolled in mathematics courses

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JRME	1970	The Relative Effectiveness of Four Strategies for Teaching Mathematical Concepts				x		Participants were university students
JRME	1970	Discovery Learning in Kindergarten Mathematics			x			Three instructional interventions and control group
JRME	1970	Some Strategies for Solving Simple Multiplication Combinations					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1970	The Feasibility of Inducing Number Conservation Through Training on Reversibility			x			Intervention provided, control and experimental groups, pre and posttest administered
JRME	1970	Differential Performance of First-Grade Children when Solving Arithmetic Addition Problems					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1970	The Effects of Studying Decimal and Nondecimal Numeration Systems on Mathematical Understanding, Retention, and Transfer			x			Intervention provided for 9 days, three experimental groups and control group
JRME	1970	Parts of a Systems Approach to the Development of a Unit in Probability and Statistics for the Elementary School			x			Intervention provided, pre and posttests administered
JRME	1970	Behavioral Objectives and Flexible Grouping in Seventh-Grade Mathematics			x		x	Intervention provided, multiple experimental groups, assessments on both achievement and attitudes
JRME	1970	Linear Measurement in the Primary Grades: A Comparison of Piaget's Description of the Child's Spontaneous Conceptual Development and the SMSG Sequence of Instruction	x					Description of the similarities and differences between Piaget's writings and SMSG sequence of instruction
JRME	1970	On Scrambling Instructional Stimuli	x					Concept development

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JRME	1973	Toward a Theory of Sequencing: An Integrated Program of Research	x					Description of a research program and theory
JRME	1973	The Effects of Test Anxiety and Success/Failure on Mathematics Performance in Grade Eight			x		x	Administration of anxiety assessments, intervention provided, multiple experimental groups
JRME	1973	A Model of Classroom Discourse for Use in Conducting Aptitude-Treatment Interaction Studies				x		Participants were teachers
JRME	1973	A Comparison of Three Strategies for Teaching a Selected Mathematical Concept to Students in College Algebra				x		Participants were college students enrolled in college algebra
JRME	1973	Research on Mathematics Education (K-12) Reported in 1972	x					Annual annotated list of research
JRME	1973	Educational Research in Mathematics at The University of Wisconsin Research and Development Center for Cognitive Learning	x					Review of past, present, and future research conducted under the auspices of the university
JRME	1973	Effects on Transfer of Training of Constant Versus Varied Training, Group Size, and Ability Level, In Elementary School Mathematics			x			Intervention provided, 3 experimental groups, posttest administered
JRME	1973	Individualized Instruction in Problem Solving in Elementary School Mathematics			x			Two different interventions provided for 16 days, 6 experimental groups, 2 control groups

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JRME	1973	Retention of Probability Concepts. A Pilot Study into the Effects of Mastery Learning with Sixth Grade Students					x	Intervention provided, pre, post, and retention tests administered, no information was provided on the intervention itself or any background information on the study, only data and results were provided in the article - this article was a continuation of a prior published article
JRME	1973	The Effectiveness of Discovery and Expository Methods in the Teaching of Fourth-Grade Mathematics			x	x		Participants were students and teachers, intervention provided for 31 weeks, 2 treatment groups
JRME	1973	The Symmetric Property of the Equality Relation and Young Children's Ability to Solve Open Addition and Subtraction Sentences					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1973	A Formative Development of an Elementary School Unit on Proof			x			Intervention provided for 17 days, experimental and control groups, pre and posttests administered
JRME	1973	The Effect of an Advance Organizer, Cognitive Set, and Post Organizer on the Learning and Retention of Written Materials			x			8 Instructional models/interventions employed, post and retention tests administered
JRME	1973	The Effect of Organizers and Knowledge of Behavioral Objectives on Learning a Mathematical Concept				x		Participants were students and preservice teachers
JRME	1973	A Study of the Ability of School Pupils to Perceive and Identify the Plane Sections of Selected Solic Figures					x	Administration of a test and classification/analysis of student responses/achievement

