HOW SCAFFOLDING RATE OF CHANGE PROBLEMS PROMOTES POSITIVE MATHEMATICAL TRANSFERABILITY

A Thesis

by

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This thesis meets the standards for scope and quality of
Texas A&M University-Corpus Christi and is hereby approved.

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ABSTRACT

This thesis reports on action research conducted by the author while teaching rate of change problems to first-semester Calculus students at a public university in the United States, with the purpose to ensure transfer of learning, and increase students' conceptual understanding of rate of change and their performance on solving rate of change problems. The action research cycles (planning, action/implementation, evaluation) involved scaffolding to promote positive mathematical transferability through conceptual understanding. The result is a Lesson Guide consisting of a learning task designed using scaffolding and abiding by principles developed and supported by the research literature on the theory of transfer, an assessment task, and a self-evaluation tool for students. Future action research cycles may be informed by the Lesson Guide developed in this study. *Calculus I* instructors can use the Lesson Guide in the teaching of rate of change problems. This study contributes to the progressive perspective of the transfer of learning in mathematics education and adds to the growing number of theses conducted with action research methodology.

DEDICATION

My studies and work on this thesis are dedicated to my grandfather who foresaw the potential in me to be this successful, of which I doubted for many years. It is the support of my family that gave me the will to return to my educational career, with new experiences and perspectives I was able to apply and will continue applying.

ACKNOWLEDGEMENTS

This study deserves the acknowledgement of the committee. They have prepared, researched, and assisted in its development towards publication. Your commitment and dedication have greatly supported the purpose and importance of this study. This study would not have been possible without participation from my own students. I learned so much from everyone. Thank you.

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CHAPTER I: Introduction

About This Study

The result of education is knowledge that is either bound to the original setting of the learning or knowledge that is available for use outside of that setting. The study reported in this thesis refers to education that has the latter goal, or transfer of knowledge. Educators agree that the transfer of knowledge, or the transfer of learning across various settings, is of the utmost importance, perhaps the most important issue, in education. Researchers in mathematics education reported on the difficulties of finding ways to ensure positive transferability of prior learning to new learning. Only about a decade ago have they started reporting successful approaches (Hohensee & Lobato, 2021), like scaffolding (Anghireri, 2006; Grothérus, Jeppsson Samuelsson, 2019; Tanner and Jones, 2000). This thesis reports on action research conducted by the author with first-semester Calculus students, with the purpose of increasing their conceptual understanding of rate of change and their performance on solving rate of change problems.

The Purpose of the Study

The main purpose of this study is to examine how action research can support development of scaffolding rate of change problems in support of *Calculus I* students' conceptual understanding that promotes positive knowledge transfer and enhance their performance on rate of change problems. A secondary goal is to design a *Lesson Guide* consisting of learning tasks using scaffolding and a set of abiding principles informed by the research literature on the theory of transfer, assessment task, and self-evaluation to foster positive transferability of students' knowledge through conceptual understanding.

Research Questions

The study sought to examine how action research can be employed to design educational lesson and assessment on rate of change that has the potential to increase *Calculus I* students' understanding of rate of change and their performance on solving rate of change problems. Specifically, the study was intended to address the following two research questions:

1. How can action research help scaffold rate of change problems as support of *Calculus I* students' conceptual understanding by promoting positive transfer of knowledge and enhance students' performance on rate of change problems?
2. To what extent does a *Lesson Guide* consisting of learning tasks using scaffolding and a set of abiding principles informed by the research literature foster positive transferability of students' knowledge through conceptual understanding?

Both the lesson and the assessment focus on rate of change problems/tasks. During the action research cycles (planning, action/implementation, evaluation), those problems/tasks went through several changes, the main change being scaffolding, to promote positive transferability of students' knowledge through conceptual understanding.

Guiding Principles

Below are three principles, developed and supported by the research literature on the theory of transfer, that guided the design of the mathematical tasks in the lesson and assessment in this study:

P1. Successful transfer of knowledge learned in one context to another context indicates the proper understanding of the concept of rate of change.

P2. Scaffolding of instruction can support understanding (procedural and conceptual) necessary for transfer.

P3. There exist different scaffolding methods intended for learners to discover and master mathematical concepts in a task and should be troubleshooted before and after instruction.

Significance of the Study

The curriculum materials (lesson guide and assessment tasks) developed in this study may be used by the *Calculus I* instructors from the school at which it was developed and other schools to improve the assessment(s) design of rate of change problems with scaffolding. Additionally, the study is significant from the researchers' perspective, in the sense that: i) it contributes to the progressive perspective of the transfer of learning in mathematics education that departs from the unsuccessful traditional perspective on transfer (Hohensee and Lobato, 2021), and ii) adds to the growing number of studies conducted with action research methodology as a viable alternative methodology in education research (Bragg, 2017, Lari, Rose, Ernst, Clark, Kelly, DeLuca, 2019).

Types of Understanding in Mathematics

During the “Back-to-Basic” movement, Skemp (1976) described relational and instrumental understanding and pointed out that although relational (conceptual) and instrumental (procedural) mathematics may refer to the same subject matter, they are, in fact, two different kinds of understanding leading to two kinds of mathematics. Instrumental (procedural) understanding refers to applying “rules without reason” (Skemp, 1976, p. 152), i.e., the student may apply an algorithm to solve a problem or execute a task, without knowing why the algorithm works. The entry point and the end point to the algorithm/rule are fixed. Relational (conceptual) understanding means “knowing what to do and why” (Skemp, 1976, p. 152). That is, the student can apply the knowledge in a situation different from the one in which it was learned, and then justify why it works. A student with conceptual understanding has a structure of knowledge that can be accessed in multiple ways, from multiple entry points, to multiple end points, at other times, in other situations, and used to construct further mathematical knowledge. This makes conceptual knowledge necessary for the transfer of learning from old situations to new situations.