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JRME	1973	Validating Learning Hierarchies for Sequencing Mathematical Tasks in Elementary School Mathematics			x			Intervention provided, 7 treatment groups
JRME	1973	The Effect of Class Size on the Learning of Mathematics: A Parametric Study with Fourth-Grade Students			x			Intervention provided, 4 treatment groups
JRME	1973	A Comparison of Initially Teaching Division Employing the Distributive and Greenwood Algorithms with the Aid of a Manipulative Material			x			Intervention provided, 2 different treatments administered
JRME	1973	Achievement Monitoring via Item Sampling: A Practical Data-Gathering Procedure for Formative Evaluation					x	Data gathering to inform revisions of an educational product
JRME	1973	The Interaction of Three Levels of Aptitude Determined by a Teach-Test Procedure with Two Treatments Related to Area			x			Intervention provided, 2 different treatments administered
JRME	1973	The Relative Effectiveness of Two Different Instructional Sequences Designed to Teach the Addition and Subtraction Algorithms			x			Intervention provided, 2 different treatments administered
JRME	1976	An Analysis of Some of Piaget's Topological Tasks from a Mathematical Point of View	x					Concept development
JRME	1976	An Experimental Study of the Effectiveness of a Formal Versus an Informal Presentation of a General Heuristic Process on Problem Solving in Tenth-Grade Geometry			x			Intervention provided, experimental and control groups
JRME	1976	An Interpretation of Advanced Organizers	x					Concept development
JRME	1976	College and Other Post-Secondary School Studies	x					Concept development
JRME	1976	Cognitive Emphasis of Geometry Teachers' Questions				x		Participants were teachers

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JRME	1976	Comparative Effects of Three Sequences of Moves for Teaching Selected Mathematical Concepts to College Students				x		Participants were college students
JRME	1976	Research on Mathematics Education Reported in 1975	x					Annual annotated list of research
JRME	1976	Brief Reports: Use and Recall of Advance Organizers in Mathematics Instruction	x					Short summary of 2 different studies, detailed information on the studies is not provided
JRME	1976	Brief Reports: Fennema-Sherman Mathematics Attitudes Scales: Instruments Designed to Measure Attitudes Toward the Learning of Mathematics by Females and Males	x				x	Description of the Fennema-Sherman Mathematics Attitudes Scales
JRME	1976	The Effect of Instructional Gaming on Absenteeism: the First Step			x			Experimental and control groups
JRME	1976	A Test with Selected Topological Properties of Piaget's Hypothesis Concerning the Spatial Representation of the Young Child					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1976	Elementary and Middle School Children's Comprehension of Euclidean Transformations					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1976	The Development of the Concept of a Standard Unit of Measure in Young Children			x		x	Administration of an interview/test and classification/analysis of student responses/achievement, 2 experimental groups
JRME	1976	Models and Applications as Advanced Organizers			x			Intervention provided, 16 treatment groups, posttest administered
JRME	1976	The Influence of an Advanced Organizer on Two Types of Instructional Units About Finite Geometries				x		Participants were university students enrolled in C26-1 Geometry

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JRME	1976	The Influence of Two Types of Advanced Organizers on an Instructional Unit About Finite Groups				x		Participants were university students enrolled in C41 Modern Algebra
JRME	1976	The Relative Effectiveness of Four Strategies for Teaching Algebraic and Geometric Disjunctive Concepts and for Teaching Inclusive and Exclusive Disjunctive Concepts			x	x		Intervention provided, 4 treatments, participants were preservice teachers
JRME	1976	An Analysis of Children's Written Solutions to Word Problems					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1976	An Analysis of the Fraction Concept into a Hierarchy of Selected Subconcepts and the Testing of the Hierarchical Dependencies					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1976	A Comparison of Two Methods of Column Addition			x			Intervention provided for 2 weeks, 2 treatment groups, posttests administered
JRME	1976	Factors Associated with Third- and Fourth-Grade Children's Performance in Solving Multiplication and Division Sentences					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1976	Effect of Interspersed Questions on Learning from Mathematical Text			x	x		Participants were college students, 2 treatment groups
JRME	1976	Retroactive Interference of Similar Methods to Teach Translation of Base Systems in Mathematics			x			Intervention provided, 2 experiments, 2 experimental groups
JRME	1976	Measuring the Effectiveness of Using Slide-Tape Lessons in Teaching Basic Algebra to Mathematically Disadvantaged Students			x	x		Participants were university students, 4 treatment groups