For example, a *Calculus I* student with instrumental (procedural) understanding of rates of change as derivatives may be able to apply the differentiation rules, and correctly differentiate $f(x) = x^2$ as $(x^2)' = 2x$ .When asked to justify the rule, the student with instrumental (procedural) understanding may say something like “this is the rule,” or perhaps refer to the immediate application of the rule $(x^n)' = nx^{n-1}$ when $n = 2$. A student with conceptual knowledge may be able to justify the rule with the definition of the derivative as a limit,

$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}=\lim_{h\to 0}\frac{(2x+h)h}{h}=2x\,,$$

and conceive of the rate of change as a derivative in the following situation adapted from the study conducted by Mamolo and Zazkis (2012):

> Use the diagram (see Figure 1, below) to show why the derivative of the area of a circle yields the formula for the circumference of the circle.

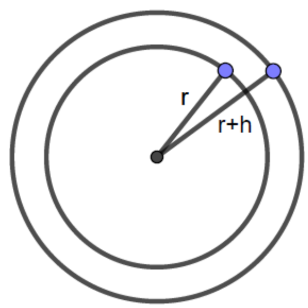


Figure 1. Diagram for the task adapted from Mamolo and Zazkis (2012)

To solve the problem, or to execute the task, the student may conceive of the rate of change of the area between the two circles as a derivative, and calculate it using the definition of the derivative as a limit:

$$\lim_{h\to 0}\frac{A_{r+h}-A_r}{h}=\lim_{h\to 0}\frac{\pi(r+h)^2-\pi r^2}{h}=\lim_{h\to 0}\frac{(2\pi r+\pi h)h}{h}=\lim_{h\to 0}(2\pi r+\pi h)=2\pi r\,.$$

Types of Transfer

Transfer of mathematical knowledge, or transfer of learning, has been described in the literature along several dimensions. Perkins (1992) explains that the mechanisms of transfer influencing positive transfer include abstraction, affordances, high road and load road transfer.

The National Research Council (2000) also considers near transfer and far transfer. Near transfer refers to transfer between very similar contexts. Low road is rather reflexive and involves triggering “of well-practiced routines by stimulus conditions similar to those in the learning context” (National Research Council, 2000, p. 8). High road transfer depends “on mindful abstraction” and “demands time for exploration and the investment of mental effort” (National Research Council, 2000, p. 8). Similarly, far transfer occurs when transferring information to rather different contexts. Pugh and Bergin (2006) found these conditions do not occur as often as near transfer and low road transfer. High road and far transfer, because of their difficulties, may lead to negative transfer.

Negative transfer occurs “when an experience with one set of events hurts the performance on related tasks” (National Research Council, 2000, p. 53). Researchers (Evans, 1999; National Research Council, 2000; Pugh and Bergin, 2006) agree that the few ways transfer can be negated include: teaching in only one context leading knowledge to be context-bound, lack of motivation, overly contextualizing knowledge, and lack of previous experience. Initial learning is necessary for positive transfer. One cannot expect to transfer knowledge if their initial understanding is not conceptual. Even if they have a conceptual understanding of the given material, they cannot transfer that knowledge to something they cannot relate to their previous experiences. If a student has previous experiences to relate to the problem given, they can still be prevented from transfer if the intended content is overly contextualized. To prevent a failure to relate, lesson needs to present an opportunity to discover and present familiarities. A learner’s familiarity and relation to a concept or context has an effect on the learner's motivational interests; if interest or motivation doesn’t exist, then it is expected to have a lack of learner application and discourse. A learner who may not relate, or have any familiarity, can still gain positive transfer. This is most likely achieved by having self- monitoring skills. Lobato (2003) reports that there should be different design cycles in which a student finds familiarities and actor-oriented transfer is distributed across mental, material, social, and cultural planes. When conducting design tasks, researchers should ask questions like "What relations of similarity are created? How are they supported by the environment?" (Lobato, 2003, p. 20). A student can also be context bound based on their previous experiences. Negative transfer happens most often when the student fails to relate a task to previous experiences, giving them different perspectives.

To achieve positive transfer, the student must achieve low road transfer. There is some abstraction and affordances provided in the task that can in turn help achieve or possibly hinder positive transfer. To detect the abstraction between the two tasks, the intended affordances from the implementer should be perceived correctly by the student. These affordances can include analogies and metaphors (Perkins 1992).

Definition of Terms

This section defines the main terms used in this paper.

Scaffolding. Scaffolding means the support consisting of instructional methods provided by the instructor to her students, with the purpose to help the students learn new content, solve a new problem, or perform a new mathematical task.

Transfer of learning. Also referred as transfer of knowledge, transfer, or transferability means using old learning or old knowledge in new situations, similar or different than the situation in which old learning occurred or old knowledge was constructed.

Positive transfer. A positive transfer of learning or knowledge takes place when one can successfully use old learning in new situations, and as a result some new learning occurs.

Conceptual understanding. A student has relational understanding or conceptual understanding when the student can apply the knowledge in other situations and justify why it works. A student with conceptual knowledge may access and use that knowledge in multiple ways, from multiple entry points, to multiple end points, any time, in any situation, and to construct further mathematical knowledge.

Mathematical task. A mathematical task is anything, with mathematical content, that we ask the students to execute during the teaching and learning process (e.g., tasks for learning or assessment tasks). Oftentimes, a mathematical task is a problem, and will be referred to as a mathematical task/problem or problem.

Rate of change (in Calculus). The rate of change of a function is the instantaneous measure of how the change of the values of the function varies with respect to changes in the independent variable. This thesis records the rate of change of $f(x)$ as $f'(x)$ and takes the computation, where it exists, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ as that measure.

Action research. Action research is a cyclical research process (planning, action/implementation, evaluation) used to improve instructional practice, curriculum materials, and student outcomes. It has a local focus, like the author's school, and takes place in natural settings, like the classes taught by the author.

CHAPTER II: Theoretical Background

The lesson guide and assessment task described in Chapter IV abide by the principles stated in Section 1.4, which have been developed using theoretical constructs by Peressini Peressini, Borko, Romagnano, Knuth, and Willis (2004). Peressini et al. (2004) propose that positive transfer is based on learning, understanding what is to be learned, then applying those concepts to similar and abstract contexts. If the distinct types of transfer are successfully completed, then it can be assumed the learner has gained conceptual understanding. But how abstract did the learner transfer? Can conceptual understanding be assessed or measured?

Peressini et al. (2004) base their theoretical framework on two assertions. The first assertion is a learner's level of conceptual understanding can be affected by their prior experiences, set of skills and knowledge, and the situation in which a person learns. As part of the design, the students are presented with a scaffolded lesson plan, collaborative activities, assessments, all to be designed for discovery and mastery. The second assertion is that the teacher's knowledge and beliefs should "interact with historical, social and political contexts to create situations in which learning to teach occurs" (Peressini et al., 2004, p. 68). This refers to the importance of a student's perspective and relation to the given task, motivating the student's interest to understand its embedded concept. If the student cannot relate to a given task, it is expected they will have less motivation to proceed. If they relate, but do not understand, motivation is expected to decrease. If they discover and understand the task's concept, they most likely were able to relate, and had motivation to complete the task then a level of understanding can be gained based on the type of transfer intended.

To achieve positive transfer of mathematical knowledge across various rate of change problems/tasks, the student must have a conceptual understanding of rates of change. To understand rates of change, the student should have a conceptual understanding, or familiarity, of rates" (Peressini et al., 2004, p. 75). Once the student has a conceptual understanding of the task, they are then expected to positively transfer that knowledge to a similar task.

A way in which conceptual understanding can be enhanced is scaffolding a task Anghileri (2006). In Anghileri's (2006) study, she recovers the process of scaffolding introduced by Wood, Bruner and Ross (1976). Anghileri (2006) identifies the various levels of scaffolding that teachers can use to promote mathematical learning. Scaffolding is intended to guide rather than "show and tell" and is "used to reflect the way adult support is adjusted as the child learns and is ultimately removed when the learner can stand alone" (Anghileri, 2006, p. 33). Anghileri (2006) reviews historic notions to support those levels she identifies. The characteristics of scaffolding are described by exploring the nature of adult interactions in children's learning. These characteristics include recruitment, reduction in degrees of freedom, direction maintenance, marking critical features, frustration control, and demonstration. There are different ways Angileri (2006) proposes to implement scaffolding, particularly for this study, I implement foothold scaffolding and strategic scaffolding. Foothold scaffolding relates to what is described to be a funnel pattern of interactions where "students are provided with leading questions in an attempt to guide them to a predetermined solution procedure" (Anghileri, 2006, p. 36).

Similarly to Anghileri (2006), Tanner and Jones (2000) compare scaffolding to adult intervention and also refers to funneling where the teacher "selects the thinking strategies and controls the decision process to lead the discourse to a predetermined solution" (p. 21). For their study, Tanner and Jones (2000) conduct a mathematical project "to improve pupils' performance in mathematics by developing their metacognitive skills" (p. 22). Learners should participate in discourse to improve critical and logical thinking skills, which are induced by the instructional design.

Referring to the levels of scaffolding necessary to promote transferability, Anghileri (2006) proposes three levels of scaffolding that "constitute a range of effective teaching strategies that may or may not be evident in the classroom" (p. 38). At level one, "environmental provisions enable learning to take place without the direct intervention of the teacher" (Anghileri, 2006, p. 38). These provisions would include surrounding artifacts, classroom organization, peer collaboration, and emotive feedback. These provisions also reflect the framework of Peressini et al. (2004), specifically their belief that one's learning environment is a major factor of positive transfer. Level two of scaffolding (Anghileri, 2006) requires explaining, reviewing and restructuring. At this level, the teacher explains the ideas to be learned, and this relates to the funnel stance concept (Wood, Bruner, Ross, 1976), "while the categories of reviewing and restructuring identify patterns of interaction that are more responsive to the learner and these expand on the idea of focusing" (Anghileri, 2006, p. 41). In turn, the student should explain and justify. Part of reviewing is prompting and probing. Prompting questions "successively lead the students towards a predetermined solution" (Anghileri, 2006, p. 42) and supports students' thinking. Probing is similar but expands students' thinking. At level three of scaffolding (Anghileri, 2006), the student is expected to be developing conceptual thinking. At this level, the teaching interactions explicitly address "developing conceptual thinking by creating opportunities to reveal understandings to pupils and teachers together" (Anghileri, 2006, p. 47). With that, teachers should engage their students in conceptual discourse. This could be implemented during the teaching and learning process for a first task on rates of change, where each group shares their perspectives in discourse, then for a second task the student has no support of the scaffolding or peer discourse.

The search for evidence of transfer revealed several types of transfer, specifically of positive transferability. Those types include near transfer, far transfer, low road transfer, and high road transfer. Usually, not just one type is needed to achieve positive transferability. When implementing these types of transfer, a conceptual understanding is required of the implementer, or commonly known as a teacher. The implementer should be aware of the learning environment and its surroundings in order to transfer their own conceptual understanding of the mathematical material. The more familiar and confident the implementer is, the more likely the students will be able to be confident in learning the material (Pugh and Bergin, 2006).

There exist multiple factors that affect one's own learning environment. Among them is the way in which one learns is based on one's previous experience and knowledge; therefore, every learner has a different perspective (Evans, 1999). It is important for the implementer to be aware of these perspectives, so the learner relates to the given mathematics problems/tasks and, of course, and improve mathematical language. There are also affordances and constraints to every given problem/task (Watson, 2004), differentiated based on one's experiences and learning environment.

The proposed scaffolded task must be designed with affordances, signifiers, different perspectives, all in intention to avoid constraints of transfer and achieve a level of conceptual understanding to positively transfer to a similar, maybe more abstract, task. Part of avoiding constraints or misinterpreted affordances, is considering some human error when developing a hypothesis for an experiment and when analyzing the results of the experiment. It should be considered that when one can relate from their previous experiences, the probability of positive transfer to similar contexts is expected to be much higher than not. This is more than common for task designers to implement, but there is a risk that even just one student will not relate. Along with the environment someone has experiences in, these can constrain one from relating them to assumed to be similar contexts. When relation is discovered, positive mathematical transfer is more likely to be achieved. “The transfer of learning refers in general to the use of ideas and knowledge learned in one context to another.” (Evans, 1999, p. 23)

To interpret mathematical language, most look for ‘signifiers’ or key words to signify what needs to be solved in any given problem/task (Evans, 1999). Although, based on one’s experience, the signifier could hinder translation. An example of this would be the word ‘more.’ Implementers encourage students to assume a word problem requires to be solved with addition or multiplication when the word ‘more’ is seen. But there is a chance that a student will not recognize this signifier like the implementer intended. For example, the phrase ‘rate of change’ should be a signifier to Calculus students that this task is going to refer to derivatives, but students may interpret it as an average rate of change, i.e., a constant ratio.