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JRME	1976	The Effects of Three Instructional Strategies on Problem-solving Behaviors in Secondary School Mathematics			x			Intervention provided, 3 treatment groups
JRME	1976	Story Problems: Merely Confusing or Downright Befuddling?			x			Intervention provided, 7 experimental groups
JRME	1976	Verbal-Nonverbal Conservation and Primary Mathematics			x		x	Administration of 2 different tests to 2 experimental groups and results compared
JRME	1976	The Introduction of Mathematics Through Measurement or Through Set Theory: A Comparison			x			Intervention provided, 2 treatment groups
JRME	1976	Relations Among Piagetian Grouping Structures: A Training Study			x			Intervention provided, 2 treatment groups
JRME	1979	An Inventory of Mathematical Thinking Done by Incoming First-Grade Children					x	Administration of an oral test and classification/analysis of student responses/achievement
JRME	1979	University Level Computing and Mathematical Problem-Solving Ability				x		Participants were university students
JRME	1979	Brief Report: Artistic Motion Cues, Number of Pictures, and First-Grade Children's Interpretation of Mathematics Textbook Pictures	x				x	Summary of a study, few details of the study provided, administration of an interview and classification/analysis of student responses/achievement
JRME	1979	Brief Report: Children's Discrimination Between Enjoyment and Value of Arithmetic	x					Summary of a study, few details of the study provided, administration of a Likert scale attitudes assessment, students and teachers assessed

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JRME	1979	Scores on Piagetian Area Tasks as Predictors of Achievement in Mathematics over a Four-Year Period					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1979	Brief Reports: Further Study of the Use of Manipulatives with Prospective Teachers	x					Summary of a study, few details of the study provided, participants were teachers
JRME	1979	Brief Reports: Student Placement - A Comparison of Traditional and Computerized Branching Test Administrations	x			x	x	Summary of a study, few details of the study provided, comparison of testing used for college student placement
JRME	1979	Brief Reports: The Symbols and Grammatical Structures of Mathematical English and the Reading Comprehension of College Students	x			x		Summary of a study, few details of the study provided, participants were college students
JRME	1979	Brief Reports: The Relationship of Field-Independent/Dependent Cognitive Style and Two Methods of Instruction in Mathematics Learning	x		x			Summary of a study, few details of the study provided, intervention provided, 2 treatments
JRME	1979	Student Perceptions of Relatedness Among Mathematical Verbal Problems					x	Multiple studies, administration of an interview/test and classification/analysis of student responses/achievement
JRME	1979	Brief Reports: High School Calculus and First-Quarter College Calculus Grades	x			x		Summary of a study, few details of the study provided, participants were college students enrolled in calculus
JRME	1979	Research on Mathematics Education Reported in 1978	x					Annual annotated list of research

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JRME	1979	Brief Reports: Incorporating Instructional Objectives into the Rules for Playing a Game	x		x			Summary of a study, few details of the study provided, intervention provided for 10 days, multiple experimental groups, 2 posttests administered
JRME	1979	Brief Reports: Some Aspects of Individual Differences in Mathematics Instruction	x			x	x	Summary of a study, few details of the study provided, participants were students and teachers
JRME	1979	Brief Reports: Hand-held Calculator Curriculum and Mathematical Achievement and Retention	x		x			Summary of a study, few details of the study provided, intervention provided, treatment and control groups
JRME	1979	The Interaction of Cognitive Aptitudes with Sequences of Figural and Symbolic Treatments of Mathematical Inequalities			x			Intervention provided, 2 treatment groups
JRME	1979	An Alternative Model Describing Children's Spatial Preferences					x	Administration of a test and classification/analysis of student responses/achievement
JRME	1979	Process, Conceptual Knowledge, and Mathematical Problem-Solving					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1979	Information Transfer in Solving Problems				x		Participants were teachers
JRME	1979	Using Games to Retrain Skills with Basic Multiplication Facts			x			Intervention provided for 10 days each year for 2 years, multiple treatment groups, pre and posttests administered each year
JRME	1979	Hand-held Calculators and the Learning of Trigonometric Ratios			x			Intervention provided for 18 days, 2 treatment groups