In short, to promote positive transfer of learning, conceptual understanding is needed, and scaffolding may assist in the developing of conceptual understanding. As such, I derived the following principles to function as a theoretical framework and guide the design of the problems/tasks from the lesson and assessment described in Chapter 4:

P1. Successful transfer of knowledge learned in one context to another context indicates the proper understanding of the concept of rate of change.

P2. Scaffolding of instruction can support understanding (procedural and conceptual) necessary for transfer.

P3. There exist different scaffolding methods intended for learners to discover and master mathematical concepts in a task and should be troubleshooted before and after instruction.

CHAPTER III: Methods

Pilot Study

The concept of rate of change has been documented to be challenging for students (Orton, 1983; Tyne, 2016; White & Mitchelmore, 1996). The objective of the pilot study was to determine the effectiveness of the instructional method of scaffolding with respect to students' performance on solving rate of change problems. Two different assignments, without and with scaffolding were administered to a class composed mostly of engineering majors (N=20), during the Spring 2019 semester. The students studied rates of change at the beginning of the semester (the pilot study was conducted roughly six weeks after).

Before proctoring each assessment, I emphasized particular steps to solving a problem (also referred to as completing a task), described in Polya's book "How to Solve It". Of note, Polya's approach to problem solving was emphasized that semester by the *Calculus I* instructors in our department, as part of our effort to improve our students' critical thinking skills and performance. The steps described by Polya include understanding the problem, devising a plan to solve the problem, executing the plan, and checking the solution. The first step to understand a problem, or task, is to understand what is being asked to solve, to identify what kind of information is being given, and perhaps, to draw a diagram representing their understanding of the problem/task. Once the student gains an understanding of the task, the student can then devise a plan to complete the task. The tasks for this study is considered complete when a solution, or result, is presented, then should be evaluated "for formal reasoning, or by intuitive insight, or both ways" (Polya, 1957, p. 35), which leads to checking for correctness.

The following assignments were given to students, two weeks apart. After the administration of the first assignment there was no discussion with the students about it, or any feedback.

<u>Assessment 1 (Without Scaffolding):</u>

a) The radius of a circle is, let us say, r, and another circle exists around that circle of width, h. The formula $\pi(r+h)^2 - \pi r^2$ describes the difference in areas between the circle with the radius $r + h$ and the circle with the radius r. The change in this difference approaches the circumference of the inner circle as h approaches 0. How would you show the derivative relationship between the area of a circle and its circumference?

b) Is it possible to represent the derivative of the area of the square as the formula for its perimeter? If so, explain how, If not, explain why not.

<u>Assessment 2 (With Scaffolding):</u>

a) The radius of a circle is, let's say, r, and another circle exists around that circle of width, h. How would you represent this?

b) Can you construct a formula to describe the difference in areas between the circle with radius $r+h$ and the circle with the radius r?

c) The rate of change in that difference approaches the circumference of the inner circle as h approaches zero. How would you represent this rate of change?

d) How would you show the rate of change to be a derivative relationship between the area of a circle and its circumference?

e) Is it possible to represent the derivative of the area of the square as the formula for its perimeter? If so, explain how, if not, explain why not.

Students' written responses on Assessments 1 and 2 were counted as 1 (completion) or 0 (no answer) for each of the steps derived from Polya's framework.: understanding the task (U), devising a plan to complete the task (D), executing that plan (E), and completing the task with a result to be checked, which does not necessarily mean correct (C). Table 1 presents those responses in aggregate form.

Table 1.
Student Performance on Assessments 1 and 2.

	U (Understanding)	**D** (Devising a plan)	**E** (Executing the plan)	**C** (Completion)
Assessment 1	8	7	4	0
Assessment 2	6	5	4	4

Analysis of Pilot Study Data

While the students who completed the assessments were all registered in the same Calculus 1 class, not all the students were present for both assessments. There were fewer students who attempted Assessment 2. So, the data cannot be used to compare performance on the two assessments for the individual categories and not for personal performance by students.

When looking across the rows, Assessment 2 had a greater percentage of students who completed the task by including executing and completion. It can be seen in Table 1 that out of 8 students who understood the problem, only 4 of them (50%) attempted to execute a plan and none (0%) completed the task. In row 2, corresponding to Assessment 2, from 6 students who understood the problem, 4 of them (66 %) completed it. The comparison between student performance on Assessments 1 and 2 shows that the drop count towards completion is significantly more for Assessment 1 (8 students understood the problem, but 0 completed it), than that for Assessment 2 (6 students understood the problem and 4 of them completed it). The ratio of completion is 0/8 (0%) for Assessment 1 without scaffolding, compared with 4/6 (66%) for Assessment 2 with scaffolding. Subsequently, this data analysis of the responses does promote integrated scaffolding for the teaching and learning of rate of change problems. Further analysis by the author of student performance showed signs of potential for positive transfer, indicating the need for further study in an action research cycle.

Action Research Cycles

The Action Research Cycle used in this study is similar to the cycle described in Bragg (2017). Bragg's research focused on students' understanding and performance of an assessment task. The type of task chosen to put into variations, annually, is given as "problem pictures", where one picture was analyzed and open-ended questions were posed, in intent to discover mathematical relations. Based on the students' performance of the task, the questions developed, and the analysis made, the students' understanding of the task's concept was determined. Variations were made to the assessment task by addressing identified issues preventing understanding. Amongst the noticed issues, Bragg implements a major variation to the task's instructions/ script to address them. The action research cycles of variations varied three critical features: 1) clarification of the task, 2) more focus on inclusive teaching instruction, and 3) professional seeing (p.128). These features mirror Bragg's methodology referencing Sagor (2014), of which defines seven steps of action research: "selecting a focus, clarifying theories, identifying research questions, collecting data, analysis of data, reporting results, and then taking informed action" (p.125).

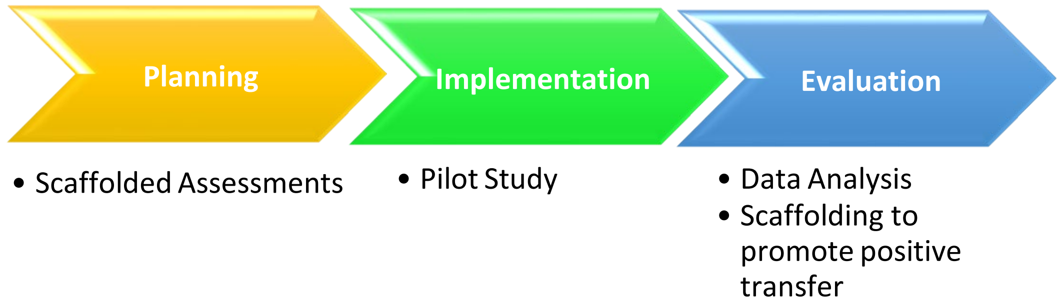
Similarly, the action research process of transfer theory mirrors those features; and this thesis evidently follows that seven-step methodology. Rather than a quantitative experiment, transfer theory promotes qualitative action research methods, given that a control group was not claimed, and given the type of data analysis. The objective of transfer theory would be to improve the quality (or effectiveness) of a task, the quality in which it is assessed, with respect to the quantity of an issue varied throughout that task.

Transfer theory of learning adheres to Bragg's variation theory of learning, given both are implemented as action research cycles with intent to improve the assessments for learning. This thesis previously described types of transfer, and the factors which prevent it, and which promote it. These are to be troubleshooted with more cycles, for qualitative purposes. Troubleshooting processes, or better yet, cycle(s) commonly consist of trial and error, where the outcome supports or negates a hypothesis. It is hypothesized for the previously defined scaffolding methods () to promote positive transferability of mathematical concepts and can be proven by merging the following action research cycle and transfer theory of learning.

Action research is cyclical processes of planning, action/implementation, and evaluation. that can be traced back to Lewin (1946). Below is a timeline of action research cycles, and the processes of troubleshooting scaffolding methods implemented in the instructional methods that support the transfer theory of learning rate of change tasks.

Figure 2

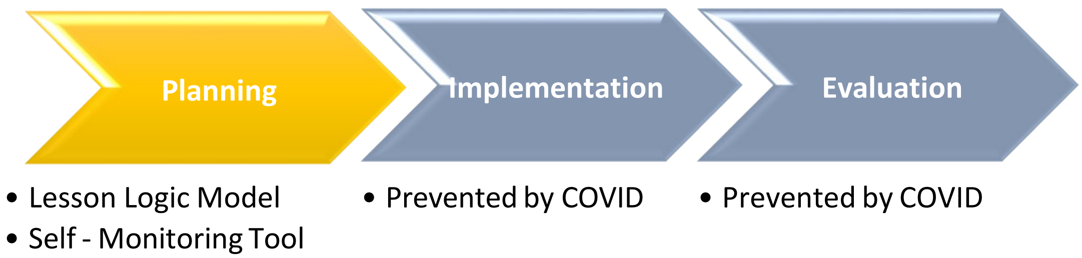
Cycle 1 Action Research Process



As seen in the Figure 2 above, the first action research process started with the subject "mathematical transferability". A great subject, but too broad for a research article. After noticing Calculus students in classes having difficulties with transferring the knowledge of rate of change to similar and sometimes abstract tasks, it was decided to narrow the search towards what is causing this negative transfer of that concept. More specifically, literature on rates of change, transfer of learning, conceptual understanding, and scaffolding/mastery goal instructional methods were consulted. Following the results of the literature review as reported in Section 3.1, the first action research cycle was constructed and conducted. The process began by creating Assessments 1 and 2 for the Pilot Study. These assessments were inspired by Mamolo and Zazkis study they had conducted. Assessment 1 is the original tasks distributed by Mamolo and Zazkis, with Task 1 and Task 2 not scaffolded (Assessment 1, Appendix 1). Given the previously stated literature research and theoretical background, Assessment 2 introduces the same tasks, but Task 1 is scaffolded into multiple sub-tasks. Then Assessments 1 and 2 were then administered by the author to her students during two regular class meetings with them. The results from Assessment 1 and 2 were evaluated and determined a scaffolding approach should continue to be troubleshooted, leading the author into Cycle 2 with the intent to modify Assessment 2 to promote its effectiveness. So, this scaffolding design was implemented again for a Lesson Guide that consists of: i) a scaffolded task for learning, ii) a "transfer" task to assess students' ability to transfer learning, and iii) a self-evaluation tool to gain insight into students' perceptions of their conceptual understanding. Figure 2, below, represents the second cycle of the action research process.

Figure 3

Cycle 2 Action Research Process



The qualitative results from the Pilot Study and the supporting research literature continued to inform the second cycle of the action research process (See Figure 3). For Cycle 2, the author developed an extremely refined scaffolded learning task based on Assessment 2 from the Pilot Study. The Lesson Guide, described in Chapter 4, is expected to gain more qualitative data to support transfer theory of learning. Research often implied self- monitoring skills as a factor towards understanding, towards positive transfer. For this cycle, the author created a self-evaluation tool so the students could monitor their own understanding during the lesson on rate of change and administration of a "transfer" task. This Lesson Guide is to be taught to Calculus students, who are expected to have a "familiar" understanding of the fundamental concepts. Implementation and Evaluation of this cycle could not be conducted because of the COVID-19 pandemic. The lesson could not be taught as intended, and the "transfer" task and the self-assessment tool could not be administered. The addition of the assessment tool and edited scaffold, is hypothesized to increase count of participants, increase count of completion, and increase chances of a positive transfer of the concept of rate of change and the derivative relationship between a shapes area and size.

CHAPTER IV: Lesson Guide

As mentioned in Chapter 3, the curriculum materials developed for this thesis, the Lesson Guide, could not be implemented and tested because of the Covid-19 pandemic.

This chapter describes the Lesson Guide that consists of a scaffolded task for learning task (Task 1), a “transfer” task (Task 2), and the self-assessment tool.