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JRME	1979	Problem Structure, Cognitive Level, and Problem-Solving Performance					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1979	Error Analysis in Mathematics Education	x					Review of the literature
JRME	1979	Explicit Heuristic Training as a Variable in Problem-Solving Performance			x			Intervention provided, 2 treatment groups, posttest administered
JRME	1979	Oral Language and Readiness for the Written Symbolization of Addition and Subtraction			x			Intervention provided for 12 weeks, 2 treatment groups, posttest administered
JRME	1979	Variables Affecting Word Problem Difficulty in Elementary School Mathematics			x		x	2 experimental groups, administration of multiple tests in varying orders and classification/analysis of student responses/achievement
JRME	1979	The Interaction of Learner Aptitude with Types of Questions Accompanying a Written Lesson on Logical Implications			x			Intervention provided, 4 treatment groups
JRME	1979	The Interaction of Field Dependence/Independence and the Level of Guidance of Mathematics Instruction				x		Participants were preservice teachers
JRME	1982	Calculator Use in the Community College Arithmetic Course				x		Participants were college students
JRME	1982	Brief Reports: Effective Mathematics Instruction for Low-Income Students: Results of Longitudinal Field Research in 12 School Districts	x					Summary of a study, few details provided, analysis of student achievement on multiple assessments
JRME	1982	Correlates and Predictors of Women's Mathematical Participation					x	Survey of career and academic plans, attitudes, etc.
JRME	1982	Mathematics Achievement and Fear of Success					x	Administration of a reasoning test, fear of success measure, and a questionnaire

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JRME	1982	Brief Reports: Nongraded Instruction, Mathematics Ability, and Mathematics Achievement in Elementary Schools	x		x			Summary of a study, few details provided, intervention provided, experimental and control groups
JRME	1982	Brief Reports: Discriminating Factors and Sex Differences in Electing Mathematics	x				x	Summary of a study, few details provided, report on the data collected about students' enrollment in mathematics courses
JRME	1982	Diagnosing Strengths and Weaknesses of Sixth-Grade Students in Solving Word Problems					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1982	Research on Mathematics Education Reported in 1981	x					Annual annotated list of research
JRME	1982	The Importance of Spatial Visualization and Cognitive Development for Geometry Learning in Preservice Elementary Teachers				x		Participants were preservice teachers
JRME	1982	Strategy Use and Estimation Ability of College Students				x		Participants were college students
JRME	1982	Brief Reports: Multisensory Information Matching Ability and Mathematics Learning	x				x	Summary of a study, few details provided, administration of multiple tests and classification/analysis of student responses/achievement
JRME	1982	Student Performance, Individual Differences, and Modes of Representation				x		Participants were preservice teachers
JRME	1982	Algebra Word Problem Solutions: Thought Processes Underlying a Common Misconception				x		Participants were college students
JRME	1982	Measures of Problem-Solving Performance and of Problem-Solving Instruction				x		Participants were college students