To move to the next level of knowledge, a learner must be able to successfully transfer the just learned concept to similar and abstract contexts. The learning task (Task 1) used in the Lesson Guide is adopted from an experiment performed and analyzed in a study by Mamolo and Zazkis (2012). After reviewing the responses from the study reported by Mamolo and Zazkis (2012), the author concluded that the students lacked the conceptual understanding of the first task (part a) of the original task, (see Appendix B), which hindered them from transferring that knowledge to the next task (part b) of the original task, (see Appendix B). The design of the original task was not sufficient for positive transfer. The intention then, was to alter the task design to better retain the proposed information, by considering instructional methods like those described by Anghileri (2006). This way the learners discover concepts and relations to other similar and abstract content and will successfully transfer that knowledge to similar contexts. As such, Task 1 (based on the original task part (a)) focuses on scaffolding in order to promote positive mathematical transferability of the rate of change concept across various tasks. Since scaffolding generates positive transfer there must be a level of conceptual understanding. This scaffolded Task 1 is expected to have a higher success rate than that of the original task (part a) from the study conducted by Mamolo and Zazski (2012) (see Appendix B) because of how the leading questioning motivates discovery.

The scaffolded form of Task 1 is described in Table 2, which contains the scaffolding steps, expected students' actions for each step, and then the referring theoretical justifications motivating each step.

Table 2.
Task 1. Scaffolded Form.

Scaffolding Steps	**Expected Student Actions**	**Theoretical Justifications**
1. Construct a circle of radius, let us say, r. Add a second circle of radius $r + h$ around that circle with the same center, and there is a width, say h, between the two circles.	Expect to see picture of circles of radius r and $r + h$ with a common center.	P1, P3 Pictures and diagramsare a visual aid that usually help understand the task.
2. For a better visualization, the width, h, between the two circles, can be seen as a ring.	Student should acknowledge. Find familiarity.	P1, P3 Furthering understanding and beginning to designaplan to execute basedon experiences and considering perspectives.
3. The area of this ring is the difference between the area of bigger circle and the area of the smaller circle. The area of a circle is $A = \pi r^2$. The inner circle has radius, r, and the bigger circle has a radius $r + h$. Now, we can construct a formula to represent the	Expect to see $A_{r+h} - A_r =$ $\pi(r+h)^2 - \pi r^2 =$ $2\pi rh + \pi h^2$	P1, P3 Visual representation of the derivative of the area of a circleas a limit.

difference between the two areas which equals the area of the ring.		
4. As this ring becomes thinner and thinner, or we can ask when the width, *h*, gets smaller, what happens to the area? The circumference of the outer circle, and the circumference of the inner circle approach each other. Therefore, as the area changes the circumferences also change, but is dependent on how thick the ring is, which is known as the width, *h*.	Expect to recognize that area becomes smaller, circumferences approach.	P1, P3 Visual representation of the derivative of the area of a circleas a limit and its evaluation.
5. Suppose the formula of a circumference is unknown. But what is known is the formula for the area of the ring, $A = Ch$, then $C = \frac{A}{h}$. Now, we can calculate the area of the ring divided by its width which is approaching zero to give us the circumference, of what was the inner circle but now the only circle.	Expect to write (with them) $\frac{A}{h} = \frac{2\pi rh + \pi h^2}{h} = 2\pi r + \pi h \to 2\pi r$ when $h \to 0$	P1, P3 The evaluation ofthelimit and the comparison with the circumference of a circle.
6. It is shown that the rate of change, or derivative, of the area of a circle is its circumference.	Prompt to verify understanding of derivative as a limit.	P1, P3
7. Allow for questions.	Expect students' questions.	

Note: P1, P3 refer to the guiding principles described in Section 1.4.

During the administration of Task 1, some high technology like GeoGebra 3D graphing applications on tablets, and data collection apps like CODAP, would be extremely helpful to promote conceptual understanding since it provides intense visual aids. Other examples of tools and sources that may be used include: any common items with commonly known societal value, student assessment logs, mostly low technology like figure shaped materials, whiteboards, graphing paper, and patty paper. This task excludes the use of calculators since their use might make measurement of conceptual understanding more difficult.

Task 2, used to assess the students' ability to transfer learning, is administered without scaffolding, abides by principles P1, P2, P3 described in Section 1.4:

> Is it possible to apply this process to a square? That is, considering two squares, one of radius r and the other of radius $r + h$, can this process help determine a derivative relationship between the "circumference" or perimeter of a square and its area? (Given that the "radius" of a square is the distance from the center of the square to a side of the square.)

The forms to be administered to instructors and students, for both Task 1 and Task 2, are found in Appendix A.

The learning task and the "transfer" task, Task 1 and Task 2, respectively, are followed by a self-monitoring tool (see Table 3, below) which enables to assess the conceptual understanding of specific concepts in relation to one's own level of familiarity with those concepts.

Table 3.
Self-Evaluation Tool for Familiarity Level.

Concept/Level of Familiarity	Level 0 **Unfamiliar**	Level 1 **Somewhat Familiar**	Level 2 **Very Familiar**
Rate of Change			
Limits and Derivatives			
Geometry of a Circle			
Geometry of a Square			

It can be seen in Table 3 that the familiarity with mathematical concepts is assessed using a three-level scale:

- Level 0 is associated with unfamiliarity with the concept, meaning that an understanding of a concept has not yet achieved or learned.
- Level 1 is associated with some familiarity with the concept, meaning the concept has been learned and some understanding of it has been attained.
- Level 2 is associated with familiarity with the concept, meaning a definite conceptual understanding.

After claiming one of the levels of familiarity for each concept, one can find the total score n, where $0 \leq n \leq 8$. The average between the minimum score ($n = 0$) and the maximum score ($n = 8$) is 4, and this is considered the score value above which one can claim a level of familiarity that may be associated to conceptual understanding and positive transfer of learning.

Students' performance on the assessment Task 2 can then be analyzed and compared with the total score, *n*. The results may be used by instructors to predict the effectiveness of actor-oriented transfer, where the learner conceptualizes and self-monitors successfully. A student who can rate themselves accurately, can be given new/different goals, depending on the type of transfer achieved. Of course, the instructor may assess the students' familiarity with concepts from the way in which they respond to questions, signifiers, and the way they use the theoretical discourse associated with concept.

CHAPTER V: Discussion and Conclusions

Discussion

Too often, has the question been asked by students "Why do we have to learn mathematics?" This is not exactly a research question, or a question that can be addressed with a simple explanation. More often, the answer to such a question would begin with "because…". The phrase "we have to" seems forced. If learning mathematics is a natural development, why do we feel forced? Why are we dismissing concepts that our brains naturally configure? Perhaps, that is too opinionated. Would it be too vague to say we are/were so focused on finding the answer, as if there is only one, rather than figuring out how to find it or how to master it? Why does this particular subject not seem important, or relational, until later years in college? There could be one answer to all of these. Perhaps, we are not learning mathematics, not naturally, like the way it should be. We are being taught it. How can mathematics be appreciated, or found interesting, when we do not have to do the work for it, when we don't discover it for ourselves? The questions above (and many others) constitute the impetus of my research study.

The primary purpose of this study, therefore, was to examine how scaffolding rate of change problems develop *Calculus I* students' conceptual understanding that promotes positive knowledge transfer and enhance their performance on rate of change problems. It was also to design a *Lesson Guide* consisting of learning tasks using scaffolding and a set of abiding principles informed by the research literature on the theory of transfer, assessment task, and self-evaluation to foster positive transferability of students' knowledge through conceptual understanding. To this end, I constructed two research questions to guide my study: First, I wanted to investigate how scaffolding rate of change problems develop *Calculus I* students' conceptual understanding that promotes positive transfer of knowledge that leads to enhanced students' performance on rate of change problems. Second, I was interested in examining the effects of a researcher developed *Lesson Guide* and *Assessment protocol* consisting of learning tasks using scaffolding and a set of abiding principles (informed by the research literature) in fostering positive transfer of students' knowledge through conceptual understanding. In order to effectively carry out the study, I began with a *pilot study* involving *Calculus I* students to determine the effectiveness of the instructional method of scaffolding on students' achievement on rate of change problems. I administered two different assignments – one with scaffolding and the other without scaffolding to a class of students composed mostly of engineering majors (N = 20), during the Spring 2019 semester. I used the data/information obtained from the pilot study to develop curriculum materials (i.e., a *Lesson Guide* and *Assessment Tasks*) intended to be used in the main study.

Unfortunately, COVID-19 pandemic began (world-wide) immediately following the pilot study and the development of the curriculum materials. This made it challenging to adequately examine my research questions in this study, due to changes in institutional policies regarding instructional delivery formats and other educational gatherings in response to the pandemic, time and resource constraint, and limited access to my research participants. The mathematical tasks developed in this study focused on scaffolding in order to develop conceptual understanding that has the potential to promote positive mathematical transferability of the rate of change concept across various contexts.

As discussed earlier in the study, the concept of rate of change has been documented to be challenging for students (Orton, 1983; Tyne, 2016; White & Mitchelmore, 1996). One way to help learner overcome these challenges and many others in mathematics is to help students develop *conceptual understanding* (i.e., an integrated and functional grasp) of these concepts. As I posited in this study, one way of developing conceptual knowledge is through the use of scaffolding with meaningful mathematical tasks. The extant research literature (e.g., NCTM, 2000, 2014; NRC, 2001) indicates that students with conceptual understanding know more than isolated facts and methods - that is, they understand why a mathematical idea is important and the kinds of contexts in which is it useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know. Conceptual understanding also supports retention, and because facts and methods learned with understanding are connected, they are easier to remember and use, and they can be reconstructed when forgotten (NCTM, 2000, 2014; NRC, 2001). This type of understanding also reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either of them – which are dominant elements in solving problems related to rates of change.

Although it was not the focus of my study, I also recognize (and want to state) that helping students to develop procedural fluency is also an essential aspect of helping students gain a solid mathematical knowledge. Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NCTM, 2000, 2014). Research suggests that the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures (NCTM, 2000, 2014). Thus, although conceptual knowledge is an essential foundation, procedural knowledge is also important in the development of students' mathematical knowledge (NRC, 2001).

Conclusions

It is not evident why learners are convinced mathematics is insignificant, but a possible solution was troubleshooted. The problem is understood to be that learners are not learning, otherwise, transferability would not evidently be at risk. However, it is at risk, it is the transfer theory that is not being implemented, nor is it learned. If we have a goal to learn a particular concept, or complete a task, it takes motivation to do so, then significance can be relational, and a plan to complete the task can be expected. Plans should consider multiple perspectives before executing it. And once a plan is executed, its result can be evaluated with logical reasoning, and understanding can be then be expected as gained. Depending on the type of task, the way in which it is implemented, affects a learners' development and improvement of procedural and conceptual understanding.

This action research of tasks that require low road or high road transfer of learning the rate of change concept, can be promoted through scaffolding instructional methods. The participants of this study had taken more initiative to complete the task, provided evidence of understanding, and the majority of them attempted to transfer their knowledge to a similar concept. So, it is evident: scaffolding instruction induces discovering the significance of a concept, rather than forcing the learner to find significance of a given computation. The scaffolding steps were to be transferred to complete the assessment, similar to the computational steps to be taken when depicting and applying a particular formula (that was given not discovered) to solve for unknown variables. The tasks that were not scaffolded from the pilot study, resulted in learners losing interest in the tasks, and is a perfect example of the computational tasks still being implemented. As long as this continues to be the case, negative transfer will continue to occur and relational understanding will cease, and more learners will be convinced of mathematical application insignificance. While scaffolding may not answer the question “why”, as long as instructional methods continue to be researched, discussed, and troubleshooted for quality improvement, it may not be asked so often.

Limitations

This study is only done within action research cycles. The Covid−19 pandemic prevented a controlled study to be conducted amongst students in the authors′ present time classes. The pilot study was conducted amongst students from one university with only one scaffolded rate of change problem. If this was to be administered to students, it would′ve had to be via online. As stated in the literature and theoretical background, negative transfer has many factors, and the students learning environment can be one. The classroom setting ensures a positive learning environment and is normally present with other learners to collaboratively discourse. A classroom where distractions can be alleviated, and instructors can monitor learners′ skills that can be accessed via online. Yes, these tasks can easily be administered in some online assessment form, and have anonymous participants, but quantitative results are not the purpose, nor the objective of this study

As mentioned in Chapter 3, the materials developed for this thesis could not be implemented and evaluated because of the COVID-19 pandemic. This Lesson Guide implements scaffolding towards mastery goals, specifically the rate of change tasks, in order to promote positive mathematical transferability of the rate of change concept.