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JRME	1982	Cognitive Development and Children's Solutions to Verbal Arithmetic					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1982	Fourth Graders' Heuristic Problem-Solving Behavior			x			Intervention provided for 9 weeks, 4 experimental and control groups
JRME	1982	Careless Errors Made by Sixth-Grade Children on Written Mathematical Tasks					x	Error analysis of student responses/achievement on a battery of tests
JRME	1982	Basic Fact Thinking Strategies for Multiplication - Revisited			x			Intervention provided for 9 weeks, 2 treatment groups
JRME	1982	The Use of Problem-Solving Heuristics in the Playing of Games Involving Mathematics					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1982	Processes Used by Good Computational Estimators					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1982	Skill in Estimation Problems of Extent and Numeracy				x		Participants were students and adults
JRME	1982	Drawn Versus Verbal Formats for Mathematical Story Problems			x			Intervention provided for 5 to 6 weeks, 2 treatment groups, aptitude measures and posttest administered
JRME	1982	The Position of the Unknown Set and Children's Solutions of Verbal Arithmetic Problems					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1982	Intuitive Functional Concepts: A Baseline Study on Intuitions					x	Administration of a questionnaire

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JRME	1982	Story Problem Solving in Elementary School Mathematics: What Differences Do Calculators Make?			x			Intervention provided for 8 to 12 weeks, experimental and control groups, posttest administered
JRME	1985	Brief Reports: Using Microcomputers with Fourth-Grade Students to Reinforce Arithmetic Skills	x		x			Summary of a study, few details provided, intervention provided, 2 treatment groups
JRME	1985	Brief Reports: The Problem of Inflated Significance When Testing Individual Correlations From a Correlation Matrix	x				x	Summary of a study, few details provided, analysis of correlation tests
JRME	1985	Research on Mathematics Education Reported in 1984	x					Annual annotated list of research
JRME	1985	Mathematics Education Research: 1984 in Review	x					Summary of annual research topics
JRME	1985	A Beginning Teacher's View of Problem Solving	x					Participant was a mathematics teacher
JRME	1985	Brief Reports: Cognitive Functioning and Performance on Addition and Subtraction Word Problems	x					Summary of a study, few details provided, administration of multiple tests and classification/analysis of student responses/achievement
JRME	1985	Brief Reports: The Acquisition of Inverse Proportionality: A Training Experiment	x		x			Summary of a study, few details provided, intervention provided, experimental and control groups, pre and posttests administered
JRME	1985	The Role of Implicit Models in Solving Verbal Problems in Multiplication and Division					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1985	Order and Equivalence of Rational Numbers: A Cognitive Analysis			x			Intervention provided for 18 weeks, full study not discussed in article, partial results reported in article

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JRME	1985	Computation Routines Prescribed by Schools: Help or Hindrance					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1985	Mastery of Basic Number Combinations: Internalization of Relationships or Facts?	x					Concept development
JRME	1985	Is It Farfetched That Some of Us Remember Our Arithmetic Facts?	x					Concept development
JRME	1985	Computational Estimation and Related Mathematical Skills					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1985	Construct a Sum: A Measure of Children's Understanding of Fraction Size					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1985	Metacognition, Cognitive Monitoring, and Mathematical Performance	x					Concept development
JRME	1985	The Influence of Training Hispanics in Test Taking on the Psychometric Properties of a Test				x		Participants were college students
JRME	1985	The Use of Spatial Visualization in Mathematics					x	Administration of an interview/test and classification/analysis of student responses/achievement
JRME	1985	Computer-Video Instruction in Mathematics: Field Test of an Interactive Approach			x		x	Multiple field tests of a computer program, administration of an interview, pre and posttests, and a questionnaire on success and failure attitudes
JRME	1985	The Comparative Effectiveness of Microcomputers and Flash Cards in the Drill and Practice of Basic Mathematics Facts			x			Intervention provided for 6 weeks, 2 treatment groups, pre and posttests administered

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JRME	1985	Instruction on Derived Facts Strategies in Addition and Subtraction			x			Intervention provided for 8 weeks, administration of interviews, pre and posttests, and observations, classification/analysis of student responses/achievement
JRME	1985	A Screening Procedure to Identify Children Having Difficulties in Arithmetic					x	Multiple administrations of probes and achievement scales and classification/analysis of student responses/achievement
JRME	1985	Rote Versus Conceptual Emphasis in Teaching Elementary Probability				x		Participants were college students enrolled in an introductory psychology course