Since scaffolding generates positive transfer there must be a level of conceptual understanding. Implementing the scaffolding design also motivates the learner toincrease their skill level and contributes to increasing conceptual understanding.

Further Study

The study continues to another cycle. Depending on the outcome of this lesson guide, its effectiveness will show as either promoting positive transfer or negative. An incomplete task does not necessarily mean that the scaffolding instructional method causes negative transfer. The self-evaluation will help determine the familiarity level which will help the instructor evaluate the students' difficulties of the task, and what needs to be assessed for the next cycle.

While the positive approach on the transfer of learning and the theory on scaffolding support the design of the Lesson Guide, further research should be conducted to account for other methods to achieve conceptual understanding. The most successful way to achieve positive transfer will always be questionable but can be discovered with enough research and implementation of possible instructional methods that are more conceptually oriented rather than procedurally taught. It has been noted, “scaffolding also presupposes that learning is hierarchical and built on firm foundations, while teachers know that elements of understanding can appear in students as an eclectic collection until connections are established” (Anghileri, 2006, p. 50).The researcher would also like to recommend that future researchers design mechanisms to enable them to conduct similar studies with participants within the online learning environment to avoid the interference of events such as the COVID-19 pandemic. Future research on this topic could explore research questions such as: Why is the concept of rate of change not being positively transferred to similar and abstract concepts and contexts? Why are mathematical applications, specifically that of the rate of change, not intriguinglearners, and hindering them from conceptual knowledge? What instructional methods are being used that are causing negative transfer of this concept, and what methods should be implemented to avoid it? Is procedural knowledge of rate of change formulas more likely to be gained after relational knowledge it is positively transferred?

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APPENDIX A

Task 1. *Scaffolded Form for Instructors.*

Scaffolding Steps	**Expected Student Actions**
1. Construct a circle of radius, let us say, r. Add a second circle of radius $r + h$ around that circle with the same center, and there is a width, say h, between the two circles.	Expect to see picture of circles of radius r and $r + h$ with a common center.
2. For a better visualization, the width, h, between the two circles, which could be seen as a ring.	Student should acknowledge, find familiarity and relativity.
3. The area of this ring is the difference between the area of bigger circle and the area of the smaller circle.The area of a circle is $A = \pi r^2$. The inner circle has radius, r, and the bigger circle has a radius $r + h$. Now, we can construct a formula to represent the difference between the two areas which equals the area of the ring.	Expect to see and discover, thedifferential equality. $A_{r+h} - A_r = \pi(r+h)^2 - \pi r^2 = 2\pi rh + \pi h^2$
4. As this ring becomes thinner and thinner, or we can ask when the width, h, gets smaller, what happens to thearea? The circumference of the outer circle, and the circumference of the inner circle approach each other. Therefore, as the area changes the circumferences also changes, but is dependent on how thick the ring is, which is known as the width, h.	Expect to recognize that area becomessmaller circumferences approach.

5. Suppose the formula of a circumference isunknown. But what is known is the formula for the area of thering, $A = Ch$. Then $C = \frac{A}{h}$. Now, we can calculate the area of the ring divided by its width that is approaching zero to give us the circumference, of what was the innercircle but nowthe only circle.	Expect to write (with them) $\frac{A}{h} = \frac{2\pi rh + \pi h^2}{h} =$ $2\pi r + \pi h \rightarrow 2\pi r$ when $h \rightarrow 0$
6. It is shown that the rate of change, or derivative,of the area of a circle is its circumference.	Prompt to verify understanding ofderivative as a limit.
7. Allow for questions.	Expect students' questions leading to logical understanding and discoveringconcepts.

Task 1. *Scaffolded Form for Students.*

Scaffolding Steps	**Students Work**
1. Construct a circle of radius, let us say, r. Add a second circle of radius $r + h$ around that circle with the same center, and there is a width, say h, between the two circles.	
2. For a better visualization, the width, h, between thetwo circles, can be seen as a ring	
3. The area of this ring is the difference between the area of biggercircle and the area of the smaller circle.The area of a circle is $A = \pi r^2$. The inner circle has radius, r, and the bigger circle has aradius $r + h$. Now, we can construct a formula to represent the difference between the two areas which equals the areaof the ring.	
4. As this ring becomes thinner and thinner, or we can ask whenthe width, h, gets smaller, what happens to the area? The circumference of the outercircle, and the circumference of the inner circle approach each other. Therefore, as the area changesthe circumferences also changes, but is dependent on how thick the ring is, which is known as the width, h.	

Task 2. *Pilot Study and Original Task 1B (Appendix B).*

Is it possible to apply this process to a square? That is, considering two squares: one of radius r, and the other, of radius $r + h$, show how this process can help determine a derivative relationship between the "circumference", or perimeter, of a square and its area? (Given that the "radius" of a square isthe perpendicular distance from the center of the square to a side of the square.)

Task 2. *Expected Student Actions.*

Students are expected to show similar actions, steps, taken in the previous task. When a picture is drawn, it is possible students will mislabel measurements and segments. May be confused by the radius term, posed to be a signifier, an affordance to help the transfer of the steps taken to depict a visual aid. It is very likely students will execute a plan and find a solution, but they should check for correctness, try to detect their error, and execute another plan and check that solution. After administration, perhaps the instructor finds benefit from mediating collaborative discourse. Or perhaps, another cycle of the same tasks, should be conducted.

APPENDIX B

Original Tasks from the study by Mamolo and Zazkis (2012):

During a calculus class, one student noticed that when working with the circle, the derivative of the area formula yields the formula for circumference. That is, $\frac{dA}{dr} = \frac{d(\pi r^2)}{dr} = 2\pi r$. The student asked why this relationship held for the circle, and not in other cases such as with the square.

1A. Use the diagram (below) to show why the derivative of the area of a circle yields the formula for the circumference.

1B. Is it possible to represent the derivative of the area of a square as the formula for its perimeter? If so, explain how. If not, explain why not.